General Theory of Space Vector Modulation for Five-Phase Inverters

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Abstract—Multiphase motor drives are a very promising technology, especially for medium and high power ranges.

As known, a multiphase motor drive cannot be analyzed using the space vector representation in a single d-q plane, but it is necessary to introduce multiple d-q planes. So far a general space vector modulation for multiphase inverters is not available due to the inherent difficulty of synthesizing more than one independent space vector simultaneously in different d-q planes.

In this paper the problem of the space vector modulation of five-phase inverters is completely solved extending the theory of space vector modulation used for traditional three-phase voltage source inverters and introducing the concept of reciprocal vector. This approach leads to the definition of a very flexible modulation strategy that allows the full exploitation of the dc input voltage and the simultaneous modulation of voltage space vectors in different d-q planes. The validity of the proposed modulation theory is confirmed by experimental tests.

I. INTRODUCTION

The basic justification for multiphase drives comes from the simple idea that variable-speed drives are invariably supplied from power electronic converters and the number of phases does not have to be equal to three any more and it can be considered a design variable.

Initially, multiphase motor drive were investigated since they represent a possible solution in high and medium power applications, but, afterwards, it has been recognized that they feature several distinguishing properties. Reduction of the amplitude and increase of the frequency of torque pulsations, reduction of the stator current per phase, and increase of the fault tolerance [1] are only some of the advantages of multiphase motors over the traditional three-phase motor drives.

Furthermore, multiphase motor drives offer a greater number of degrees of freedom compared with three-phase motor drives, which can be utilized to improve the drive performance [2]. An interesting possibility is the independent control of the low order spatial harmonic components of the magnetic field in the air gap of the machine. If the harmonic components of order greater than one are set to zero, the torque pulsation can be strongly reduced. On the other hand, if all the spatial harmonics are synchronized, the torque production capability of the machine can be increased [3], [4].

Another possibility is related to the so-called multi-motor drives. A well-defined number of multiphase machines, having series-connected stator windings with an opportune permutation of the phases, can be independently controlled with a single multiphase inverter [5]-[6].

It is worth noting that, whereas traditional three-phase motors are analyzed representing the main motor quantities as space vectors on a single d-q plane, for multiphase electromagnetic system this representation is insufficient and it is necessary to introduce other d-q planes (multiple space vectors) [7].

The exploitation of the potential of a M-phase motor is possible only if the modulation strategy for the M-phase Voltage Source Inverters (VSIs) can generate the output voltages, whatever the reference voltage space vectors are.

Two different methods are usually adopted, i.e. Space Vector Modulation (SVM) [8]-[16], and carrier-based Pulse Width Modulation (PWM) [6], [17]-[20]. For three-phase VSIs the two methods have been proved to be equivalent, and they can be interchangeably implemented. To the contrary, in the case of multiphase VSIs the carrier-based PWM method seems to be the most effective approach. This is due to the inherent difficulty of synthesizing more than one independent space vector, in different d-q planes simultaneously.

Nevertheless, there are several reasons to develop a SVM technique. The main reason is that SVM is well-known for three-phase inverters and it has been integrated in a number of different solutions also in the logic devices that manage the turn-on and turn-off of the inverter switches, such as FPGAs and CPLDs. For reasons related to the technical experience or just for economic convenience, a company could find preferable to update the available SVM algorithms for three-phase inverters rather than to completely renounce to its previous know-how.

Despite several attempts, so far a general SVM technique for M-phase VSIs has not yet been proposed. Even for $M = 5$, a fully satisfactory solution is not available [9]-[12], [14]. The SVM techniques proposed in [9], [10], [13] require the second voltage space vector to be always zero. The SVM technique defined in [11] allows the modulation of the first and the second voltage space vectors, but only with rigorous constraints between their arguments and frequencies. Finally, the SVM techniques presented in [12], [14] and [15] can independently control the two voltage space vectors, but do not fully utilize the dc input voltage, leading to reduced output voltage capability.

In this paper the problem of the SVM of 5-phase inverters is solved representing the inverter voltages by means of complex variables [21]-[23]. This approach allows to generalize the SVM used for three-phase inverters and to adapt it also for five-phase inverters, thus obtaining a very flexible modulation strategy that allows the full exploitation of the dc input voltage,
and the simultaneous modulation of voltage space vectors in different d-q planes.

