15th ESICUP Meeting
Zoetermeer, The Netherlands, May 23-25, 2018
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Organised by:

ESICUP – EURO Special Interest Group on Cutting and Packing
ORTEC

Supported by:

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Welcome

Dear Friends,

Welcome to the 15th Meeting of ESICUP - The EURO Special Interest Group on Cutting and Packing. Since its formal recognition as a EURO Working Group in 2003, ESICUP has run a series of annual meetings which have successfully brought together researchers and practitioners in the field of cutting and packing. Previous meetings have been organized in Wittenberg (Germany), Southampton (United Kingdom), Porto (Portugal), Tokyo (Japan), L'Aquila (Italy), Valencia (Spain), Buenos Aires (Argentina), Copenhagen (Denmark), La Laguna (Spain) and Lille (France), Beijing (China), Portsmouth (United Kingdom), Ibiza (Spain), Liège (Belgium) and this 15th meeting is now held in Zoetermeer (Netherlands).

Once again, this meeting will serve as an instrument for the development of research and the dissemination of knowledge in our field. Twenty eight papers have been accepted for presentation, allowing for clear insights into the current state-of-the-art of cutting and packing and preparing the ground for fruitful discussions.

We are delighted to be holding the meeting at the offices of ORTEC. ORTEC is one of the world’s leaders in optimization software and analytics solutions. Turning complex challenges into easy-to-use solutions. ORTEC was founded on April 1st, 1981 by five innovative thinkers. These Econometrics students at Erasmus University Rotterdam believed the mathematical theories and algorithms they worked on could be practically applied to significantly improve business performance. Two of those founders, Aart van Beuzekom and Gerrit Timmer, still work side by side today as CEO and CFO. Their global team includes many PhDs and professors in Econometrics and Applied Mathematics – all devoted to innovation and the success of every customer – providing world class technology and consulting services.

Zoetermeer is a city in the western Netherlands. A small village until the late 1960s, it had 6,392 inhabitants in 1950. By 2013 this had grown to 123,328, making it the third largest population centre in the province of South Holland, after Rotterdam and The Hague. The name Zoetermeer (Dutch for "freshwater lake") refers to the former lake north of the town (reclaimed in 1614). Because the name literally translates as "sweet lake" local residents have dubbed Zoetermeer "Sweet Lake City".

We wish all of you a successful conference and a very pleasant stay in Zoetermeer!

Julia Bennell
University of Southampton
Program Chair

Joaquim Gromicho
ORTEC
Local Organizer Chair

Zoetermeer, The Netherlands, May 23–25, 2018
Information for Conference Participants

MEETING VENUE
The 15th ESICUP Meeting will be held at the headquarters of ORTEC B.V. in Zoetermeer, The Netherlands. The institute is located close to the main train station (Zoetermeer).

Address:
ORTEC B.V., Houtsingel 5, 2719 EA Zoetermeer, The Netherlands

REGISTRATION
The registration desk will be located in the meeting venue where you will collect your name badge and registration pack for the event. Registration will be open from 8.30am to 9.30am, May 24, 2018 and during session breaks.

YOUR NAME BADGE
You should wear your name badge at all times during the event. It is your admission to the venue (includes coffee breaks and lunch).

NOTES ON PRESENTATION
- **Equipment**
  The conference room is equipped with an overhead projector and a laptop computer will be provided. We suggest that you bring your own computer and/or transparencies as a backup.

- **Length of Presentation**
  20 minutes for each talk, including discussion. Please note that we are running on a very tight schedule. Therefore, it is essential that you limit your presentation to the time which has been assigned to you. Session chairpersons are asked to ensure that speakers observe the time limits.

INTERNET ACCESS
Further details on how to access wireless network at the conference venue will be given at registration.

DIETARY, MOBILITY AND OTHER REQUIREMENTS
Please let the registration desk know if you have any additional special requirements.
## Program Overview

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<td>Get-together (optional)</td>
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**Note:**
- Session 5 (5 talks) on May 24
- Session 6 (4 talks) on May 25

Zoetermeer, The Netherlands, May 23–25, 2018
### Scientific Program Schedule

**Thursday, May 24th**

#### 9:00 – 9:15

**Opening Session**

Welcome Address

#### 9:15 – 10:20

**Session 1**  
*Chair: Jelke J. van Hoorn*

| 1.1 | Challenge ROADEF / EURO 2018 Cutting Optimization  
Lydia Tlilane*, Quentin Viaud* |
| 1.2 | Real life benchmarks for load building  
Jelke J. van Hoorn*, Lina Rahali*, Ties Westendorp* |
| 1.3 | Bin Packing With Trapezia: Methods and Applications  
Rhyd Lewis* |

#### 10:50 – 12:20

**Session 2**  
*Chair: Fran Parreño*

| 2.1 | PackageCargo - Open Source Tool based on Optimization and Simulation for the Container Loading Problem with Dynamic Stability  
Juan Camilo Martínez*, Daniel Cuellar*, David Álvarez-Martínez* |
| 2.2 | An integer linear model for the pallet-loading vehicle routing problem with practical constraints  
M.T. Alonso*, A. Martinez-Sykora*, J.A. Bennell†, Ramon Álvarez-Valdés‡, Francisco Parreño* |
| 2.3 | Packing of arbitrary polyhedra with continuous rotations using nonlinear optimisation  
T. Romanova*, J.A. Bennell†, Y. Stoyan†, A. Pankratov* |
| 2.4 | GRASP with ejection chains for Pallet Building and Truck Loading with practical constraints  
Francisco Parreño*, María Teresa Alonso*, Ramon Álvarez-Valdés‡ |

#### 13:30 – 15:00

**Session 3**  
*Chair: Toni Martinez-Sykora*

| 3.1 | A practical view on packing and loading problems with ‘surprising’ impacting variables and constraints from real life  
Stefan Althoff* |
| 3.2 | Packing of ellipses in a convex polygon of minimum area  
A. Pankratov*, T. Romanova*, I. Litvinchev* |
| 3.3 | Multi-objective Build Orientation and Bin Packing of Parts in Selective Laser Melting  
V. Griffiths*, A. Martinez-Sykora*, J. P. Scanlan*, M. H. Eres*, P. Chinchapatnam† |
| 3.4 | Upper Bounds for Heuristic Approaches to the three-dimensional Guillotine Strip Packing Problem  
Torsten Buchwald*, Guntram Scheithauer* |
15:30 – 17:00

Session 4  
Chair: Tony Wauters

4.1 – Beam Search for the 2D-Single Knapsack Problem  
Carlos Alegría*, Julia A. Bennell†, Marta Cabo∗, A. Martínez Sykora†

