

Static Synchronous Generators for Distributed Generation and Renewable Energy

Qing-Chang Zhong and George Weiss

Abstract—In this paper, the idea of operating an inverter as a synchronous generator is developed after establishing a model for synchronous generators to cover all dynamics without any assumptions on the signals. The inverters which are operated in this way are called static synchronous generators (SSG). This means that the well-established theory/device for synchronous generators can still be used for inverters, which will dominate the power generation in the future because of the increasing share of distributed generation sources and renewable energy sources utilised. The power of an SSG can be regulated using the well-known frequency and voltage drooping mechanism. SSGs can also be easily operated in grid-connected mode or island mode. Simulation results are given to verify the idea.

Index Terms—Distributed generation, renewable energy, DC-AC converters, frequency drooping, voltage drooping, load sharing, static synchronous generators, virtual synchronous generators, parallel inverters, inverters-dominated power system

I. INTRODUCTION

The current paradigm in the control of wind or solar power generators is to extract the maximum power from the power source and inject it all into the power grid; see for example [1], [2]. This is a good policy as long as such power sources constitute a negligible part of the grid power capacity. Indeed, any random power fluctuation of the renewable power generators will be compensated by the controllers associated with the large conventional generators, and some of these generators will also take care of overall system stability

This work was initiated when G. Weiss was visiting Q.-C. Zhong in August 2008.

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and fault ride-through. When renewable power generators (especially the solar ones) will provide the majority of the grid power, such “irresponsible” behaviour will become untenable. Thus, the need will arise to operate them in the same way as conventional power generators function today; see also [3], [4], [5]. This requires first of all large and high efficiency energy storage units, so that the random fluctuations of the prime power source can be filtered out. In solar power generators, and also in variable-speed synchronous generators based wind turbines, the power is present in DC form and enters the grid via DC/AC power converters (also called inverters).

In this paper, we propose that these inverters can be operated to mimic the behaviour of a synchronous generator (SG). They are called static synchronous generator (SSG). In this way, the conventional control algorithms and equipment that have been developed for SG driven by prime movers (and which have reached high level of maturity over 100 years) can be applied to inverters.

While we were working on this paper, we discovered the conference paper [6], in which a short-term energy storage is added (outside of the inverter) to provide virtual inertia to the system so that a generator can behave like a virtual synchronous generator during short time intervals and contribute to the stabilisation of grid frequency. In this paper, we are concerned about the inverter itself, which behaves like a synchronous generator all the time, not just during short-time intervals. The whole system may include an energy storage subsystem, but this is irrelevant.

The rest of the paper is organised as follows. In Section II, a dynamic model of synchronous generators is established under no assumptions on the signals. Although the model of an SG has been well described in the literature, the way the model described here is somewhat fresh. Then,

a static synchronous generator is implemented in Section III, followed by its operation in Section IV. Simulation results are given in Section V and conclusions are made in Section VI.

II. MODELLING SYNCHRONOUS GENERATORS

The model of a synchronous generator can be found in many sources, such as [7], [8], [9], [10]. Most of these make various assumptions, such as steady state and/or balanced sinusoidal voltages/currents, to simplify the analysis. Here, we briefly outline a model that is a dynamic system without any assumptions on the signals, from the perspective of system analysis and controller design for a round rotor machine (without damping windings), with p pairs of poles per phase, with no saturation effects in the iron core.

A. The electrical part

For details on the geometry of the windings, we refer to [9], [8]. The field and the three identical stator windings are distributed in slots around the periphery of the uniform air gap. The stator windings can be regarded as concentrated coils having self-inductance L and mutual inductance $-M$ ($M > 0$ with a typical value $\frac{1}{2}L$, the negative sign is due to the $\frac{2\pi}{3}$ phase angle), as shown in Figure 1. The field/rotor winding can be regarded as a concentrated coil having self-inductance L_f . The mutual inductance between the field coil and each of the three stator coils varies with respect to the (electrical) rotor angle θ as follows:

$$\begin{aligned} M_{af} &= M_f \cos(\theta), \\ M_{bf} &= M_f \cos\left(\theta - \frac{2\pi}{3}\right), \\ M_{cf} &= M_f \cos\left(\theta - \frac{4\pi}{3}\right). \end{aligned}$$

The flux linkages of the windings are

$$\begin{aligned} \Phi_a &= Li_a - Mi_b - Mi_c + M_{af}i_f, \\ \Phi_b &= -Mi_a + Li_b - Mi_c + M_{bf}i_f, \\ \Phi_c &= -Mi_a - Mi_b + Li_c + M_{cf}i_f, \\ \Phi_f &= M_{af}i_a + M_{bf}i_b + M_{cf}i_c + L_f i_f. \end{aligned}$$