II. REVIEW OF SVM FOR THREE-PHASE INVERTERS

Before illustrating the SVM for multiphase inverters, it is convenient to present a short review of the SVM for traditional three-phase inverters, in order to emphasize the common underlying principles.

A. Space Vectors

The basic scheme of a three-phase inverter is shown in Fig. 1. The signals \( s_1, s_2, s_3 \) are switch commands of the three inverter branches, and can assume only the values 0 or 1. The inverter output pole voltages are

\[
\begin{align*}
v_{1p} &= s_1 E_{DC} \\
v_{2p} &= s_2 E_{DC} \\
v_{3p} &= s_3 E_{DC}.
\end{align*}
\]

where \( E_{DC} \) is the dc-link voltage.

The main problem is to control the load phase voltages \( v_{1N}, v_{2N}, \) and \( v_{3N} \) according to the requirements imposed by the application, e.g., vector control of ac machines.

An elegant solution to this problem is the space vector representation of the load voltages, which describes the underlying principles. Three-phase inverters, in order to emphasize the common principles.

The best choice is given by the two vectors delimiting the sector in which the reference voltage vector lies. Since two consecutive vectors differ only for the state of one switch, this choice allows ordering the active and the zero vectors so as to minimize the number of switch commutations in a switching period. For example, if the desired voltage vector lies in sector 1, as shown in Fig. 2, the two adjacent voltage vectors are \( v_1 \) and \( v_2 \), whose configurations \((0,0,1)\) and \((0,1,1)\) differ for only one bit.

After the active vectors have been chosen, the requested voltage can be expressed as a combination of them as follows:

\[
\overline{v}_{ref} = \delta_1 \overline{v}_1 + \delta_2 \overline{v}_2
\]

where \( \delta_1 \) and \( \delta_2 \) are the duty-cycles of \( \overline{v}_1 \) and \( \overline{v}_2 \) in the switching period.

The space vector and the zero sequence component can be expressed as functions of \( s_1, s_2, \) and \( s_3 \) by substituting (1) in (2) and (3).

\[
\begin{align*}
\overline{v}_{pole} &= \frac{2}{3} \left(v_{1p} \overline{y}_1 + v_{2p} \overline{y}_2 + v_{3p} \overline{y}_3\right) \\
\overline{v}_{pole,0} &= \frac{1}{3} \left(v_{1p} + v_{2p} + v_{3p}\right)
\end{align*}
\]

where the coefficients \( \overline{y}_k \) \((k=1,2,3)\) are defined as follows:

\[
\overline{y}_k = e^{(k-1)\frac{2\pi}{3}}\quad (k=1,2,3)
\]

The space vector and the zero sequence component can be expressed as functions of \( s_1, s_2, \) and \( s_3 \) as follows:

\[
\begin{align*}
\overline{v}_{pole} &= \frac{2}{3} E_{DC} \left(s_1 \overline{y}_1 + s_2 \overline{y}_2 + s_3 \overline{y}_3\right) \\
\overline{v}_{pole,0} &= \frac{1}{3} \left(s_1 + s_2 + s_3\right) E_{DC}
\end{align*}
\]

It is well-known that the load voltages depend only on the space vector of the pole voltages, whereas the inverter zero sequence component affects only the potential of the load neutral point. In other words, to control the load, it is sufficient to control the vector \( \overline{v}_{pole} \).

B. SVM for Three-Phase Inverters

The modulation problem consists in controlling the switch states such that the mean values of the inverter output voltages are equal to the desired values in any switching period \( T_p \).

There are eight (namely \( 2^3 \)) possible configurations for a three-phase inverter, depending on the states of the three switch commands \( s_1, s_2, \) and \( s_3 \). Six configurations correspond to voltage vectors with non-null magnitudes. These vectors, usually referred to as active vectors, are represented in Fig. 2, where the configurations of each vector are also expressed in the form \((s_1,s_2,s_3)\). Two configurations, i.e. \((s_1,s_2,s_3)=(0,0,0)\) and \((s_1,s_2,s_3)=(1,1,1)\), lead to voltage vectors with null magnitudes, usually referred to as zero vectors.

The space vector modulation selects two active vectors and applies each of them to the load for a certain fraction of the switching period. Finally, the switching period is completed by applying the zero vectors.