4.2 – An archaeological irregular packing problem  
J.A. Bennell*, M. Cabo Nodar†, C. Lamas Fernandez*, A. Martinez-Sykora†

4.3 – A cutting plane approach for the two-dimensional rectangular strip packing problem  
Isabel Friedow*, Guntram Scheithauer*

4.4 – Improving a semi-discrete lines representation for the 2D irregular strip packing problem  
Wojciech Kuberski*, Dirk Roose*, Tony Wauters*

Friday, May 25th

8:45 – 10:40

Session 5  
Chair: José Fernando Oliveira

5.1 – The method of artificial expansion of space in the optimal problem of balls  
Sergey Yakovlev*, Oleksii Kartashov*, Kyryl Korobchynskyi*

5.2 – On problems of optimization in the configuration space of spherical objects with variable radius  
Sergey Yakovlev*, Sergey Shekhovtsov*, Bogdan Skrypka*

5.3 – The manufacturer’s pallet loading problem with stability constraints – A case study  
António G. Ramos*, Elsa Silva†, Pedro Lopes†, Pedro Ribeiro†

5.4 – The Skiving Stock Problem and its Application to Wireless Communications  
John Martinovic*, Eduard Jorswieck*, Guntram Scheithauer*, Andreas Fischer*

5.5 – A matheuristic approach for stochastic problems with scarce information on uncertainty: paving the way to deal with uncertainty in Cutting and Packing?  
Beatriz Brito Oliveira*, Maria Antónia Carraúl†, José Fernando Oliveira*, Alysson Machado Costa†

11:10 – 12:40

Session 6  
Chair: Antonio G. Ramos

6.1 – Metaheuristic solution of the two-dimensional strip packing problem  
Rosephine Georgina Rakotonirainy*, Jan H van Vuuren*

6.2 – A parallel branch and bound for the 3D Cutting and Packing Problem  
Everton Fernandes da Silva*, Tony Wauters*

6.3 – Improved Layout structure with Complexity measure for the Manufacturers pallet-loading problem (MPLP) using a Mega-box Approach  
Deemah Aljuhani*, Lazaros G. Papageorgiou*

6.4 – Load Balance recovery for multi-drop distribution problems – a mixed integer linear programming approach  
Elsa Silva*, António G. Ramos†, José F. Oliveira†

12:40 – 12:50

Closing Session

Zoetermeer, The Netherlands, May 23–25, 2018
Social Program

- Get-together Evening
  May 23, 2018, from 18.00,
  Golden Tulip Hotel Zoetermeer,
  Address: Kinderen van Versteegplein 18, 2713 HB Zoetermeer
  Note: Drinks and buffet-style dinner are available on a pay-yourself basis.

- Conference dinner
  May 24, 2018, from 19:30,
  Euromast
  Address: Parkhaven 20, 3016 GM Rotterdam
  Three courses (starter + main + dessert), drink included.
  Fee included in the registration fee
  This restaurant is located at a height of 100 meters with a spectacular view. During the evening it is possible to visit the top to have an even better view over Rotterdam
  A coach is arranged to bring you from Zoetermeer to the Euromast between 18:30 and 18:45 and back again after the dinner (fee included in the registration fee)
Abstracts

1.1 Challenge ROADEF / EURO 2018 Cutting Optimization
Lydia Tlilane*, Quentin Viaud*
* Datalab, Saint-Gobain

The French Operational Research and Decision Support Society (ROADEF) organizes jointly with the European Operational Research Society (EURO) the ROADEF/EURO challenge 2018 dedicated to glass cutting problem in collaboration with Saint-Gobain Glass France (SGGF). SGGF, part of the Saint-Gobain Group, is one of the world’s leading at glass manufacturers. It specializes in float glass manufacture and magnetron coated glass for a wide variety of domestic and commercial applications including housing equipment (windows, bay windows, interior design), facade, urban development and the realization of major projects. The goal of this challenge has multiple aspects. First, it allows industrial partners to follow recent developments in the field of Operations Research and Decision Analysis. Second, researchers have the opportunity to face up to a complex industrial optimization problem, demonstrate their knowledge and share their know-how and expertise on practical problems.

The subject of this challenge is to tackle an industrial glass cutting problem faced by SGGF. Flat glass is mostly produced through a process called the “float process”. At the end of this process, an infinite glass ribbon is cut into large glass sheets (bins). To facilitate the storage, bins are stacked after cutting from the ribbon. These bins are most of the time cut into smaller rectangular pieces (items) adapted to customer needs. These items are cut according to a cutting pattern which satisfies a certain amount of constraints related to the customer (production plan, ...) or to the physics of glass (guillotine cut, cut length limitation). Bins are not perfect and may contain defects inherent to the float process. The goal of this challenge is, for a given sequence of bins and their defects, as well as a given item batch to cut, to propose an algorithm allowing to reduce as much as possible the glass losses of the cutting process.

This problem is a two-dimensional bin-packing problem with additional constraints. They include bin stack limitation, item production plan and also technical constraints related to glass cutting. A more detailed subject, realistic instances and solution checker can be found at challenge.roadef.org. An overall amount of 45.000 Euros will be distributed to the best contributions. Registration is possible until September 2018.

Keywords: Challenge

1.2 Real life benchmarks for load building
Jelke J. van Hoorn*, Lina Rahali*, Ties Westendorp*
* ORTEC, The Netherlands

At ORTEC we have a wealth of practical load building cases. We would like to share this data with the academic community whenever this is possible. This year we started to create set of benchmarks instances using real data with practical constraints. We present the tools we created and benchmark instances we can already make available. With this presentation we also launch our new benchmark website. Furthermore, we present the plans we have for adding even more types of constraints and objectives.

Keywords: benchmarking, container loading, pallet loading, practical constraints

1.3 Bin Packing With Trapezia: Methods and Applications
Rhyd Lewis*
* School of Mathematics, Cardiff University, CF24 4AG, Wales, UK

In this research we consider an extension of the one-dimensional bin packing problem in which items have a fixed height, vary in width, and are trapezoidal in shape. This problem is of particular interest in the construction industry, where we wish to cut trapezium-shaped roof trusses from fixed-length rectangular stocks. It also arises when laying decked flooring in outdoor areas, particularly when boards are required to travel diagonally across a workspace.

Consider a set $U$ of trapezium-shaped items, each that has a fixed height $H$. Each item $i \in U$ is defined as having a base width $b_i$, and two projection widths $x_i$ and $y_i$ that determine the angles of its lateral sides. An item’s “central width” is then simply $c_i = b_i - x_i - y_i$. Each trapezium can also take one of two forms: acute, where projections occur on the same side of the shape, or obtuse, where they occur on alternate sides. In either case, the area of an item $i$ is simply $A(i) = \frac{1}{2} H (b_i + c_i)$. Zoetermeer, The Netherlands, May 23–25, 2018
Our task in this problem is to pack all items of $U$ into a minimal number of $H \times W$ bins such that no bin is over-filled. The NP-hard one-dimensional bin packing problem is therefore a special case of this problem in which all items are rectangular ($x_i = y_i = 0, \forall i \in U$); hence this current problem is also NP-hard.

