Numerical, experimental and hybrid methods for the prediction of railway-induced ground vibration

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ABSTRACT: Ground-borne noise and vibration due to railway traffic are important environmental issues in the development of new railway lines and land adjacent to existing lines. Prediction models are indispensable to address properly these issues and to design efficient mitigation measures for both existing and new-build situations. The aim of the present contribution is to provide a comprehensive overview of methods which have been developed for the prediction of railway-induced ground vibration. A distinction is made between numerical models, based on physical or mechanical models, empirical models, derived from measured data, and hybrid models that combine elements from numerical and empirical models.

Numerical models are particularly useful in new-build situations. They can be used to assess the environmental impact of new railway lines or to develop and design mitigation measures. Numerical predictions are generally characterized by significant model and parameter uncertainty, however, which need to be considered for robust engineering and design. In engineering practice, empirical methods are mostly used, which are based on an experimental characterization of the source strength and vibration transmission. These methods have the advantage of accounting for the particularities of the vibration transmission at a given site, but can only be applied in those cases for which suitable data and prior experience are available. In order to overcome the limitations of numerical and empirical methods, these methods can be combined in a hybrid or semi-empirical prediction procedure. It is shown how such a hybrid method is obtained by replacing the experimental source strength or vibration transmission term by an equivalent quantity which is computed using numerical simulations.

KEY WORDS: railway; ground vibration; wave propagation.

1 INTRODUCTION

Ground-borne noise and vibration due to railway traffic is a problem of large societal and economical relevance. A large number of high speed, freight and urban railway lines are presently planned or under construction, particularly in Europe, Asia and North-America (e.g. Crossrail in London, Regional Express Network in Brussels, High Speed Railway in China, California High Speed Rail) to meet increasing demands for passenger and freight transport. Railway noise and vibration also needs to be addressed for the development of land in urban areas adjacent to railway lines.

Railway-induced vibration is generated by quasi-static and dynamic axle loads; the latter are due to excitation mechanisms such as wheel and rail unevenness, impact excitation due to rail joints and wheel flats, and parametric excitation due to spatial variation of the dynamic track stiffness [1], [2]. These loads are transferred to the track, its supporting structure and the soil, where vibration propagates as elastic waves and excite the foundations of nearby buildings. In the most common case where the train speed is below the wave velocities in the track and the soil, the dynamic load component is the dominant cause of railway-induced ground vibration [3], [4], [5].

In the frequency range between 1 and 80 Hz, building vibration is perceived as mechanical vibration of the human body, whereas between 16 and 250 Hz, ground-borne vibration can cause structure-borne noise by vibrating walls and floors. Norms and guidelines recognise discomfort to people malfunctioning of sensitive equipment and damage to buildings as possible consequences of vibration.

Guidance on the prediction of ground-borne vibration arising from rail systems is provided in a recent ISO standard [6]. A distinction is made between numerical models, based on physical or mechanical models, empirical models, derived from measured data, and hybrid models combining elements from numerical and empirical models.

The aim of the present contribution is to provide a comprehensive overview of numerical and empirical methods for the prediction of railway-induced ground vibration. The paper focuses on the prediction of low frequency feelable vibration and the case of railway traffic at grade, although many of the conclusions can be readily generalized to ground-borne noise and railway traffic in tunnels. Sections 2 and 3 of the paper discuss advantages and drawbacks of numerical and empirical methods, respectively. For illustration, results obtained for a site located at Lincent (Belgium), which has been considered in previous research [5], [7], [8], are included. Section 4 presents a novel hybrid or semi-empirical prediction procedure which combines elements from numerical and empirical methods and allows some limitations of these methods to be overcome.
2 NUMERICAL PREDICTION MODELS

2.1 Overview

The prediction of railway-induced vibrations in buildings (figure 1) is a complex three-dimensional (3D) coupled problem, involving moving loads and dynamic soil-structure interaction at the source (the track) and the receiver (the building where vibration is perceived or should be avoided).