The active vectors and their duty-cycles are determined so that the mean value of the output voltage vector in the switching period is equal to the desired voltage vector.

The best choice is given by the two vectors delimiting the sector in which the reference voltage vector lies. Since two consecutive vectors differ only for the state of one switch, this choice allows ordering the active and the zero vectors so as to minimize the number of switch commutations in a switching period. For example, if the desired voltage vector lies in sector 1, as shown in Fig. 2, the two adjacent voltage vectors are \( v_1 \) and \( v_2 \), whose configurations \((0,0,1)\) and \((0,1,1)\) differ for only one bit.

After the active vectors have been chosen, the requested voltage can be expressed as a combination of them as follows:

\[
\overline{v}_{ref} = \delta_1 \overline{v}_1 + \delta_2 \overline{v}_2
\]

where \( \delta_1 \) and \( \delta_2 \) are the duty-cycles of \( \overline{v}_1 \) and \( \overline{v}_2 \) in the switching period.

Fig. 1. Schematic of a PWM-VSI connected to a three-phase load.

Fig. 2 - Voltage vectors used in SVM technique, represented in d-q reference frame.
The explicit expressions of $\delta_1$ and $\delta_2$ can be easily calculated evaluating the following dot products:

$$\delta_1 = \frac{v_{\text{ref}} \cdot w^{(1)}}{v_{\text{ref}} \cdot v_{\text{ref}}}$$

$$\delta_2 = \frac{v_{\text{ref}} \cdot w^{(2)}}{v_{\text{ref}} \cdot v_{\text{ref}}}$$

where

$$w^{(1)} = \frac{jv_0}{v_1} = \frac{1}{E_{\text{dc}}} (\gamma_1 - \gamma_2)$$

$$w^{(2)} = \frac{jv_1}{v_1} = \frac{1}{E_{\text{dc}}} (\gamma_2 - \gamma_1)$$

Once $\delta_1$ and $\delta_2$ have been calculated, the designer can still choose in which proportion the two zero vectors are used to fill the switching period.

Fig. 3 shows the vector sequence corresponding to the example of Fig. 2. In the sequence of Fig. 3, the zero vectors are equally distributed in the switching period.

### C. Identification of the Sector

The determination of the sector $S$ of the reference vector, with modern floating-point DSP or high-frequency fixed-point DSP, is very simple because it is sufficient to calculate the argument $\theta$ of $v_{\text{ref}}$ using inverse trigonometric functions. Assuming $\theta$ in the range $0^\circ \leq \theta < 360^\circ$, the sector number is given by

$$S = \text{int}\left(\frac{\theta}{60^\circ}\right) + 1$$

where the function $\text{int}(\cdot)$ provides the integer part of the argument.

A second method to determine the sector of the reference vector is explained hereafter. This method can be used also for low-cost fixed point DSP, due to the fact that it does not require the evaluation of any inverse trigonometric function, but only the calculation of dot products.

The main idea is that each sector can be represented univocally as the intersection of three half-planes. For example, Fig. 4 shows that Sector 1 is the intersection of the three half planes highlighted in grey.

To check if the reference voltage vector lies in a certain sector, it is sufficient to verify that it belongs to the three half planes whose intersection is the given sector. This result is clearly depicted in Fig. 5, where the correspondence between sector numbers and sector codes is shown. Furthermore, Fig. 5 shows also the three vectors $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$ defined by (14).

After the calculation of $L_k$ with (13), the sector of $\vec{v}_{\text{ref}}$ can be identified by using Table I, that relates the sector codes $(L_3, L_2, L_1)$ to the sector numbers. The entries of Table I have been ordered so that the sector code, interpreted as a binary number and converted to its decimal representation, can be used as address for identifying the table entry of the sector number.

### III. FIVE-PHASE INVERTERS

Fig. 6 represents a schematic drawing of a five-phase VSI supplying a star connected balanced load.

The study of three-phase systems, in steady-state and transient operating conditions, takes advantage of the space vector representation. This powerful tool can be usefully extended to the analysis of five-phase systems [20].

