Modelling of saturated reactor compensator for system studies

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Abstract: A method of modelling multi-limbed saturated reactors under dynamic and steady-state conditions, allowing for both the electrical interconnection of the windings and the core construction, is presented. A selection of results from a digital computer program based on the simulation are given, illustrating the dynamic response of the reactor model to parameter changes under system energisation. The principle of harmonic series compensators in saturated reactors is described, and the harmonic analysis of the model's steady-state current waveforms for various loading is discussed.

1 Introduction

There is an increasing demand for reactive power control in order to increase the power capability of transmission networks and to suppress the rapid voltage fluctuations caused by some loads such as arc furnaces.

One effective method of VAR and system voltage control is the inclusion on the system of saturated reactors. Essentially, these are a fixed voltage device consisting of a multi-limbed core working in magnetic saturation, having interconnected windings to give internal harmonic compensation. The inherent slope reactance can be compensated by series capacitors, and, in combination with a shunt capacitor bank, VAR flow control can be achieved over a wide range. A typical voltage/current characteristic for a saturated reactor compensator is shown in Fig. 1 and has been described in various references [1-4].

![Fig. 1 V/I characteristic of static reactive compensator](image)

This paper presents a method of modelling the multi-limbed saturated reactor under transient and steady-state conditions, allowing for both the electrical interconnections of the winding and the core construction, and including the non-linear magnetic characteristics. Computer results are presented showing the reactor model's response under different operating conditions.

2 Operational principles of reactor

Saturated reactors take advantage of the saturation characteristic of iron cores to achieve large changes in current with small changes in terminal voltage. However, due to the non-linear magnetic characteristic, harmonics are generated which can be reduced to an extremely low level by internal compensation. This was exploited particularly by Friendlander [5] with his treble-triple reactor. The primary winding arrangement of the reactor as shown in Fig. 2 consists of a combination of plain star and ±20° phase shifting zig-zag windings in series, embracing three groups of 3-phase saturated core limbs. For a symmetrical 3-phase supply, this configuration, in conjunction with secondary harmonic compensation, virtually eliminates harmonics up to the 17th.

![Fig. 2 Primary winding arrangement of the treble-tripler reactor](image)

The primary winding arrangement of a treble-triple reactor may be expressed in terms of the magnetomotive force (MMF) equation;

$$[F]_{1-9} = [N][I]_{RTB}$$

where

$$[N] = \begin{bmatrix}
K_1 & 0 & -K_2 \\
-1 & 0 & 0 \\
K_1 & -K_2 & 0 \\
-K_2 & K_1 & 0 \\
0 & -1 & 0 \\
0 & K_1 & -K_2 \\
0 & -K_2 & K_1 \\
0 & 0 & -1 \\
-K_2 & 0 & K_1
\end{bmatrix}$$

1, K_1 and K_2 are the p.u. turns on windings as indicated in Fig. 2.

For the 9-limb reactor the resultant MMF in each limb is equal in magnitude and displaced by 360°/9 = 40°, as shown in Fig. 3A. From Fig. 3A, the phasor diagram of
limb 1 can be redrawn, as shown in Fig. 3B, and $K_1$ and $K_2$ calculated as

$$K_1 = \frac{\sin 40^\circ}{\sin 120^\circ} = 0.742, \quad \text{and} \quad K_2 = \frac{\sin 20^\circ}{\sin 120^\circ} = 0.395$$

![Fig. 3A Phasor diagram of the fundamental limb currents](image)

![Fig. 3B Ampere-turn diagram for limb 1 in p.u. $|I_1| = |I_a|$](image)

Neglecting losses, the fundamental primary voltages in each limb ($V_1, \ldots, V_9$) can be considered in quadrature with the primary currents ($I_1, \ldots, I_9$). Taking into account the phase shift of these limb voltages, the line voltages can be drawn as shown in Fig. 4. From the analysis it can be shown that several harmonic components are zero, as given in Table 1, where 1 p.u. voltage represents the voltage across the 1 p.u. turn winding.

### Table 1: Harmonic components

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase voltage (p.u.)</td>
<td>3</td>
<td>1.3</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>1.3</td>
<td>3</td>
</tr>
</tbody>
</table>

With an isolated star point the triple harmonics are eliminated in the line voltages, the 17th harmonic being present as expected.

### 3 Reactor model

A realistic representation of the saturated reactor, where the core passes through conditions of severe overfluxing, is important, and a model based on the magnetic equivalent circuit was therefore used. A magnetic equivalent circuit for the core may be postulated and one leakage path included for each winding. Such a circuit is shown in Fig. 5 for a treble-tripler reactor, where $g_1, g_2, \ldots, g_9$ are the inverse reluctances of the nine limbs and $g_{10}, \ldots, g_{17}$ the corresponding values for the yokes, the remaining branches being the leakage paths. In general, $g = Ba/\ell$, where $a$ and $\ell$ are the effective area and length of the magnetic branch, respectively.