In addition to deciding which items to assign to which bins, in this problem we must consider how items are to be packed into a bin, preferably so that wastage between successive items is minimised. This can be characterised by the following decision problem:

Let $U \subseteq \mathbb{A}$ be a set of trapezia whose total area is less than or equal to a single bin's area (that is, $A(U) = \sum_{i \in U} A(i) \leq HW$). The trapezium packing sub-problem (TPSP), involves determining whether an arrangement of the items in $U$ exists such that the “inter-item wastage” is less than or equal to $HW - A(U)$. Here, inter-item wastage is defined as the total area of all triangular spaces between each pair of adjacent items, plus the left- and right-most triangles of wastage. If the inter-item wastage is indeed less than or equal to $HW - A(U)$, then an arrangement of all items in $U$ exists that allow them to be packed into a single bin.

In our presentation we will show how the TPSP can be solved in all cases using a polynomially bounded algorithm. This algorithm is based on the graph-theoretical concepts of Eulerian paths, minimum-weight matchings, and minimum spanning trees, all of which are known to be identifiable in polynomial time. We will then discuss ways that this exact algorithm can be combined with heuristics to help identify high-quality, but possibly approximate, solutions to the NP-hard multiple-bin version of this problem. Finally, we will also review some special cases of the multiple-bin version, focussing particularly on those arising when laying decking boards diagonally across a regular shaped workspace. It could be that some of these cases are actually solvable in polynomial time, particularly as they bear striking resemblances to certain subclasses of the one-dimensional bin packing problem that are themselves known to be polynomially solvable.

Keywords: Bin Packing, Trapezia, Eulerian Cycles, Decking

2.1 PackageCargo - Open Source Tool based on Optimization and Simulation for the Container Loading Problem with Dynamic Stability

Juan Camilo Martínez*, Daniel Cuellar*, David Álvarez-Martínez*
* School of Engineering, Universidad de Los Andes, Colombia

Traditional considerations related to dynamic stability constraint in the Container Loading Problem (CLP), such as boxes with insufficient lateral support, have proven to be inaccurate when compared with the results of dynamic simulations where the boxes are subjected to real world conditions. In response to this, two indicators for dynamic stability were introduced by Ramos et al. (2015); where the number of fallen boxes and the number of boxes likely to be damaged in case of acceleration were predicted using a linear regression model and some physical characteristics of the cargo simulated with the Bullet physics library. These indicators, however, have not yet been estimated using a mechanical approach.

An open source application (PackageCargo) was developed to calculate, visualize, and save efficient packing patterns to instances of the CLP, and now has been enhanced with a dynamic simulation environment to obtain performance indicators related the dynamic stability of such patterns. The physics engine used in the application (Physx® SDK 3.0) is used mostly in the real-time simulation of physics and is frequently implemented in videogames. This means that the engine is capable of trading accuracy for simulation speed and is in most cases non-deterministic. However, a simple benchmark test comparing Physx with software designed for accuracy in dynamic simulations (Autodesk Inventor® and ANSYS®) shows that the developed application has adequate precision, while producing the results in a fraction of the time used by the standard simulation software. The application has many available functions for the end user, but in this project, it will be used mainly to evaluate the accuracy of new dynamic stability metrics. These metrics are found using the kinetic parameters of the load such as mass distribution, coefficient of friction, and rigidity; and are similar to the indicators proposed by Ramos et al. Through a dynamic analysis of the forces and accelerations acting upon the boxes, it is possible to predict loss of support, tipping, or the reach of critical velocity that would damage the cargo.

The process by which the new indicators are obtained lead to the development of a mechanical model that measures dynamic stability without the need to subject the packing pattern to a physics simulation. The resulting model was proven to be accurate at predicting the percentage of fallen boxes of a packing pattern, and the dynamic stability indicators were confirmed to be related with volume utilization, as suggested in previous work. The possibility of predicting the number boxes within the damage boundary curve and the integration of the mechanical model with the container loading algorithm are presented to be explored in future works.

Keywords: Dynamic Stability, Decision Support Systems, Container Loading Problem
2.2 An integer linear model for the pallet-loading vehicle routing problem with practical constraints

M.T Alonso*, A. Martinez-Sykora†, J.A. Bennell†, Ramon Álvarez-Valdés‡, Francisco Parreño*

* University of Castilla-La Mancha, † University of Southampton, ‡ University of Valencia

We will present our mathematical model and results for a distribution problem that includes both the packing and routing of customer orders. The model needs to determine the packing of products onto pallets, assign pallets to trucks, decide the position of the pallets on the trucks so that a customer’s pallets are together and finally the route the trucks will take.

For the real problem that inspired this study, the demand of a customer can exceed the truck capacity and therefore we allow solutions that use more than one truck to visit any given customer. This situation has not been addressed in the existing literature on routing and packing, which always consider that every client is visited once. Another distinctive feature of the problem addressed here is that, besides the standard geometric constraints we also consider other practical constraints concerning the total weight, the weight on the axles and the dynamic stability of the load.

We propose an integer linear model that includes the constraints of both problems, routing and packing, with the objective of minimizing the total transportation cost.

The constraints are:

- All trucks start and end at the depot.
- The products have been previously put into pallets and the demand of each customer is given as a set of heterogeneous pallets.
- All demands must be supplied. Sometimes that requires that a customer is served by more than one truck.
- The pallets of each customer have to be accessed to be unloaded without moving other pallets on the same truck.
- Each truck has a maximum weight and there are limits on the weight each axle can support.
- The centre of gravity of the loaded truck has to be as near as possible to the geometric centre of the truck and always between its axles.
- The load has to be dynamically stable, so the pallets will not be displaced when the truck is moving and subject to acceleration and braking forces.

We present an extensive computational study where we vary the number and locations of the customers and their demand. Our computational results show that the proposed model has difficulties in solving to optimality even small size problems. We then propose a decomposition algorithm in which some of packing constraints are first relaxed in the model and then considered by a heuristic packing algorithm. If the heuristic fails, an auxiliary model is used to ensure the optimality of the solution obtained.