![Fig. 4 - Representation of Sector 1 as intersection of three half-planes.](image)

![Fig. 5 - Relationship between sector numbers and sector codes for three-phase inverters, and representation of the voltage vectors $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$ in the d-q stationary reference frame.](image)

**TABLE I - SECTOR NUMBER AS A FUNCTION OF THE SECTOR BINARY CODE**

<table>
<thead>
<tr>
<th>Sector</th>
<th>$L_3, L_2, L_1$ (decimal)</th>
<th>$L_3, L_2, L_1$ (binary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3</td>
<td>001 010 011</td>
</tr>
<tr>
<td>2</td>
<td>4 3 6</td>
<td>100 101 110</td>
</tr>
<tr>
<td>3</td>
<td>6 1 5</td>
<td></td>
</tr>
</tbody>
</table>
For a given set of five real variables $y_1, ..., y_5$, a new set of variables $x_0, \bar{x}_i, \bar{x}_i$ can be obtained by means of the following symmetrical linear transformations:

$$x_0 = \frac{1}{5} (y_1 + y_2 + y_3 + y_4 + y_5)$$

$$\bar{x}_i = \frac{2}{5} (y_i \bar{a}_i + y_{i+1} \bar{a}_{i+1} + y_{i+2} \bar{a}_{i+2} + y_{i+3} \bar{a}_{i+3} + y_{i+4} \bar{a}_{i+4})$$

$$\bar{x}_i = \frac{2}{5} (y_i \bar{a}_i + y_{i+1} \bar{a}_{i+1} + y_{i+2} \bar{a}_{i+2} + y_{i+3} \bar{a}_{i+3} + y_{i+4} \bar{a}_{i+4})$$

where

$$\bar{a}_k = e^{j \frac{2\pi}{5} (k-1)} \quad (k=1,2,...,5).$$

Similarly to the case of three-phase inverters, the quantity $x_0$ defined by (15) is usually called zero-sequence component, whereas the variables $\bar{x}_i$ and $\bar{x}_i$ are usually referred to as multiple space vectors. According to (15) - (17), a general five-phase system can be represented by two space vectors and the zero-sequence component. Since $\bar{x}_i$ and $\bar{x}_i$ are independent variables, $\bar{x}_i$ is assumed to be a vector moving in the plane $d_1-q_1$ and $\bar{x}_i$ a vector moving in the plane $d_3-q_3$.

The transformations (15)-(17) can be applied to the main quantities of a multiphase inverter (output voltages and output currents), thus defining the multiple voltage vectors $\bar{v}_i$ and $\bar{v}_i$, and the multiple output current vectors $\bar{i}_i$ and $\bar{i}_i$.

IV. SVM FOR MULTIPHASE INVERTERS

Likewise the three-phase case, the SVM for multiphase inverters should determine a sequence of switch configurations and their duty-cycles, which is able to approximate the desired output voltages in a switching period.

For the definition of a SVM strategy for multiphase inverter, it could appear obvious to proceed in a similar way, but this cannot be done without solving some difficult problems.

The basic steps of the SVM for three-phase inverters are as follows: a) determination of the sector of the desired voltage vector, b) selection of the nearest voltage vectors, c) calculation of the duty-cycles.

First of all, the concepts of "sector" and "nearest voltage vectors" are not univocal in multiphase inverters. In fact, the figures that represent the admissible voltage vectors in the planes $d_1-q_1$ and $d_3-q_3$, shown in Fig. 7, are much more complex than the simple hexagon of Fig. 2. In addition it is not possible to decouple the modulation of a voltage vector in the plane $d_1-q_1$ from the modulation of a voltage vector in the plane $d_3-q_3$.

Due to these difficulties, up to now, the problem of SVM has been solved only for specific cases, i.e. when the magnitude of the voltage vector in the plane $d_1-q_1$ is zero, or when it is acceptable that the dc-link voltage is not fully exploited.

A. Multidimensional Space Vectors and Multidimensional Sectors

In order to find a modulation technique that is able to synthesize the voltage space vectors of all the d-q planes simultaneously, the analysis carried out on bi-dimensional planes should be abandoned in favor of a multidimensional point of view, that has been found to some extent in a recent paper [14], [15]. According to this new approach, the output voltages of the inverter are represented by a multidimensional vector $\bar{V}$, which is defined as follows:

$$\bar{V} = \begin{pmatrix} \bar{v}_1 \\ \bar{v}_s \end{pmatrix}$$

It is worth noting that $\bar{V}$ is a vector in a 4-dimensional space.
space, since both \( \vec{v}_1 \) and \( \vec{v}_3 \) have two scalar components.