Crucial in the prediction of railway-induced ground vibration is the wave propagation in the soil. Apart from a small zone immediately underneath the track, the strain levels in the soil remain relatively low during the passage of a train, so that a linear elastic constitutive behaviour can reasonably be assumed in the study of ground-borne vibration. Furthermore, soils often vary in the vertical direction only.

The prediction of ground-borne vibration in buildings is usually performed in a two step procedure, where the free-field response due to a running train is calculated first and subsequently used for computing the building response. This is due to interference of waves transmitted by different parts of the contact area. An accurate prediction of the stress distribution at the track-soil interface is therefore essential for predicting ground-borne vibration due to railway traffic, in particular when the wavelength in the soil is comparable or smaller than the characteristic dimension of the stress distribution area [14].

General purpose 3D finite element (FE) methods offer the largest flexibility in modelling, but require appropriate procedures to avoid spurious reflections at the boundaries of the finite volume of soil accounted for in the analysis [15], [16]. Alternatively, 3D coupled finite element - boundary element (BE) methods [17], [18] or spectral element [19] formulations can be used. The versatility of 3D FE and 3D FE/BE models comes at a very high computational cost, however. Dedicated models have therefore been developed that exploit the (assumed) regularity of the track and the underlying soil.

When the geometry of the track and the soil are uniform in the direction along the track $e_y$, the motion of the load in equation (1) can be replaced by an equivalent reverse motion of the receiver:

$$u(x',t) = \sum_{k=1}^{n_a} \int_{-\infty}^{t} H_0(x_k, x', t - \tau) g_k(\tau) d\tau$$

where $g_k(\tau)$ is the time-dependent load at axle $k$ and where the transfer function $H_0(x_k, x', t - \tau)$ relates the response at a point $x'$ to the load at one of the time-dependent positions $x_k(\tau)$ of the $n_a$ axles of the train. The coupling between the position $x_k(\tau)$ of the $k$-th axle ($k = 1, \ldots, n_a$) and the time history of the load $g_k(\tau)$ through the time $\tau$ gives rise to the Doppler effect.

The transfer of vibrations, as characterised by the transfer function $H_0(x, x', t)$ in equation (1), depends on how the load applied to the track is transferred to the soil. Generally, a distribution of the load over a larger area of soil, will result in a reduction in the high frequency vibration transmitted to the free field. This is due to interference of waves transmitted by different parts of the contact area. An accurate prediction of the stress distribution at the track-soil interface is therefore essential for predicting ground-borne vibration due to railway traffic, in particular when the wavelength in the soil is comparable or smaller than the characteristic dimension of the stress distribution area [14].

Assuming a linear behaviour of the coupled track-soil system, the prediction of the incident wave field $u(x',t)$ induced by a train pass-by can be formally written as follows:

$$u(x',t) = \sum_{k=1}^{n_a} \int_{-\infty}^{t} H_0(x_k, x', t - \tau) g_k(\tau) d\tau$$

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where $x_k(\tau)$ in equation (1) has been elaborated as $x_k(\tau) = x_{k0} + v\tau e_y$, with $x_{k0}$ the position of the axle at $t = 0$ and $v$ the train speed. In this case, it suffices to compute the transfer function $H_0(x, x', t)$ for a fixed source position $x$ from a large number of receivers $x'$ along the track. Omitting the (fixed) source coordinates as arguments of the transfer function, and replacing $x'$ by $x$ for notational convenience, the latter is rewritten as $H_0(x, y, z, t)$. Instead of computing the transfer function for a large number of receivers $(x, y, z)$ along the track, it is more convenient from a computational point of view, however, to compute the transfer function in the frequency-wavenumber domain by applying a Fourier transform with respect to the coordinate $y$ along the track [20], [21]:

$$H_0(x, k_y, z, \omega) = \int_{-\infty}^{+\infty} H_0(x, y, z, \omega) \exp(ik_y y) dy$$