Denote

$$\Phi = \begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \end{bmatrix}, \quad i = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

and

$$\widetilde{\cos\theta} = \begin{bmatrix} \cos\theta \\ \cos\left(\theta - \frac{2\pi}{3}\right) \\ \cos\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}, \quad \widetilde{\sin\theta} = \begin{bmatrix} \sin\theta \\ \sin\left(\theta - \frac{2\pi}{3}\right) \\ \sin\left(\theta - \frac{4\pi}{3}\right) \end{bmatrix}.$$

Since

$$i_a + i_b + i_c = 0,$$

(the neutral line is not connected), the stator flux linkages can be rewritten as

$$\Phi = L_s i + M_f i_f \widetilde{\cos\theta}, \quad (1)$$

where $L_s = L + M$, and the field flux linkage can be rewritten as

$$\Phi_f = L_f i_f + M_f \langle i, \widetilde{\cos\theta} \rangle, \quad (2)$$

where $\langle \cdot, \cdot \rangle$ denotes the conventional inner product. The second term $M_f \langle i, \widetilde{\cos\theta} \rangle$ is constant if the three phase currents are sinusoidal and balanced. Assume that the resistance of the stator windings is R_s , then the phase terminal voltages $v = [v_a \ v_b \ v_c]^T$ can be obtained from (1) as

$$v = -R_s i - \frac{d\Phi}{dt} = -R_s i - L_s \frac{di}{dt} + e, \quad (3)$$

where $e = [e_a \ e_b \ e_c]^T$ is the back emf due to the rotor movement given by

$$e = M_f i_f \dot{\theta} \widetilde{\sin\theta} - M_f \frac{di_f}{dt} \widetilde{\cos\theta}. \quad (4)$$

We mention that, from (2), the field terminal voltage is

$$v_f = R_f i_f + \frac{d\Phi_f}{dt}, \quad (5)$$

where R_f is the resistance of the rotor winding. However, we shall not need the expression for v_f because we shall use i_f , instead of v_f , as an adjustable constant input. This completes modelling the electrical part of the machine.

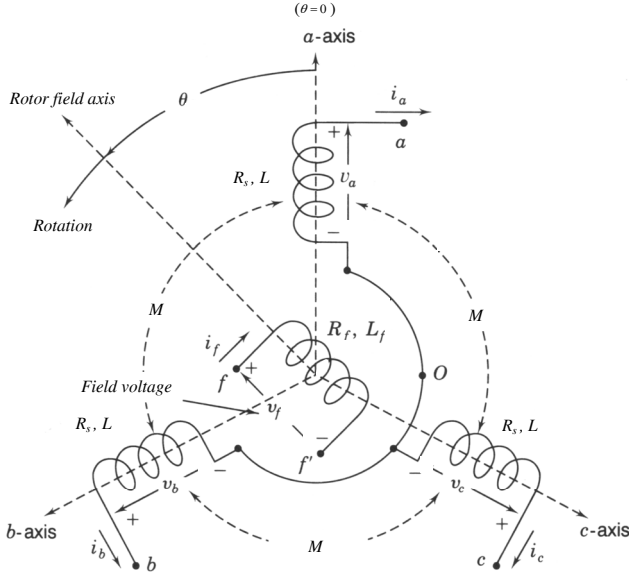


Figure 1. Structure of idealised three-phase round-rotor synchronous generators with $p = 1$, modified from [7, Figure 3.4].

B. The mechanical part

The mechanical part of the SG is governed by

$$J\ddot{\theta} = T_m - T_e + D_p\dot{\theta}, \quad (6)$$

where J is the moment of inertia of all parts rotating with the rotor, T_m is the mechanical torque, T_e is the electromagnetic torque and D_p is a damping factor. T_e can be found from the total energy E stored in the machine, which is the sum of the magnetic energy stored in the stator and rotor magnetic fields and the kinetic energy stored in the rotating parts, *i.e.*,

$$\begin{aligned} E &= \frac{1}{2} \langle i, \Phi \rangle + \frac{1}{2} i_f \Phi_f + \frac{1}{2} J \dot{\theta}^2 \\ &= \frac{1}{2} \langle i, L_s i \rangle + M_f i_f \langle i, \widetilde{\cos\theta} \rangle + \frac{1}{2} L_f i_f^2 + \frac{1}{2} J \dot{\theta}^2. \end{aligned}$$