As an immediate consequence, the space can be divided in sectors likewise the three-phase case, but the concept of sector used for three-phase inverters must be replaced by the concept of multidimensional sector. Furthermore, whereas a three-phase inverter has 6 sectors, i.e. \( 3! \) sectors, a five-phase inverter has 120 multidimensional sectors, i.e. \( 5! \) sectors.

The vectors constituting each multidimensional sector can be obtained by means of a combination of four adjacent multidimensional vectors (i.e. differing one another only in the state of one inverter leg), as follows:

\[
\vec{v} = \delta_1 \vec{v}^{(1)} + \ldots + \delta_4 \vec{v}^{(4)}
\]

where \( \delta_1, \delta_2, \delta_3 \) and \( \delta_4 \) are positive duty-cycles.

Equation (20) represents a set of four scalar equations, and this is clear if (20) is rewritten in terms of multiple space vectors instead of multidimensional vectors, as follows:

\[
\left( \begin{array}{c}
\vec{v}_1 \\
\vec{v}_3 \\
\vec{v}_2 \\
\vec{v}_4
\end{array} \right) = \delta_1 \left( \begin{array}{c}
\vec{v}^{(1)}_1 \\
\vec{v}^{(1)}_3 \\
\vec{v}^{(1)}_2 \\
\vec{v}^{(1)}_4
\end{array} \right) + \ldots + \delta_4 \left( \begin{array}{c}
\vec{v}^{(4)}_1 \\
\vec{v}^{(4)}_3 \\
\vec{v}^{(4)}_2 \\
\vec{v}^{(4)}_4
\end{array} \right)
\]

The first advantage of this approach over the traditional one is that all the multidimensional sectors have the same shape, thus introducing a symmetry property that cannot be found analyzing the problem on multiple d-q planes.

B. Calculation of the Duty-Cycles

In literature the duty-cycles for the SVM of a 5-phase inverter are generally calculated solving the set of linear equations obtained from (21). Although this approach is theoretically correct, it is not optimal from a computational point of view, because the coefficients of the linear equations depend on the voltage sector and they have always to be recomputed.

A more elegant solution can be proposed introducing the concept of reciprocal vectors [24].

Given the multidimensional vectors \( \vec{v}^{(1)}, \ldots, \vec{v}^{(4)} \), by definition, the reciprocal vector \( \vec{w}^{(k)} \) satisfies the following constraints:

\[
\vec{w}^{(k)} \cdot \vec{v}^{(k)} = 1 \\
\vec{w}^{(k)} \cdot \vec{v}^{(h)} = 0 
\]

(22)

(23)

where the dot product is calculated by summing the result of the dot product between the components in the plane \( d_1-q_1 \) and the result of the dot product between the components in the plane \( d_3-q_3 \).

Equation (22) and the three equations in (23) form a set of four linear equations where the unknowns are the four scalar components of \( \vec{w}^{(k)} \). If the vectors \( \vec{v}^{(1)}, \ldots, \vec{v}^{(4)} \) are linearly-independent, this set of equations has one and only one solution.

This procedure can be repeated four times, for \( k=1, \ldots, 4 \), thus leading to four reciprocal vectors \( \vec{w}^{(1)}, \ldots, \vec{w}^{(4)} \).

The usefulness of the reciprocal vectors is evident in the calculation of the duty-cycles in (20). In fact, supposing that \( \vec{w}^{(1)}, \ldots, \vec{w}^{(4)} \) are the reciprocal vectors of \( \vec{v}^{(1)}, \ldots, \vec{v}^{(4)} \), each duty-cycle can be calculated simply with a dot product, similarly to the three-phase case, as follows:

\[
\delta_1 = \vec{w}_{refk} \cdot \vec{w}^{(k)} \quad (k=1, \ldots, 4)
\]

(24)

where \( \vec{w}_{refk} \) is the desired multidimensional vector.