$$H_0(x, y, z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_0(x, k_y, z, \omega) \exp(-ik_y y) dk_y$$

where a hat and a tilde above a variable denote its representation in the frequency domain and frequency-wavenumber domain, respectively.
In this so-called 2.5D methodology, a problem with 2D geometry is solved for each frequency \( \omega \) and wavenumber \( k_y \) to compute \( \tilde{H}_u(x, k_y, z, \omega) \) and the 3D solution \( \tilde{H}_u(x, y, z, \omega) \) is recovered by an inverse Fourier transformation with respect to the wavenumber \( k_y \). The response in the free field is computed from the transfer function in the frequency-wavenumber domain as follows [5], [7]:

\[
\tilde{u}(x, y, z, \omega) = \sum_{k_y=1}^{n_w} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left( -i k_y \tilde{H}_u \left( x, \frac{\omega - \tilde{\omega}}{v}, z, \omega \right) \right) \times \exp \left[ -i \left( \frac{\omega - \tilde{\omega}}{v} \right) (y - y_0) \right] \tilde{g}_k(\tilde{\omega}) \mathrm{d}\tilde{\omega}
\]

(4)

In the 2.5D approach, the dynamic load component \( \tilde{g}_{d}\kappa(\omega) \) of the total load \( \tilde{g}_k(\omega) \) in equation (4) is generally computed from the combined wheel and track unevenness [3], [7], [22] assuming a perfect contact between the train and the track:

\[
\tilde{u}_a(\omega) = \tilde{u}_1(\omega) + \tilde{u}_{w/r}(\omega)
\]

(5)

where \( \tilde{u}_a(\omega) \) and \( \tilde{u}_1(\omega) \) collect the displacements of the axles and the track at the \( n_w \) moving contact points. The vector \( \tilde{u}_{w/r}(\omega) \) collects the combined wheel and rail unevenness perceived by the axles.

Next, the displacements \( \tilde{u}_a(\omega) \) and \( \tilde{u}_1(\omega) \) are expressed in terms of the dynamic train loads by means of the track and vehicle compliance matrices \( C(\omega) \) and \( \tilde{C}(\omega) \) that relate the displacements at the multiple moving contact points to the vector \( \tilde{g}_k(\omega) \) of dynamic train loads [7]:

\[
[C(\omega) + \tilde{C}(\omega)] \tilde{g}_k(\omega) = -\tilde{u}_{w/r}(\omega)
\]

(6)

An additional compliance matrix representing the contact spring is in some cases considered in equation (6) to account for Hertzian contact between the wheel and the rail [23]. In the frequency range of interest for ground-borne vibration, the contact stiffness is relatively high compared to the track and vehicle stiffness, so that the corresponding compliance can generally be disregarded. Note that equation (6) is only valid when the geometry of the track and the soil is uniform along the track. Because of the assumed perfect contact, equation (6) does not allow accounting for large deflections or loss of contact, e.g. in the presence of wheel flats [24]; this would require a non-linear model.

Since 2.5D methods have a high computational efficiency and require only a relatively modest modelling effort, these methods have been applied by a large number of researchers to study dynamic train-track interaction [25] as well as ground-borne vibration due to railway traffic at grade, for example the research groups at ISVR [26], [27], [28], NGE [29], NTU [30], BAM [22], FEUP [31], [32], Chalmers [33], and KU Leuven [7], as well as for underground railway traffic, for example at Cambridge University [34], [35], [36], NTU [37], TU Munich [38], and ISVR [39], [40].