Keywords: Routing, packing, trucks, pallets, stability

2.3 Packing of arbitrary polyhedra with continuous rotations using nonlinear optimisation

T. Romanova*, J.A. Bennell†, Y. Stoyan†, A. Pankratov*

* Department of Mathematical Modeling and Optimal Design, Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Pozharsky Str., 2/10, Kharkov, 61046, Ukraine, † Southampton Business School, University of Southampton, Highfield, Southampton SO17 1BJ, UK

We study the problem of packing a given collection of arbitrary, in general non-convex, polyhedra into a cuboid of minimal volume. This problem has a wide spectrum of applications, for example in 3D-printing, modern biology, mineralogy, medicine, materials science, nanotechnology, space engineering. Continuous rotations and translations of polyhedra are allowed. In addition, minimal allowable distances between polyhedra are taken into account. We derive an exact mathematical model using adjusted radical free quasi phi-functions for non-convex polyhedra to describe non-overlapping and distance constraints. The model is a nonlinear programming formulation. We develop an efficient solution algorithm, which employs a fast starting point algorithm and a new compaction procedure that involves a fast starting point algorithm. We also propose the COMPOLY procedure to search for “good” local optimal solutions. It can be used as a compaction algorithm, starting from a feasible point found by any algorithm published before. The COMPOLY procedure reduces our problem to a sequence of nonlinear programming sub-problems of considerably smaller dimension and a smaller number of nonlinear inequalities and, therefore, allows us to reduce computational costs (time and memory) considerably. This reduction is of a paramount importance, since we deal with nonlinear optimisation problems. Our results
on new instances and instances from the literature show our approach has superior performance. The benefit of this approach is borne out by the computational results, which include a comparison with previously published instances and new instances.

**Keywords:** Packing, Polyhedra, Continuous rotations, Distance constraints, Mathematical modeling, Non-linear optimisation

### 2.4 GRASP with ejection chains for Pallet Building and Truck Loading with practical constraints

Francisco Parreño*, Maria Teresa Alonso*, Ramon Álvarez-Valdés†

* University of Castilla-La Mancha, Department of Mathematics, Albacete, Spain, † University of Valencia, Department of Statistics and Operations Research, Burjassot, Valencia, Spain

A logistics company has to send all the products demanded by a customer using the minimum number of trucks. Products have to be packed into pallets and the company prefers first to build homogeneous pallets, then pallets in which each layer is composed of one product, and finally pallets containing the remaining units of each product. Then, these pallets have to be loaded into trucks, satisfying packing constraints.

To produce solutions useful in practice, our problem considers five types of constraints: geometric constraints, so that pallets lie completely inside the trucks and do not overlap; weight constraints, defining the maximum weights supported by a truck and by each axle, constraints concerning the position of the centre of gravity of the cargo; and finally dynamic stability constraints. These last constraints forbid empty spaces between pallets to avoid cargo displacement when the truck is moving. We also consider extensions to the case in which the demands must be served over a set of periods to meet delivery dates.

Exact mathematical approaches are not able of finding solutions within a limited time, and therefore we use a metaheuristic algorithm for obtaining high quality solutions in short computational times. A greedy randomized adaptive search procedure (GRASP) for the multi container-loading problem is presented, including a constructive procedure, and some improvement moves including an adaptation of ejection chains. We explore the adaptation of the ejection chain methodology as an improvement phase in GRASP.

The computational results on a real set of test instances show that the algorithm is very efficient for a wide range of multi container loading problems and can be very easily adapted to other constraints of the problem. We also provide a comparison with an integer programming formulation. The comparison justifies the use of a metaheuristic algorithm.

**Keywords:** GRASP, metaheuristics, packing, trucks, pallets

### 3.1 A practical view on packing and loading problems with ‘surprising’ impacting variables and constraints from real life

Stefan Althoff*

* ORTEC (Senior Product Manager), The Netherlands

This presentation informs about some examples of business problems from real life, where the basic problem of packing and/or loading is extended with other constraints and objectives than traditional cutting and packing algorithms usually address. The aim of this session is to inform and challenge the scientific community to think and potentially work on solutions, which can also support those kind of additional constraints and objectives. Examples to be shown are, e.g. the conflicts in objectives of different business partners in the supply chain about what a ‘good’ truck load means (Sales department, Warehouse, Carrier, Legal authorities, etc.). Another example is the conflict of minimizing the amount of pallets needed to ship one or multiple orders and the effect on the warehouse efficiency when packing those pallets. As a third example, it will be presented how other organizational constraints or variables can add complexity in finding a satisfying packing or loading solution for a business, which performs Cross-Docking in their Supply Chain and which have the flexibility of combing package items with different, partially overlapping allowed loading time windows.

**Keywords:** Container loading, Practitioners, Business problems

### 3.2 Packing of ellipses in a convex polygon of minimum area

A. Pankratov*, T. Romanova*, I. Litvinchev†

* Department of Mathematical Modeling and Optimal Design, Institute for Mechanical Engineering Problems of the National Academy of Sciences of Ukraine, Pozharsky Str., 2/10, Kharkov, 61046, Ukraine, † Nuevo Leon State University (UANL), Mexico

Zoetermeer, The Netherlands, May 23–25, 2018
The paper studies a packing problem of ellipses with continuous rotations within an arbitrary convex polygon of minimum area, which is a part of operational research and computational geometry. The problem is NP-hard and has multiple applications, e.g., in molecular dynamics (structure and properties of liquid crystals, the structure of liquids and glasses, chromosome packing in cell nucleus mineral, powder metallurgy, mineral industries), logistics (packing rolls of wallpaper, transportation of the pipes, transportation of the paint buckets), rational cutting of industrial materials (mirror manufacturers, furniture making, 3D modeling of granular, structures and substances), robotics, additive technologies (topology optimization of products for 3D printing).

We propose new tools of mathematical modeling of relations between ellipses using the phi-function technique. We introduce new quasi-phi-functions and phi-functions to describe non-overlapping and containment constraints with the smaller number of variables than already published quasi-phi-functions. Our tools allow us to formulate the ellipse packing problem in the form of a non-linear programming problem. We propose a new starting point algorithm and a new modification of the LOFRT procedure to search for efficient locally optimal solutions of the ellipse packing problem. Our solution algorithm has an evaluation cost that is linear in the number of ellipses.

We provide two groups of instances: new instances and those taken from the recent related papers. The comparison results for 210 instances show benefits of our algorithm both in the value of the objective function and computational time.

**Keywords:** Ellipse packing, continuous rotations, non-overlapping, containment

### 3.3 Multi-objective Build Orientation and Bin Packing of Parts in Selective Laser Melting

V. Griffiths*, A. Martinez-Sykora*, J. P. Scanlan*, M. H. Eros*, P. Chinchapatnam†

* University of Southampton, Southampton, SO17 8BJ, UK , † Rolls-Royce plc, PO Box 31, Derby, DE24 8BJ, UK

We present a metaheuristic model for the three-dimensional rotation, bin assignment and bin packing of parts across identical Selective Laser Melting (SLM) machines, with the aim of reducing the overall production cost. Selective Laser Melting (SLM) is an additive manufacturing (AM) method, which creates metal parts by iteratively depositing thin layers of metal powder on a build platform and fusing the powder particles in each layer with a laser, until the part is fully formed. Since it is not limited by traditional manufacturability constraints, SLM can build different part geometries in a single process and in almost any build orientation, which is the 3D rotational position of each part relative to the build platform.