Since the mechanical rotor position θ_m satisfies $\theta = p\theta_m$, we have

$$\begin{aligned} T_e &= -\frac{\partial E}{\partial \theta_m} = -p \frac{\partial E}{\partial \theta} \\ &= -p M_f i_f \left\langle i, \frac{\partial \widetilde{\cos\theta}}{\partial \theta} \right\rangle \\ &= p M_f i_f \left\langle i, \widetilde{\sin\theta} \right\rangle. \end{aligned} \quad (7)$$

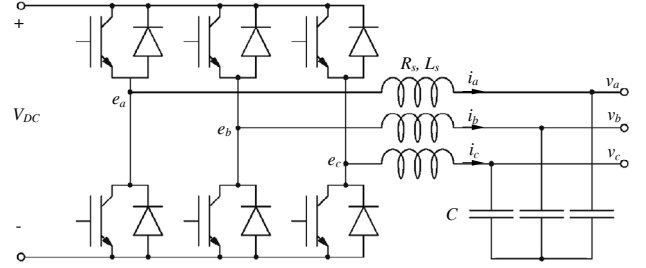


Figure 2. A simple DC-AC converter that mimics a synchronous generator

III. IMPLEMENTATION OF A STATIC SYNCHRONOUS GENERATOR

In this section, the details of how to implement an inverter as an SSG will be described. A simple DC/AC converter (inverter) used to convert the DC power supply V_{DC} obtained from renewable/distributed energy sources into three-phase AC (v_a, v_b, v_c) is shown in Figure 2. It consists of three phase legs and a three-phase LC filter, which is used to suppress the switching noise. If a neutral line is needed, then the strategy proposed in [11], [12] to provide a neutral line without affecting the control of the conventional three phase legs can be used. The filtering capacitors C should be chosen such that the resonant frequency $\frac{1}{\sqrt{L_s C}}$ is approximately $\sqrt{\omega_n \omega_s}$, where ω_n is the nominal angular frequency of the voltage and ω_s is the angular switching frequency used to turn on/off the switches (IGBTs are shown in the figure but other power semiconductors can be used as well).

An SSG can be implemented according to the mathematical model developed in the previous section. As explained in details later in this subsection, an SSG consists of a hardware part, *i.e.*, the inverter shown in Figure 2, and a software part shown in Figure 3. These two parts are interfaced via the signals e and i (and v to be used for regulating the terminal voltages).

A. The software part

It is advantageous to assume that the field (rotor) winding is fed by an adjustable DC current source i_f instead of a voltage source v_f . In this case, the terminal voltage v_f varies, but this is

irrelevant. As long as i_f is constant, the generated voltage from (4) is

$$e = \dot{\theta} M_f i_f \widetilde{\sin\theta}. \quad (8)$$

Define the generated real power and reactive power as

$$P = \langle i, e \rangle, \quad Q = \langle i, e_q \rangle,$$

where e_q has the same amplitude as e but with a phase delayed from that of e by $\frac{\pi}{2}$, *i.e.*,

$$e_q = \dot{\theta} M_f i_f \widetilde{\sin(\theta - \frac{\pi}{2})} = -\dot{\theta} M_f i_f \widetilde{\cos\theta}.$$

Then, the real power is

$$P = \dot{\theta} M_f i_f \langle i, \widetilde{\sin\theta} \rangle$$

and the reactive power is

$$Q = -\dot{\theta} M_f i_f \langle i, \widetilde{\cos\theta} \rangle. \quad (9)$$

Note that if $i = i_0 \widetilde{\sin\varphi}$ (as would be the case in sinusoidal steady state), then

$$P = \dot{\theta} M_f i_f \langle i, \widetilde{\sin\theta} \rangle = \frac{3}{2} \dot{\theta} M_f i_f i_0 \cos(\theta - \varphi),$$

$$Q = -\dot{\theta} M_f i_f \langle i, \widetilde{\cos\theta} \rangle = \frac{3}{2} \dot{\theta} M_f i_f i_0 \sin(\theta - \varphi).$$

These coincide with the conventional definitions for real power and reactive power (note that the coefficient 3 is because there are three phases). When the voltage and current are in phase, *i.e.*, $\theta - \varphi = 0$, the product of the rms values of the voltage and current gives the real power P . When the voltage and current are $\frac{\pi}{2}$ out of phase, this product gives reactive power Q . Moreover, inductors absorb reactive power with a positive Q (since $\theta - \varphi = \frac{\pi}{2}$, *i.e.*, the voltage leads the current by $\frac{\pi}{2}$) while capacitors generate reactive power with a negative Q (since $\theta - \varphi = -\frac{\pi}{2}$, *i.e.*, the voltage lags the current by $\frac{\pi}{2}$). The above two formulae about P and Q are very important when regulating the active and reactive power of a SG. However, it seems that the formula for reactive power is not well documented in the literature and the reactive power is not regarded as an important part of the model.