One of the drawbacks of these 2.5D models is that, due to the assumed translational invariance, they do not allow account to be taken of periodic rail support as in a conventional ballasted track. This implies that the stress distribution under the sleepers is not entirely correctly predicted, which is important at high frequencies when wavelengths in the track and the soil are of the same order of magnitude as the sleeper dimensions. For the same reason, 2.5D models cannot account for parametric excitation, unless represented by an equivalent geometric unevenness [22]. In order to resolve these issues and take into account the periodicity of the track structure, a similar methodology may be used which is based on the Floquet instead of the Fourier transform [41], [42]. In this case, the 3D solution is obtained based on the discretization of a single periodic cell.

Still other types of models are required for the analysis of vibration generated by trains crossing transition zones, switches and crossings [17], [43]. Limitations arising from linear 2.5D or periodic models for the track can be partially circumvented [44] by using different models for the train-track and track-soil interaction problems, respectively. The use of more detailed models of the train and the track [45], [46], [47], [48] in the first step allows for the consideration of parametric excitation, nonlinear behaviour of track components as ballast and rail pads, as well as loss of contact between the wheel and the rail [47].

### 2.2 Case study: site at Lincent

Notwithstanding the large progress which has been made in the development of models for the prediction of railway induced ground vibration, simplifications are often needed to limit the computational cost or simply because detailed input data is not available. These simplifications lead to significant prediction uncertainty, however. This is now illustrated for a site at Lincent, Belgium, located next to the high speed line between Brussels and Køln, where ground vibration measurements have been made for validation of numerical models [5], [7], [8].

The track in Lincent is a classical ballasted track with UIC 60 rails supported every 0.60 m by rubber pads on monoblock concrete sleepers. The rails are continuously welded and are fixed with a Pandrol E2039 rail fastening system and supported by resilient studded rubber rail pads (type 5197) with a thickness of 11 mm. Each rail pad is preloaded with a clip toe load of about 20 kN per rail seat. The track is supported by a porphyry ballast layer (calibre 25/50, thickness \( d = 0.35 \) m) and a limestone sub-ballast layer (thickness \( d = 0.60 \) m). Boring performed prior to the construction of the high speed line show that the soil consists of a shallow quaternary top layer of silt with a thickness of 1.2 m, followed by a layer of fine sand up to a depth of 3.2 m. Between 3.2 m and 7.5 m is a sequence of stiff layers of arenite (a sediment of a sandstone residue) embedded in clay. Below the ballast, the soil has been mixed with lime to improve its mechanical properties.

The small strain dynamic soil characteristics at the site have been determined by means of two spectral analysis of surface wave tests and five seismic cone penetration tests. These results show that, in the frequency range of interest for railway-induced vibration, the soil can be represented by a single layer with a thickness of 3.0 m and a shear wave velocity of about 150 m/s on top of a stiffer halfspace with a shear wave velocity of 280 m/s. A summary of the dynamic soil characteristics at this site as assumed in previous studies [5], [7] is shown in table 1.

The track at the site is located in a shallow excavation, cutting partly through the top layer of the profile identified in table 1. In
Table 1. Dynamic soil characteristics of the site at Lincent, Belgium.

<table>
<thead>
<tr>
<th>Layer</th>
<th>d</th>
<th>C₁</th>
<th>C₂</th>
<th>ρ</th>
<th>β</th>
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<tr>
<td></td>
<td>[m]</td>
<td>[m/s]</td>
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<td>[kg/m³]</td>
<td>[-]</td>
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<tr>
<td>1</td>
<td>3</td>
<td>150</td>
<td>300</td>
<td>2000</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>∞</td>
<td>280</td>
<td>560</td>
<td>2000</td>
<td>0.03</td>
</tr>
</tbody>
</table>

(a) Cross section of (a) simplified track model with reduced top layer and (b) more detailed model including excavation and stiffened subgrade.