The cost of SLM can be minimised through minimising the number of machines (i.e. bins) required to complete an order; minimising the total material used; and minimising the build time of each machine, which is directly proportional to the height of the tallest part in each machine. Additionally, in order to prevent significant distortion and potential failure during SLM, steep overhanging sections of the part must either be avoided or reinforced with sacrificial support structure, which increases the volume of material used.

Thus, we propose an Iterative Tabu Search Procedure (ITSP) to solve the above problem in two stages – firstly, we select the build orientation of each part, which is described by two angles of rotation; secondly, we vertically project a two-dimensional concave polygon of each part (i.e. a 2D piece) and solve a two-dimensional irregular bin packing problem (2DIBBP) to pack the parts into SLM machines.

The ITSP consists of six stages, where each stage addresses one of the above drivers of SLM cost. The first stage finds an initial solution with a minimum number of bins. The second stage aims to minimise the maximum part height in each bin. The third stage aims to minimise the total volume of sacrificial support structure in each bin. Similarly to the second and third stages, the fourth and fifth stages minimise the maximum part height and support structure volume, respectively; however bins are addressed and repacked in pairs, in order to improve the bin assignment of parts. The final stage adds a new empty bin to the solution and attempts to randomly remove parts from the old bins and place them into the new bin. Every time a part is moved in this way, the remaining parts in the disrupted bin are rotated, in order to reduce the cost of the overall solution. The final stage terminates when the new bin is full or after a certain number of no-improvement iterations.

By focusing on a different cost driver in each stage, the ITSP aims to address different neighbourhoods in the solution space. Each stage uses a Tabu search to find a new build orientation for a selected part (for instance, the second stage selects the tallest part in each bin, and uses the Tabu search to minimise its height). Also, each time a part build orientation is changed, the bin must be re-packed to ensure the rotated part still fits.

Finally, we present our computational results based on 27 new test instances, evaluate the effectiveness of each stage in the procedure, and benchmark our results against a commercial software, Magics v.15. The test instances were created using 68 realistic geometries, six of which are provided by companies from the aerospace industry, including Rolls Royce and General Electric.

**Keywords:** Selective Laser Melting, Additive Manufacturing, Irregular, Bin Packing
3.4 Upper Bounds for Heuristic Approaches to the three-dimensional Guillotine Strip Packing Problem

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We consider the COMB-3D heuristic for the three-dimensional Strip Packing Problem. This heuristic combines an implementation of a First-Fit Decreasing-Height, called FFDH-3D heuristic, and the algorithm of Steinberg. Furthermore, it is known that the absolute worst-case performance ratio of this heuristic is at most 5.

In this presentation, we show an example which proves that this absolute worst-case performance bound is tight. It reveals that this absolute worst-case performance bound can only be reached by instances which fulfil a certain property. Using induction, we succeeded to prove an improved absolute worst-case performance bound for the case that this property is violated. Again we construct examples to show that this absolute worst-case performance bound is tight and is at least 4.5.

Furthermore, we show that the absolute worst-case performance of the COMB-3D heuristic is at most 4.25 if the length of each item is not smaller than its width and the lengths of the container is not greater than its width. This conditions can be fulfilled for the z-oriented three-dimensional Strip Packing problem, where length and width of items can be interchanged but the height of the items is fixed. We also proved that this absolute worst-case performance bound improves to 4, if the container has a squared base area. This theorem is proved by a comprehensive case-by-case analysis and a special kind of induction, where the induction index $i$ increases from $i = 1$ to $i$ or $i + 1$.

Furthermore, we show that if all items and the container have squared base area, the absolute worst-case performance ratio of the COMB-3D heuristic is at most 3.6875 and if all items are cubes the absolute worst-case performance of the COMB-3D heuristic is at most 3.561. For proving these theorems we had to consider an unsolved problem for the two-dimensional Bin Packing Problem:

Can a set of items of length and width at most 1/2 and total area at most 5/9 be packed into a bin of length and width 1?

By a comprehensive case-by-case analysis we proved that such a set of items can be packed into the bin if all items are squares. This result solves the problem for a special case and is also tight, which means that 5/9 is the maximum total area bound. Finally we show that all pattern of the COMB-3D heuristic have the guillotine-property.

**Keywords:** Cutting and Packing, Bin Packing Problem, Strip Packing Problem, Heuristics, Performance Bounds, Guillotine Packing

4.1 Beam Search for the 2D-Single Knapsack Problem

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The Two-Dimensional Single Knapsack Problem is a well-known problem in the cutting and packing literature. It is an output maximization problem, where the decision maker must choose between a finite set of rectangular pieces to be placed in a rectangular bin, so that the profit of the selected pieces is maximized. This problem appears in many industries, such as glass, steel or plywood companies, where sheets of different sizes must be cut from a bigger sheet to serve as raw material to other companies. It also has applications in the hardware industry, where small components must be placed in an electrical circuit or tablet so to maximise the performance of the hardware.

We present a beam search heuristic for this problem. Beam search is a heuristic that acts as a tree search, where each node represents a partial solution to the problem. In this case, since we are working on maximizing the output over a single bin, a partial solution consists on a partially packed bin with a selected set of pieces. At each level, all partial solutions will have the same number of pieces placed, and all nodes emanating from the same parent will only differ in one piece.

One of the main characteristics of beam search is the two evaluation functions used to prune branches. The first evaluation is made locally and help decide which children will remain from a given parent node. The rationale behind this evaluation is to allow the creation of multiple child nodes, and quickly evaluating which ones may be immediately discarded. Thus a quick heuristic that creates these child nodes is required.