The equation (6), together with (7), (8) and (9), can be easily implemented in the software part of an SSG shown in Figure 3. Thus, the state variables of the SSG are i (which has only two independent components and is an actual current), θ and $\dot{\theta}$ (which are a virtual angle and a virtual angular speed). In order to avoid numerical overflow, the integrator that produces θ can be reset when $\theta = 2\pi$. The control inputs of the SSG are T_m and $M_f i_f$. In order to operate the SSG in a useful way, we need a controller that generates the signals T_m and $M_f i_f$ such that system stability is maintained and the desired values of active and reactive power are followed. The significance of Q will be seen in the next section.

B. The hardware part

The terminal voltages $v = [v_a \ v_b \ v_c]^T$ given in (3) can be obtained from the output terminals of the DC-AC converter shown in Figure 2. The inductance L_s and resistance R_s of the inductor can be chosen to represent the stator impedance of a synchronous generator. The switches in the inverter are operated so that the average values of e_a , e_b and e_c over a switching period should be equal to e given in (8) and, hence, no special pulse-width-modulation (PWM) techniques, *e.g.* the space vector PWM, are necessary. Any sinusoidal PWM technique will do the job as the three legs can be treated phase by phase.

IV. OPERATION OF AN SSG

A. Frequency drooping and regulation of active power

For synchronous generators, the rotor speed is maintained by the prime mover and it is known that the damping factor D_p is due to mechanical friction etc. An important mechanism of SG to share load evenly is to droop frequency according to the active power it delivers. When the active power demanding increases, the speed of the prime mover drops. The speed regulation system of the prime mover will then increase the mechanical power, *e.g.* widening the throttle valve of an engine, so that a new balance is achieved. Here,

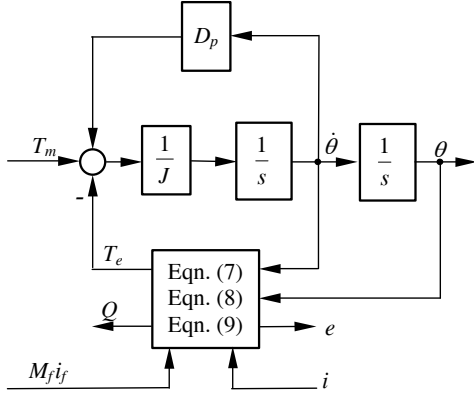


Figure 3. The software part of an SSG

this mechanism can be easily implemented by comparing the virtual angular speed $\dot{\theta}$ with the nominal angular speed $\dot{\theta}_n$ before feeding it into the damping block D_p ; see the upper part of Figure 4. As a result, the damping factor D_p actually behaves as the frequency drooping coefficient, which is defined as the ratio of the required change of torque ΔT to the change of speed (frequency) $\Delta \dot{\theta}$. That is,

$$D_p = \frac{\Delta T}{\Delta \dot{\theta}} = \frac{\Delta T}{T_{mn}} \frac{\dot{\theta}_n}{\Delta \dot{\theta}} \frac{T_{mn}}{\dot{\theta}_n},$$

where T_{mn} is the nominal mechanical torque. Note that in many literature, *e.g.* [3], D_p is defined as $\frac{\Delta \dot{\theta}}{\Delta T}$. Because there is no delay involved in the frequency loop, the time constant τ_f of the frequency loop can be made much smaller than a physical synchronous generator. When $i = 0$, *i.e.*, when there is no load connected, τ_f is given by

$$\tau_f = J/D_p.$$

When the inverter is loaded ($T_e \neq 0$), the time constant τ_f increases. However it is difficult to calculate τ_f because of the nonlinearity in (7). The mechanical torque T_m can be obtained from the set point of active power P_{set} after dividing the nominal mechanical speed $\frac{\dot{\theta}_n}{p}$. This completes the feedback loop for active power; see the upper part of Figure 4. Because of the built-in frequency drooping mechanism, this inverter automatically shares the active load with other inverters of the same type connected on the same bus.