As an illustration of the incurred prediction uncertainty, figure 4 compares the measured and predicted free field velocity at 12 m from the track during the pass-by of an InterCity (IC) train at a speed of 198 km/h [8]. The predictions have been obtained using the detailed model shown in figure 2b, tuning the track characteristics by fitting the predicted track receptance to the one measured in loaded track conditions. The dynamic axle loads have been computed based on a simplified unsprung mass model for the wheelsets and the unevenness measured by a track recording car. The track unevenness was statistically processed to yield a power spectral density (PSD) function, allowing the extrapolation of the unevenness data beyond the measured range of wavelengths. The multiple predictions shown in figures 4c and 4d have been obtained for different realizations of track unevenness which all match the PSD function of the track unevenness [5]. These results reveal significant prediction uncertainty. Since the range of results spanned by the predictions does not include the measured results, at least part of the mismatch between the latter is due to other model or model parameter errors.

2.3 Discussion

Numerical models have undoubtedly contributed to a better understanding of physical mechanisms in the generation of ground-borne vibration, providing insight into results of field measurements and vibration problems encountered in practice. In design, numerical models are particularly useful in new-build situations where a new building is to be built close to an existing track or tunnel or where a new railway line is to be constructed close to existing buildings. They can be used to assess the environmental impact of new railway lines or to develop and design mitigation measures aimed at reducing vibration nuisance to an acceptable level.

There are also, however, important limitations in the use of numerical models which can be attributed to model and parameter uncertainty. First, the simplifications introduced for modelling may be too restrictive for the model to be useful. The geometry of the track and the soil may not be translationally invariant or periodic, e.g. due to the presence of transition zones in the track, inclined soil layers or heterogeneities in the soil. Hunt and his co-workers have recently assessed a number of simplifying assumptions in the prediction of ground-borne vibration due to underground railway traffic. Deviations of up to 10 dB have been found at particular locations and frequencies, when assuming horizontal soil stratification and disregarding the slightly inclined nature of soil layers [49], disregarding voids at the tunnel–soil interface [50] or not considering a neighbouring
tunnel [51]. Second, even when situations are adequately represented by simple model geometries, the model parameters are also subject to significant uncertainty. For new-build situations, estimations have to be based on prior experience or engineering judgement. In existing situations, measurements of track unevenness [5] or in situ tests for identification of track and soil properties [7, 44] will not allow all model parameter uncertainty to be eliminated. Schevenels et al. [52] have recently quantified the uncertainty in the prediction of ground vibration transmission that arises from the limited resolution of a Spectral Analysis of Surface Waves test. Hunt and Hussein [53] have shown that deviations due to model parameter uncertainty are expected to be of the same order of magnitude as those arising from model uncertainty. Although the importance of prediction uncertainty is generally recognized, its quantification in the numerical prediction of ground-borne vibrations has seldom been addressed. This is essential, however, in order to arrive at robust predictions as needed in engineering practice.

3 EMPIRICAL PREDICTION MODELS

3.1 General principle

Despite the large recent progress in the development of numerical models, engineering practice mostly makes use of empirical methods. The ISO 14837-1 standard that provides general guidance on ground-borne noise and vibration arising from rail systems [6] indicates that requirements for absolute predictions change during the various stages of development. It distinguishes between scoping models (earliest stage), environmental assessment models (planning process) and detailed design models (part of construction and design). This categorization is seen in many of the empirical models in use but, of course, also applies to numerical models.

Examples of empirical methods include the procedures developed by the Federal Railroad Administration (FRA) and the Federal Transit Administration (FTA) of the U.S. Department of Transportation [54], [55], the method developed by the Swiss Federal Railways (SBB) [56], the method of Madshus et al. [57] which was based on measurements in Norway and Sweden, and the method of Hood et al. [58] which was developed within the frame of the Channel Tunnel Rail Link project in the UK. The procedures developed by FRA and FTA distinguish between the three different levels of assessment of the ISO 14837-1 standard [6]. The Detailed Vibration Assessment is based on a prediction technique developed by Bovey [59] and Nelson and Saurenman [60] and presents a more elaborate method for the prediction of ground-borne vibration and re-radiated noise in buildings. The method developed by SBB [56] distinguishes between two prediction models, VIBRA-1 and VIBRA-2, where the latter is more detailed and considers, for example, frequency-dependent attenuation models. The empirical methods by Madshus et al. [57] and Hood et al. [58] follow a similar structure to the one by SBB, and additionally consider the issue of prediction uncertainty.