We developed a heuristic that places one piece at a time, and will be used to create the child nodes. This heuristic will find the best position to place a single selected piece into the bin. For each of the child nodes, a different piece will be selected, and the evaluation function must decide which solution to keep for further exploration. The placing procedure will take advantage on the fact that we are only placing rectangles, so once a piece is placed, the available space can be divided into small rectangles where to place the next piece. We define a maximal rectangle, as a rectangular area in the bin, where each of its sides is concurrent with either an edge of a piece or an edge of the bin. The selected piece will be placed in the best maximal rectangle.
The second evaluation is made globally over all the child nodes from each level. The goal of this evaluation is to perform a drastic prune on the branches that will potentially lead to bad solutions at the end of the algorithm. This evaluation is needed due to the greediness of the local evaluation, who does not take into account future decisions to keep one child node over the others. To perform the global evaluation, we need to construct the entire solution taking the current child node as starting point. That is, with a partially packed bin, and a set of available pieces, decide how the rest of the bin will be packed. To construct such solution, we adapt the best-fit heuristic to the needs of our problem. Originally, this heuristic was intended to solve the strip packing problem: it finds the lowest gap available on the bin and select the lowest piece that fit into that gap. Since in our problem, the height of the bin is fixed and our objective is to maximize profit, the selection of the pieces, will take into account, not only the horizontal gap, but also the vertical space available, and among all the pieces that fit in that area, we choose one that will give the maximum profit.

During this talk, we will explain in detail the two selected heuristic and how they are embedded into the beam search framework. We will present some preliminary results, and compare them to the best ones from the literature.

**Keywords**: beam search, knapsack problem, best-fit

### 4.2 An archaeological irregular packing problem

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The paper explores the use of cutting and packing methodologies in a new and innovative way to support archaeological research. Pre-hispanic cultures in Mexico used codices to register different aspects of their everyday life including their lands and crops. Each family register the number of pieces of land that they owned together with some measures of its dimension in a codex. These codices have been deciphered and we have accurate information about the length of each side of the pieces of land and the area they covered. Using the given dimensions, each terrain can be reconstructed as polygons. In addition, the dimensions and location of the settlements are known. This description equates to a two dimensional packing problem with an irregular bin and irregular pieces that can be rotated. While irregular shape packing has been an active research area for many years, the work focuses on packing a single strip for cutting material for manufacturing. However, while the problem at hand is quite different, we draw on the techniques arising from this research area in both geometry and algorithm design. In the presentation we will present the problem and some specific complexities that lead to the solution methodology needing to generate a range of different solutions. We present a formulation of the problem and use some simple heuristics to find a set of solutions.

**Keywords**: irregular shape, single stock sheet, strongly heterogeneous small items, covering

### 4.3 A cutting plane approach for the two-dimensional rectangular strip packing problem

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We consider the two-dimensional rectangular strip packing problem (SPP). Given a set of rectangles, the objective of the SPP is to place these rectangles within a strip of given width and unrestricted height in such a way that the strip height needed becomes minimal. A relaxation of this problem is the one-dimensional contiguous bin packing problem (CBP), in which each rectangle is represented by a unit-height item-type with an order demand corresponding to the rectangle height. All items of the same type have to be packed into consecutive one-dimensional bins under the objective to minimize the overall number of used bin patterns.

We investigate a cutting plane approach in which sets of bin patterns fulfilling this contiguity property of item-types, called sequences, are generated to obtain a CBP solution. Within this approach, linear problems are solved. Starting from the linear programming relaxation of the Gilmore-Gomory model of the one-dimensional binary cutting stock problem, a relaxation of the CBP, additional constraints are included successively to the linear problems due to contiguity. More precisely, each partial or completed sequence of bin patterns fulfilling the contiguity property is related to a subset of the newly introduced constraints.

A solution of the CBP, i.e., a sequence of bins which contains all item-types where the number of bins is minimal, only represents a solution of the SPP if the constant location property is fulfilled. This means, all items representing one rectangle need to have the same x-position in each bin in which they are contained. In our approach, we do not fix any x-positions of items. We only ensure that there exists at least one feasible (partial) two-dimensional packing for each sequence described by the current linear problem.
The considered linear problems (relaxations) are solved with the column generation technique. By the solution of appropriate slave problems feasible packings of bins are obtained which satisfy both desired properties: the continuation of a sequence fulfilling the contiguity property and the constant location property.

We investigate several strategies to reduce the number of constraints, or rather cutting planes, involving aspects of symmetry and dominance as well as lower bounds. We tested our cutting plane approach for a wide range of test instances and the results obtained are competitive to those of other exact methods proposed in literature.

**Keywords:** strip packing problem, contiguous bin packing problem, column generation, cutting plane approach

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### 4.4 Improving a semi-discrete lines representation for the 2D irregular strip packing problem

Wojciech Kuberski*, Dirk Roose*, Tony Wanters*

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The 2D irregular strip packing problem deals with placing a number of irregular pieces on a rectangular sheet of material, where the size in one dimension is fixed—the height—while the size in the other dimension—the width—is minimized. This problem is encountered in e.g. the furniture, textile, metal and 3D printing industries. We assume that pieces are polygons. Efficient collision checking of pieces is essential for this problem.

The present work proposes two improvements to the semi-discrete lines representation described in by Akunuru and Babu, where pieces are represented as a set of equidistant vertical line segments. These line segments are used for collision checking, hence it is necessary that the polygon implicitly defined by these lines fully contains the original polygon. One of the most important parameters of the representation is the resolution—the distance between the lines, which strongly affects the accuracy and the computational cost. Decreasing the distance between the lines improves the accuracy of the representation, but also increases the computational cost of each step of the strip packing algorithm (creation of the semi-discrete representation, collision checking, etc.). The algorithm presented by Akunuru and Babu does not describe in detail how to handle edges for which the length of their projection on the x-axis is smaller than the resolution and which may fall between the discretizing lines. We present a refinement of the algorithm dealing with these situations, ensuring that the resulting semi-discrete representation always fully contains the original polygon.

Additionally, we suggest a better method to determine an appropriate resolution, by considering more information from the dataset. This allows to achieve a better tradeoff between the solution quality and the computational cost. More specifically, we consider the expansion ratio, which is the ratio between the area of the polygon obtained using the semi-discrete representation and the area of the original polygon for a given resolution as a proxy for the solution quality. This expansion ratio is estimated using features from the given dataset: the probability that an edge falls between the lines, the expected polygon expansion for a number of cases, etc. This estimate is computed without explicitly creating the semi-discrete representation, which allows to select a resolution that corresponds to a user-selected expansion without significantly increasing the computational cost. This estimate is validated using the datasets from ESICUP. The final aim is to use the relationship between the resolution and the polygon expansion to determine a suitable resolution to find an accurate solution to the 2D irregular strip packing problem within a given time constraint.

**Keywords:** strip packing, discretization, nesting problems, discrete representation, geometry

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### 5.1 The method of artificial expansion of space in the optimal problem of balls

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The problem of optimal placement of unequal balls is considered. Its model is mathematical programming problem with a linear objective function and quadratic constraints. The problem is multi-extremal. To find its local solutions, there is a wide range of methods. The paper proposes a method for improving the local solutions called by the method of artificial expansion of the space of variables. The main idea of the method is as follows. The combinatorial structure of the problem is singled out, in which the radii of the balls are constants.