![Fig. 5 Magnetic circuit](image)

Following Cherry [6], the magnetic circuit may be replaced by its dual, the dual forming the basis of the electrical equivalent circuit of Fig. 6, the parameters being the differential inductances $L$, where $L = (N^2 dB/dH) (a/l)$ and $N$ represents the chosen number of base turns. In addition to relating the actual winding turns to the base turns, the ideal transformers ensure that the constraints imposed by the magnetic circuits are not invalidated by the constraints introduced by the interconnection of the windings. The inductance terms are variables for the iron paths, the non-linearity being defined by the $B/H$ curve for the core material.

![Fig. 6 Electrical equivalent circuit](image)

Core losses may be divided into hysteresis and eddy current loss components, although, for most applications, the former is small and may be neglected. It should be included when residual flux conditions are important, and, with low-loss steel, a simple two-slope boundary representation based on the major $B/H$ loop is considered adequate.
Eddy current losses under high saturation conditions are important and are simulated by a shunt resistance across the windings. Whereas the inductance parameters are readily evaluated from available data, this is not true of the parameters representing eddy current losses, especially under conditions of high and variable saturation. The present approach is an empirical one using a constant value of loss resistance derived from available test data.

4 Reactor study

To simulate the performance of the reactor under transient and steady-state conditions a simplified system was studied which included a voltage source inductance, slope correction capacitors and the treble-tripler reactor.

For this study a simple, single value relationship between \( B \) and \( H \) was used of the form

\[
B = d_1 H + d_2 \tanh d_3 H
\]

where \( d_1, d_2 \) and \( d_3 \) are constants determined for the particular core steel. Using core dimensions and winding for a particular reactor unit, the relationship between \( \lambda \) and \( i \) for a branch \( n \) in the magnetic equivalent circuit becomes

\[
\lambda_n = K_{n1} i_n + K_{n2} \tanh K_{n3} i_n
\]

where

\[
K_{n1} = \frac{N a_n d_1}{l_n}, \quad K_{n2} = N a_n d_2 \quad \text{and} \quad K_{n3} = \frac{N}{l_n} d_3,
\]

\( a \) and \( l \) are the cross-sectional area and the length of the magnetic path, respectively).

A set of equations can be derived from this, expressing the phase currents in terms of the source voltages, taking into account the isolated neutral point of the star connected windings and the magnetic constraints introduced by the reactor as derived in Appendixes. The magnetic branch currents determined from the phase currents are used to set the differential inductances at each step in the numerical integration.

5 Results

Based on the mathematical model presented, a computer program was developed and results obtained indicating the current response to changes in terminal conditions and core characteristics under energisation and steady-state conditions.

System data

Base values = 100MVA (total), inst. voltage = \((33/\sqrt{3})\) \(1.15\) kV, time = 0.01 s.

Source voltage level = 1.0 p.u.

Source resistance = 0.011 p.u.

Point on wave switching angle = 0° on red phase

Sequence of switching: simultaneous

Magnetisation characteristic of limbs and yokes based on the \( B/H \) magnetic curve of Alphasil: 35M6

Core loss series resistance for reactor = 0.5 p.u.

Slope correction capacitance = 103 p.u.

The above parameters were used unless otherwise indicated in the Figure.

Fig. 7a shows the current waveforms at the reactor terminals following energisation, illustrating the dynamic characteristic associated with switching magnetic circuits operating in high saturation. Initially the currents are highly distorted due to the unbalanced fluxes and the mutual coupling of the core limbs, but damp out to steady state in three cycles demonstrating the inherent fast response of the reactor.

The results due to the change in the value of the slope correction capacitor are shown in Fig. 7b. The initial currents on all phases are small but rapidly attain comparatively high steady-state values attributable to the change in the effective reactance of the reactor.

Reducing the core loss resistance results in a reduction in the damping and, consequently, a slower response, as indicated in Fig. 7c.

To assess the accuracy of the model under steady-state conditions with regard to harmonic content of the current waveform, results were computed for various voltage levels, thus achieving a wide variation of loading. The \( V/i \) characteristic is shown in Fig. 8 indicating that, above the

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knee part, small changes in voltage produce large changes in reactive current, offering the useful advantage in limiting dynamic overvoltages because of the inherent and fast nature of its response time overload capacity. The respective waveforms and harmonic content of current for different voltage levels are shown in Figs. 9 and 10.