The aforementioned empirical methods aim at predicting the quasi-stationary response during a train passage and can be cast in the following general form of the ISO 14837-1 standard [6]:

\[ A(f) = S(f)P(f)R(f) \]  

where \( A(f) \) is the magnitude of ground vibration, typically a root mean square (RMS) value in one-third octave bands for detailed design situations, \( S(f) \) is the source strength, \( P(f) \) characterizes the propagation path, and \( R(f) \) the receiver. ISO 14837-1 [6] stipulates that each of these terms should be further divided into relevant components, which interact and can only be assumed uncoupled in some situations for simplified models. Since this paper focusses on the prediction of the free field velocity, the receiver term \( R(f) \) is not discussed in the following.

In the methods by SBB [56] and Madshus et al. [57], the source magnitude \( S(f) \) in equation (7) is eliminated by taking measurements at a reference distance \( r_0 \), leading to:

\[ A(r,f) = A(r_0,f) \frac{P(r,f)}{P(r_0,f)} \]  

and the ratio \( P(r,f)/P(r_0,f) \) is computed assuming attenuation with distance of the form \( r^{-n} \) for some value of \( n \) which may depend on frequency. Madshus et al. [57] propose a reference distance of 15 m whereas a much shorter distance of 3 m is given in Kuppelwieser and Ziegler [56]. At such a small distance from the source, however, quasi-static excitation will significantly contribute to the response and equation (8) does not allow for a valid extrapolation of the response to larger distances. Again, vibration velocities \( A(r_0,f) \) known from past measurements can be used for predictions at other sites using equation (8), correcting for train speed [57] and track quality [56], [57]. Madshus et al. [57] also indicate that the reference vibration level \( A(r_0,f) \) and the exponent \( n \) in the attenuation law depend on the soil type.

The Detailed Vibration Assessment of the U.S. Department of Transportation provides a prediction of the one-third octave band RMS values of the vibration velocity on a logarithmic dB scale. This transforms the product in equation (7) in a summation:

\[ L_v(x') = L_F(X,x') + T_{ML}(X,x') \]  

where \( L_v(x') \) is the vibration velocity level at a location \( x' \), \( T_{ML}(X,x') \) the line source transfer mobility characterizing the propagation path similarly to \( P(f) \) in equation (7) and \( L_F(X,x') \) is equivalent to the force density \( S(f) \) of equation (7). The line source transfer mobility \( L_F(X,x') \) is determined from field measurements by adding contributions from incoherent point sources at different positions \( x_k \) along the track, which are collected in the vector \( X \) (figure 5a):

\[ T_{ML}(X,x') = 10 \log_{10} \left[ \frac{L_F(X,x')}{n_s} \sum_{k=1}^{n_s} 10^{\frac{3dB}{10} |x_k-x'|} \right] \]  

The source positions \( x_k \) are typically chosen uniformly spaced along the track and should cover the total length of the train. When the spacing between the source points is chosen to be sufficiently small, the line source transfer mobility for a given receiver position \( x' \) will converge to a value which only depends on the total train length.

The force density \( L_F(X,x') \) is determined indirectly by subtracting the line source transfer mobility from the vibration velocity level:

\[ L_F(X,x') = L_v(x') - T_{ML}(X,x') \]
Due to this indirect determination of the force density, the latter depends on the receiver position $x'$, as well as on the positions $X$ of the incoherent point sources considered for the determination of the line source transfer mobility. This is counter-intuitive, as one would expect the force density to depend only on the excitation of the track by the train, in a similar way as the axle loads considered in the numerical prediction of the vibration velocity in equation (1). However, equation (9) or, equivalently, equation (7) that provides the general framework for empirical models, only hold approximately in case of a moving load due to the Doppler effect. As a result, the source magnitudes $L_f(X,x')$ and $S(f)$ in equations (9) and (7), respectively, provide distance dependent equivalent strengths for a stationary load to result in the same vibration level as the passing train.