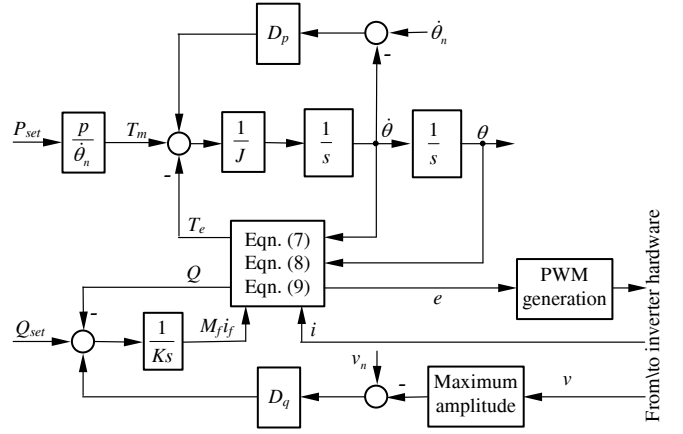


Figure 4. SSG: Complete scheme to regulate active and reactive power

B. Voltage drooping and regulation of reactive power

The regulation of reactive power Q flowing out of the SSG can be realised similarly. Define the voltage drooping coefficient D_q as the ratio of the required change of reactive power ΔQ to the change of voltage Δv , *i.e.*,

$$D_q = \frac{\Delta Q}{\Delta v} = \frac{\Delta Q}{Q_n} \frac{v_n}{\Delta v} \frac{Q_n}{v_n},$$

where Q_n is the nominal reactive power, which can be chosen as the nominal power, and v_n is the nominal amplitude of terminal voltage v . Again note that in many literature, *e.g.* [3], D_q is defined as $\frac{\Delta v}{\Delta Q}$. The feedback loop for the reactive power can be realised as shown in the lower part of Figure 4. The difference between the amplitude v_n of the nominal voltage and that of the terminal voltage v is amplified with the coefficient D_q before adding to the difference between the set point Q_{set} and the current reactive power Q . The resulting signal is then fed into an integrator with a gain $\frac{1}{K}$ to generate $M_f i_f$. It is important to note that there is no need to measure reactive power as this is available internally.

V. SIMULATION RESULTS

The idea described above has been verified with simulations. The parameters of the inverter for carrying out the simulations are given in Table I.

Table I
PARAMETERS OF THE INVERTER-INFINITE BUS SYSTEM

Parameters	Values	Parameters	Values
L_s	0.15 mH	L_q	0.0534 mH
R_s	0.045 Ω	R_q	0.06 Ω
C	22 μ F	Frequency	50 Hz
R (parallel to C)	1000 Ω	Voltage (line-line)	17 Vrms
Rated power	100 W	Initial grid phase	0 $^\circ$
Inertia J	0.01 Kg m^2	K	13580

The frequency drooping coefficient is chosen as $D_p = 0.2432$ so that the frequency drops 0.5% when the torque (power) increases 100%. The inverter is connected to the grid via a circuit breaker and a step-up transformer. In this case, $D_q = 0$. The SSG was connected to the grid at $t = 1$ second. The real power $P = 80$ W was applied at $t = 2$ second and the reactive power $Q = 60$ Var was applied at $t = 3.5$ second.

The output power of the SSG is shown in Figure 5. During the synchronisation period, there were some oscillations in the power (which is inside the controller as the breaker is not yet turned on and hence it does not cause any problem). Before the SSG was requested to deliver power (*i.e.*, before $t = 2$ seconds), the active power and reactive power were zero. Then, the active power delivered to the grid gradually increased to the set point 80 W. During this transient process, the SSG initially took reactive power from the grid but returned to normal. At $t = 3.5$ second, the reactive power delivered by the SSG increased to the setpoint 60 Var gradually. During this period, the active power increased slightly but then returned to the set point 80W very quickly.

VI. CONCLUSIONS

In this paper, the idea of operating an inverter as a synchronous generator has been developed after establishing a model for synchronous generators to cover all the dynamics without any assumptions to the signals. This model can be used to investigate the stability of power systems, in particular, those dominated by parallel converters in distributed generation. It has been shown that it is very easy to control the power delivered to the grid. Also, because of the built-in frequency and voltage drooping mechanism, these SSGs are able to share load automatically.

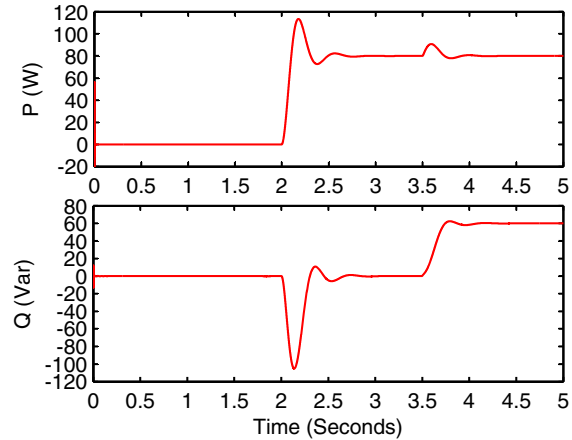


Figure 5. Response of active and reactive power

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