![Figure 9](image_url)  
**Fig. 9** Steady-state current waveforms with source voltage at 1.2 p.u.  
Harmonic 3 5 7 9 11 13 15 17 19 21 23  
percentage of fundamental current 0 1 0.7 0.1 0 0 0 6 1.6 0 0

![Figure 10](image_url)  
**Fig. 10** Steady-state current waveforms with source voltage at 1.5 p.u.  
Harmonic 3 5 7 9 11 13 15 17 19 21 23  
percentage of fundamental current 0.4 0.5 0.6 1 0 0 0 2.4 2.1 0 0

6 Conclusions

The currents on a treble-tripler saturated reactor following switching operations are of a complex nature, and their prediction requires a realistic model based on the magnetic equivalent circuit. This paper presents a method for simulating the reactor under wide-ranging conditions, and allows for the electrical constraints imposed by the interconnection of windings and the non-linear magnetic constraints imposed by the core construction.

The transient current waveforms demonstrate the typical characteristics associated with switching magnetic circuits, and the results of the harmonic analysis of the steady-state waveforms for various loads indicate that the model provides a reasonable representation of the reactor in both steady and transient regions. To improve the accuracy of representation it is necessary to acquire accurate numerical data for the reactor, especially loss data under highly saturated conditions.

Further work is now being undertaken to study the performance of the reactor when faults occur on the transmission system.

7 References


8 Appendix

The constraint equations for the reactor may be derived from the electrical equivalent circuit shown in Fig. 6.

\[
\begin{bmatrix}
L_1 + L_{10} & -L_{10} \\
-L_{10} & L_2 + L_{10} + L_{11} \\
-L_{11} & L_3 + L_{11} + L_{12} \\
-L_{12} & L_4 + L_{12} + L_{13} \\
-L_{13} & L_5 + L_{13} + L_{14} \\
-L_{14} & L_6 + L_{14} + L_{15} \\
L_7 + L_{15} + L_{16} & -L_{16} \\
-L_{16} & L_8 + L_{16} + L_{17} \\
-L_{17} & L_9 + L_{17}
\end{bmatrix}
\]

\[
p = \begin{bmatrix}
i_{10} \\
i_1 \\
i_{11} \\
i_2 \\
i_{12} \\
i_3 \\
i_{13} \\
i_4 \\
i_{14} \\
i_5 \\
i_6 \\
i_{15} \\
i_7 \\
i_{16} \\
i_8 \\
i_{17} \\
i_9 \\
i_{18}
\end{bmatrix}
\]

\[
\begin{bmatrix}
L_1 \\
L_2 \\
L_3 \\
L_4 \\
L_5 \\
L_6 \\
L_7 \\
L_8 \\
L_9
\end{bmatrix} = \begin{bmatrix}
i_{10} \\
L_1 \\
L_2 \\
L_3 \\
L_4 \\
L_5 \\
L_6 \\
L_7 \\
L_8 \\
L_9
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
i_6 \\
i_7 \\
i_8 \\
i_9 \\
i_{10}
\end{bmatrix}
\]

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or

\[ [L_2] p[I_2] - [L_p] p[I_p] = 0 \]

The limb voltages are given by

\[
\begin{align*}
V_1 &= L_{11} p(i_1 - i_{10}) \\
V_2 &= L_{12} p(i_2 - i_{11}) \\
V_3 &= L_{13} p(i_3 - i_{12}) \\
V_4 &= L_{14} p(i_4 - i_{13}) \\
V_5 &= L_{15} p(i_5 - i_{14}) \\
V_6 &= L_{16} p(i_6 - i_{15}) \\
V_7 &= L_{17} p(i_7 - i_{16}) \\
V_8 &= L_{18} p(i_8 - i_{17}) \\
V_9 &= L_{19} p(i_9 - i_{18})
\end{align*}
\]

or

\[ [V_p] = [C][V_p] \]  

Similarly,

\[ [I_p] = [C]^T[I_p] \]

From eqns. 2, 4, 6 and 7, an expression can be derived for the phase voltages in terms of the phase currents:

\[
[V_p] = [C]\{[L_1] + [L_p][L_p]^{-1}[L_p]\}[C]^T[p[I_p]]
\]

\[ [V_p] = [L] p[I_p] \]

At each step in the numerical integration, eqn. 8 is solved to give increments in phase currents, and the increments in the magnetic branch currents are determined from eqns. 7 and 2.

\[
\begin{align*}
V_k &= \frac{N_1}{N} - \frac{N_2}{N} & - \frac{N_3}{N} & - \frac{N_4}{N} \\
V_T &= - \frac{N_3}{N} & \frac{N_1}{N} & - \frac{N_2}{N} & \frac{N_4}{N} & - \frac{N_3}{N} & - \frac{N_4}{N} \\
V_g &= - \frac{N_3}{N} & - \frac{N_3}{N} & \frac{N_1}{N} & - \frac{N_2}{N} & \frac{N_4}{N} & - \frac{N_3}{N}
\end{align*}
\]

\[
\begin{align*}
V_1 &\  \\
V_2 &\  \\
V_3 &\  \\
V_4 &\  \\
V_5 &\  \\
V_6 &\  \\
V_7 &\  \\
V_8 &\  \\
V_9 &\  \\
\end{align*}
\]