The main advantage of reciprocal vectors is that they can be calculated off-line and stored in look-up tables depending on the multidimensional sector, thus reducing the computation time. For this purpose, it is possible to demonstrate by applying (22)-(23) that the reciprocal vectors necessary for SVM can be expressed in the following form, which is analogous to that of (10) and (11):

\[
\vec{w}_{kj} = \frac{1}{E_{dc}} \left( \frac{\alpha_i - \overline{\alpha}_j}{\overline{\alpha}_i - \overline{\alpha}_j} \right), \quad (i \neq j \text{ and } i,j=1, \ldots, 5).
\]

(25)

From (25) it follows that there are 20 different vectors \( \vec{w}_{kj} \), but it is sufficient to store in look-up tables only ten of them, because swapping the subscripts reverses the vector, as follows:

\[
\vec{w}_{kj} = -\vec{w}_{jk}.
\]

Table II shows a possible look-up table for the storage of the ten reciprocal vectors.

Like the four active multidimensional vectors \( \vec{v}^{(1)}, \ldots, \vec{v}^{(4)} \), the selection of reciprocal vectors depends on the sector in which the desired voltage vector is placed. Therefore, in the next section, it will be shown how this sector can be identified.

C. Determination of the Multidimensional Sector

To apply the SVM, it is necessary to identify the sector in which the reference multidimensional voltage vector is placed. The solution of this task is not as immediate as in the three-phase case, because the orientation of the multidimensional sectors in the space cannot be traced back to a simple principle.

The solution to this problem can be found by extending the method of space partitioning presented for three-phase VSI.

It can be demonstrated that a multidimensional sector for 5-phase inverter is the intersection of 10 half spaces at most.

The logic functions that express the belonging of the reference multidimensional vector to a half space can be written as follows:

<p>| TABLE II - LOOK UP TABLE OF THE RECIPROCAL VECTORS |
|-----------------------|-------|-------|-------|-------|</p>
<table>
<thead>
<tr>
<th>Subscript /</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subscript</td>
<td>( \vec{w}_{r1} )</td>
<td>( \vec{w}_{r2} )</td>
<td>( \vec{w}_{r3} )</td>
<td>( \vec{w}_{r4} )</td>
</tr>
<tr>
<td>1</td>
<td>( \vec{w}_{s1} )</td>
<td>( \vec{w}_{s2} )</td>
<td>( \vec{w}_{s3} )</td>
<td>( \vec{w}_{s4} )</td>
</tr>
<tr>
<td>2</td>
<td>( \vec{w}_{s2} )</td>
<td>( \vec{w}_{s3} )</td>
<td>( \vec{w}_{s4} )</td>
<td>( \vec{w}_{s5} )</td>
</tr>
<tr>
<td>3</td>
<td>( \vec{w}_{s3} )</td>
<td>( \vec{w}_{s4} )</td>
<td>( \vec{w}_{s5} )</td>
<td>( \vec{w}_{s6} )</td>
</tr>
<tr>
<td>4</td>
<td>( \vec{w}_{s4} )</td>
<td>( \vec{w}_{s5} )</td>
<td>( \vec{w}_{s6} )</td>
<td>( \vec{w}_{s7} )</td>
</tr>
</tbody>
</table>
where \( \mathbf{w}_i \) are the same vectors introduced in (25).

Once the logic functions \( L_{ij} \) have been calculated, it is possible to compare them with the sector codes and to identify the sector of the reference multidimensional vector.

It is worth noting that this process could be time-expensive, since it could require up to 120 comparisons. In order to improve the computation efficiency, it is convenient to treat the sector codes as binary numbers, to sort them in ascending or descending order and to apply a binary search algorithm.

D. Look-up Table for SVM

The look-up table for the implementation of SVM is shown in Table III. The column entitled "Sector code" reports the sector codes calculated with the algorithm proposed in the previous section. The sector codes are in the form \((L_{12},L_{13},L_{14},L_{15},L_{23},L_{24},L_{25},L_{34},L_{35},L_{45})\) and have been converted to decimal numbers. It is worth noting that the list of configurations is ordered, since two consecutive configurations differ only for the state of one bit.

Finally, the columns \( R_1, \ldots, R_4 \) show the reciprocal vectors that must be used for the calculation of the duty-cycles. The values of these entries, which refer to Table II, vary from \(-10\) to \(+10\). A negative number means that the subscripts of the values of these entries, which refer to Table II, vary from 10 to 120 configurations is ordered, since two consecutive configurations differ only for the state of one bit.