Then an equivalent mathematical model with variable radii is constructed. At the same time, an additional system of restrictions on the radii values is formed, which ensures their convergence to the initial fixed values. Thus, due to the variable radii, it is possible to overcome the region of attraction of the local extrema. The paper describes and analyzes the results of numerical experiments, depending on the number of balls and the choice of the starting point.

**Keywords:** packing problem, optimization, local extrema, combinatorial set, packing of balls, expansion of space

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5.2 On problems of optimization in the configuration space of spherical objects with variable radius

Sergey Yakovlev*, Sergey Shekhovtsov*, Bogdan Skrypka*

* Department of computer sciences of National Aerospace University, Kharkov, Ukraine

We consider the configuration space of spherical objects. The generalized variables of this space are the radii and the parameters of the placement of balls. A general mathematical model of the problem is build. Classification of optimization placement problems of balls with variable radii is carried out and numerical methods for their solution are proposed. In addition to the classical packing problem of nonequil balls in a ball of minimum radius, the packing problem of composite balls are considered. Particular attention is paid to the optimal placement of balls of variable radius in regions of complex shape. In this case, the total volume of the balls is maximized. The general unbalance of the set of balls of a given density and the total length of the network connecting the objects are also considered as optimization criteria. The paper describes the results of numerical experiments.

Keywords: packing problem, mathematical model, optimization, composite balls, variable radius

5.3 The manufacturer’s pallet loading problem with stability constraints – A case study

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The manufacturer’s pallet loading problem is one of the most studied problems in the cutting and packing field in the past 50 years. The problem is frequently addressed by reducing the three-dimensional problem to a two-dimensional one, since usually boxes can only rotate about a vertical axis. However, this means that practical additional constraints such as stability are not taken into consideration when determining the two dimensional layout arrangement, limiting the applicability of such solutions in practice.

This work presents a flexible heuristic algorithm that enforces stability constraints based on criteria defined by the user. The algorithm was developed as part of the Adapt Pack project which developed a framework focused on the design and development of new highly flexible modular robotic packaging and palletizing systems.

The algorithm was tested using a large set of real world instances from an industrial company, and the results were compared to the current solutions of the company and were also validated by the company.

Keywords: pallet loading, stability

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5.4 The Skiving Stock Problem and its Application to Wireless Communications

John Martinovic*, Eduard Jorswieck*, Guntram Scheithauer*, Andreas Fischer*

* Technische Universität Dresden

We consider a threshold length \( L \in \mathbb{N} \), and \( m \in \mathbb{N} \) item types that are specified by their length \( l_i \) and frequency (of occurrence) \( b_i \), where \( i \in I := \{1, \ldots, m\} \). Constituting a natural (but independent) counterpart of the well-known cutting stock problem, the one-dimensional skiving stock problem (SSP) requires to recompose the given items in order to obtain a maximal number of objects each having a total length of at least \( L \).

Besides its original application in industrial production and recycling, the SSP plays an important role whenever an efficient and sustainable use of given resources is desired, e.g., for the allocation of wireless users within a given frequency range. Due to the considerably increased demand for wireless connectivity, in recent years, the natural radio spectrum became an important and scarce resource. Normally, it is regulated by governmental entities and fixed parts of it are assigned to licensed holders for a long time. However, for large parts of the licensed spectrum the utilization may be very low leading to many wasted vacant frequency intervals. Since these spectrum holes are typically too small to single-handedly satisfy the bandwidth requirements of (given) secondary users (SUs), several of them have to be combined in order to obtain sufficiently large transmission channels. Without further conditions, the homogeneous case of this allocation problem corresponds to an ordinary skiving stock problem. However, due to hardware limitations, further practical constraints have to be incorporated leading to a generalized problem setting.

This talk presents an introduction to the skiving stock problem and discusses its application to resource allocation problems arising in the field of wireless communications. Besides stating some basic theoretical properties and
different modeling approaches, we provide numerical simulations that contribute to underline the potentials of efficient spectrum utilization.

**Keywords:** Skiving Stock Problem, ILP, Resource Allocation, Wireless Communication

### 5.5

**A matheuristic approach for stochastic problems with scarce information on uncertainty: paving the way to deal with uncertainty in Cutting and Packing?**

Beatriz Brito Oliveira*, Maria Antónia Car ravilla*, José Fernando Oliveira*, Alysson Machado Costa†

* INESC TEC and Faculty of Engineering, University of Porto, † The University of Melbourne

Uncertainty has been scarcely dealt with in Cutting and Packing. These are rather hard to solve problems in their deterministic form, what has hindered the consideration of uncertainty in Cutting and Packing when uncertainty is more a rule than an exception in all real-world optimization problems. Additionally, when dealing with stochastic problems in real-world contexts, there is often little information available on uncertainty. Stochastic programming techniques can be applied but they require the definition of scenarios or probability distributions to describe the realizations of uncertainty, which is hindered when there is scarce information about it.

Using the example of the car rental capacity-pricing stochastic problem, we propose a methodology that tackles this problem by simultaneously generating good solutions and representative scenarios. Nevertheless, this method can be easily applied to other two-stage stochastic problems, namely to cutting and packing problems. This problem concerns the decisions made by a car rental company, when planning a selling season, regarding the number of vehicles of each group, or capacity, it will have to meet demand and the prices to charge for each rental product, which are two connected decisions given the highly price-sensitivity of demand, which by nature is already uncertainty.

The methodology involves modelling the problem as a two-stage stochastic program. The overall goal is to provide decision-makers with profitable solutions to the first stage decisions, describing their ability to deal with the different realizations of uncertainty, represented by scenarios. The solution method developed is based on a co-evolutionary genetic algorithm, where parallel populations of solutions and scenarios co-evolve, depending on each other for the fitness evaluations. Solutions are evaluated according to the risk profile of the decision-maker and their performance against the scenarios. Scenarios are evaluated based on their contribution in making the scenario population representative and diverse, in terms of impact on solutions. The performance of solutions against scenarios (and the corresponding impact of scenarios on solutions) is measured by solving the mathematical programming model of the recourse problem, making this method a matheuristic.

On the one hand, this method obtains a representative and diverse population of scenarios, in relation to the impact they have on solutions. On the other hand, solutions converge to well-performing decisions, based on different risk profiles. This method does not require scenarios or probabilities to be known a priori. Moreover, it provides the decision-maker with a set of possible solutions, clearly associated with the impact of the different scenarios.