V. EXPERIMENTAL RESULTS

To verify the effectiveness of the proposed modulation strategy some experimental tests have been carried out.

The experimental setup consists of a five-phase voltage source inverter feeding a five-phase symmetrical series-connected R-L load. The load parameters in nominal condition are 11.5 Ω and around 11 mH. The dc bus voltage is around 100 V. The control algorithm is implemented in a DSP TMS320F2812 and an Altera FPGA Cyclone EP1C6. The switching period is 100 μs, the binary search algorithm about 12 μs.

The total computational time of the SVM algorithm is lower than 20μs. The calculation of the logic functions requires about 5 μs, the binary search algorithm about 12 μs and the calculation of the duty-cycles about 2 μs.

Fig. 8 shows the waveforms of four load currents when a voltage reference vector of 50 V is rotating at \( 2\pi \cdot 50 \) rad/s in plane \( d_1-q_1 \), whereas the voltage reference vector in plane \( d_1-d_3 \).

Table III - Look-up Table for SVM of 5-phase Inverters

<table>
<thead>
<tr>
<th>Id</th>
<th>Sector Code</th>
<th>Sector Number</th>
<th>( C_1,C_2,C_3,C_4 )</th>
<th>( R_1,R_2,R_3,R_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>45</td>
<td>( 16,24,28,30 )</td>
<td>(-10,8,5,1)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>56</td>
<td>( 16,24,28,29 )</td>
<td>(-10,8,3,1)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>25</td>
<td>( 16,24,28,20 )</td>
<td>(-10,3,5,1)</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>26</td>
<td>( 16,27,29,25 )</td>
<td>(-4,3,8,5)</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>27</td>
<td>( 16,28,25,29 )</td>
<td>(-4,10,8,5)</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>96</td>
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is zero. As can be seen, the waveforms are nearly sinusoidal, except for the current ripple due to the switching process.

Fig. 9 shows the behavior of the 5-phase inverter in the same operating condition. In particular tracks 2, 3 and 4 show the sector number, the entry address in Tab. III, and the index $R_1$ of the reciprocal vector used for the calculation of the first duty-cycle. It is evident that the sector number assumes ten different values in a period. However its waveform is not regular even in this simple case, because it often jumps from a value to another value that is not adjacent.

Finally, Fig. 10 shows the waveforms of the duty-cycles $\delta_1$, $\delta_2$, $\delta_3$, $\delta_4$ and $\delta_5$ for the same case of Fig 8 and 9. It is interesting to note that the waveforms of $\delta_1$ and $\delta_2$ or $\delta_3$ and $\delta_4$ are nearly opposite.

Afterwards, some tests have been carried out to verify the capability of the proposed SVM to generate simultaneously multiple voltage vectors in the planes $d_1-q_1$ and $d_3-q_3$.

Fig. 11 shows the behavior of the inverter when a voltage reference vector of 30 V is rotating at $2\pi \cdot 50$ rad/s in plane $d_1-q_1$ and a voltage reference vector of 30 V is rotating at $2\pi \cdot 150$ rad/s in plane $d_3-q_3$.

This choice for the reference voltages may appear obscure. However, it can be verified that the third time harmonic of the supply voltages can be utilized, in multi-phase machines with concentrated windings, for the excitation of the third spatial harmonic of the mmf, thus yielding an average torque component that enhances the torque production. This property is utilized in multi-phase ac machines for improving the torque density [3]-[4].

The first trace of Fig. 11 shows the current waveform, which
is evidently distorted by the presence of the third harmonic. The other traces show the behavior of the sector number, the entry address in Table III, and the index R1 of the first reciprocal vector.

Finally, Fig. 12 shows the waveform of the duty-cycles. The comparison between Fig. 12 and Fig. 10 highlights that $\delta_1$ and $\delta_3$ are now very similar.

VI. CONCLUSION

In this paper, a general solution to the problem of space vector modulation for a five-phase inverter has been presented.

The new concepts of multidimensional space vector and reciprocal vector have been introduced. By means of them it is possible to consider the well-known three-phase space vector modulation as a particular case of the proposed approach.

The feasibility of the SVM algorithm for five-phase motors is confirmed by experimental tests.

REFERENCES