**Keywords:** Stochastic programming, genetic algorithms, scenarios, car rental

### 6.1

**Metaheuristic solution of the two-dimensional strip packing problem**

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We consider the two-dimensional strip packing problem in which a given set of items has to be packed into a single object of unlimited length but fixed width. A new systematic way is introduced for comparing the performance of strip packing algorithms. A large, single set of strip packing benchmark instances from the literature is clustered into different classes of test problems based on their underlying features. We compare two improved strip packing metaheuristics with a representative sample of heuristics and metaheuristics from the literature in terms of both solution quality and execution time in respect of the clustered benchmark data. The effectiveness of the designs of the different algorithms are contrasted for each of the data categories. It is found that the aspects of the test problems affect the solution qualities and relative rankings of the various packing algorithms. Furthermore, the improved algorithms compare favourably to other existing strip packing metaheuristics in the literature in terms of both solution quality and execution time.

**Keywords:** metaheuristics, clustering, strip packing problem
6.2 A parallel branch and bound for the 3D Cutting and Packing Problem
Everton Fernandes da Silva*, Tony Wauters*
* KU Leuven

3D Cutting and Packing Problems consist of a set of items that must be placed inside one or more larger items, known as containers. The items considered are cuboid-shaped, and consequently referred to as boxes. The primary constraints related to this problem ensure that boxes are placed entirely inside the container’s dimensions (bounding constraints) and that boxes do not overlap with one another. A set of additional constraints, those related to rotations, are also considered.

Of the many subproblems emerging from the 3D Cutting and Packing Problem, the 3D Single Knapsack Problem (SKP) and 3D Single Large Object Placement Problem (SLOPP) are addressed by the present research. While both variants consist of maximizing the value of boxes placed inside a single container, the former one considers a set of strongly heterogeneous boxes whereas the latter considers a set of weakly heterogeneous ones.

This study proposes a parallel approach for solving both the SKP and the SLOPP. The strategy consists of applying a specialized branch and bound to solve multiple feasibility problems concerning combinations of various subset of boxes. To avoid solving symmetrical problems, a list of feasible and infeasible solutions already found is maintained and each node requiring investigation is first checked whether there are any symmetrical solutions already present in any of these lists. If such a solution is detected then the node is automatically pruned or branched according to the list within the symmetrical solution occurs.

The parallelization of the proposed branch and bound, which solves multiple nodes concurrently, is employed for three purposes: (i) to improve the solution performance of the entire approach, (ii) to best utilize the feasible and infeasible lists by employing a depth-first strategy and (iii) to verify the performance associated with solving mathematical formulations using all available threads compared against multiple formulations being solved concurrently with fewer threads. Experimental tests are executed on instances generated by a generator from the literature, with the obtained results compared against those attained by mathematical formulations from the literature which, in a previous comparative analysis, performed best for the considered instance sets.

Keywords: 3D cutting and packing, parallel approach


6.3 Improved Layout structure with Complexity measure for the Manufacturers pallet-loading problem (MPLP) using a Mega-box Approach
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Pallet loading is a fundamental function in the Industrial area; it has wide applications in packing, cargo, transportation, warehouse management and in such problems it is classified as a cutting and packing, bin packing, strip packing, and knapsack problems. Pallet loading contributes directly to the cost of the supply chain, and if managed properly it will lead to reduction in unnecessary operating costs including but not limited to; loading, storage, transportation, and workforce, it will also have an effect on customer service rating. In this paper we cover the area of the Manufacturers pallet-loading problem (MPLP), where small identical boxes are required for loading onto pallets of a rectangular size. The aim is to find an optimal loading configuration by reaching an optimal utilization percentage. In such problems, the main constraints are related to the size of the boxes and pallets, such as having the boxes packed orthogonally with the edges parallel to the pallet’s edges. Also the boxes must not exceed the pallet dimension and cannot overlap with one another. It is important to remark that the arrangement of boxes into pallets is a time consuming process and we aim by the proposed layouts to reduce the operating time.

A novel MILP model has been proposed to generate improved structures of pallet layouts that group boxes in a certain orientation along the x and y axis of the pallet based on a Mega-box representation. Such proposal reduces the loading complexity and drives faster operational times; which are essential in current daily operations. The problem has been tackled from a two- dimensional aspect taking into account the graphical layout structure of the proposed model and comparing it to literature graphical layouts. Where a set of Mega-boxes is defined with a selection of orientations, these Mega-boxes include small individual boxes of the same orientation by grouping them along the x and y axis. The model aims to maximise the utilisation percentage of the pallet by reducing the number of Mega-boxes used while increasing the number of grouped individual boxes in each Mega-box. The output of the model is then presented using a visualisation tool to produce the graphical layouts of the pallet; these graphical layouts are then compared with the literature, where a selection of small, medium to high number of boxes and sizes of pallets are being compared. And as a tool of analysis and evaluation, a term known as the complexity index is introduced to compare the complexity of two pallets which have the same size but different
box arrangements. Where the complexity index is defined as a numerical measure of a pallet loading complexity, it defines the average number of changes between boxes inside the pallet. This complexity index takes a value in the range between 0 and 1, the closer the complexity index to 0 the less complex the box arrangement is, and the closer the complexity index to 1 the most complex the box arrangement is. Such comparison helps to assist the pallet loading complexity and identify the differences in current and proposed layouts. As a result of the above proposed model the computational and graphical layouts show the superiority of the proposed approach compared with existing MPLP layouts. 

**Keywords:** Pallet packing, manufacturer’s pallet loading problem, mathematical programming, mixed integer optimisation

### 6.4 Load Balance recovery for multi-drop distribution problems – a mixed integer linear programming approach

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In road freight transportation, a loaded vehicle with a distribution route and a compliant load balance at the departing depot, can become non-compliant during the route, since the total weight of the cargo and its centre of gravity change with each delivery to a customer.

The current situation consists on vehicles circulating on roads undermining safety regulations or on lack of operational efficiency in customers’ visits, when these are taken into consideration and cargo is extensively rearranged at each costumer. This problem has been completely ignored both in the vehicle routing literature and in the container loading literature.

This work will contribute on ensuring that a cargo arrangement is load balanced in the complete distribution journey, i.e., from the depot until the last customer, by addressing the multi-drop Load Balance Recovery problem (MDLBRP). In the MDLBRP, given a complete route and the respective cargo arrangement, the boxes to be removed from the cargo arrangement and the boxes to be moved at each costumer must be identified as to ensure that cargo remains balanced after the delivery at the various costumers.

A MILP model is proposed to balance the cargo at each costumer stop. The MILP model incorporates load distribution diagrams constraints to determine the feasible domain for the location of the centre of gravity of the cargo arrangement according to the vehicle technical characteristics and regulatory requirements. The extensive computational experiments have shown that the MILP model can be used in practical context since it was able to find a solution in less than 10 minutes in 93% of the unbalanced test problems.

**Keywords:** Transport, Container Loading, Multi-drop, Load balance
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