A prediction at a given assessment site according to equation (9) is obtained by determining the line source transfer mobility and adding a suitable force density, based on data from another site or previous experience. When a prediction is needed prior to construction of the track at the assessment site, the line source transfer mobilities can only cover vibration transmission in the soil (figure 5b) and will not account for the load distribution by the track. This needs to be accounted for when determining the force density required for the prediction of the vibration velocity level according to equation (9). Furthermore, the indirect determination of the force density according to equation (11) results in a dependence on the receiver distance $x'$, requiring matching receiver positions in the line source transfer mobility and the force density in equation (9). In the next subsection, these two particular aspects are investigated for the site at Lincent.

3.2 Case study: site at Lincent

At Lincent, the line source transfer mobility has been determined by means of hammer impacts on the track (figure 5a) as well as by means of hammer impacts on a small aluminum foundation placed adjacent to the track (figure 5b) [8]. Figure 6a shows the force density for an Intercity train passing at 198 km/h as determined from the line source transfer mobilities obtained by means of impacts on the track. Leaving out of consideration the point at 6 m from the track where a significant contribution from quasi-static excitation is expected, the force density levels only show a relatively small dependence on the distance from the track in the frequency range between 16 Hz and 125 Hz. The observed differences are generally smaller than the differences between individual passages for the same train type and differences found for multiple measurement lines at the same site. These results suggest that adopting a force density level independent of the distance to the track will not significantly deteriorate the prediction accuracy in this case.

Figure 6b shows the force density for a similar train type and speed, but now determined from line source transfer mobilities obtained by means of impacts on a small foundation adjacent to the track. These force densities will also compensate for the fact that the line source transfer mobilities do not account for the load distribution by the track. It is expected that this results in smaller force density levels which, when added to the line source transfer mobilities, result in the same vibration velocity level during a train pass-by. As the receiver locations $x'$ are the same in both cases, the force densities will also compensate, however, for the different distances between the line source on the track (figure 5a) and the line source adjacent to the track (figure 5b). Since this distance is much smaller in the latter case, it is expected that this will lead to a further reduction of force density levels. This is confirmed by comparing the force density levels at 6 m from the track in figures 5a and 5b. At larger distances from the track, the difference in distance between the line source and the receiver point becomes small relative to the total distance. In this case, the force density levels in figures 6a and 6b are similar. This suggests that the effect of track filtering at the site is not very large.

3.3 Discussion

Empirical prediction methods have largely shown their value in practice by providing reasonable estimates of vibration velocity levels. Compared to numerical methods, empirical methods relying on measured transfer functions have the advantage of inherently accounting for the characteristics of the vibration transmission at a given site. Simplifying assumptions in numerical models, such as the horizontal nature of the soil stratification, and identification of the dynamic soil characteristics from in situ geophysical tests are avoided in this way.

Crucial for the prediction quality is the availability of a suitable source characterization, either in terms of a force density or a reference vibration level. Equation (6) that relates the dynamic train loads to the wheel and track unevenness shows that this requires a match in the type and level of excitation, train characteristics, as well as characteristics of the track and the soil. Matching soil conditions are particularly important in the low frequency range where they significantly affect the track receptance [61] and, therefore, the dynamic train loads. This implies that the method cannot be used in
situations where new train or track types are implemented or at sites with deviating soil conditions. A study of prediction errors arising from a mismatch in soil conditions for the Detailed Vibration Assessment of the U.S. Department of Transportation was recently presented by Verbraken et al. [62].

4 HYBRID PREDICTION MODELS

4.1 General principle

The use of empirical methods is limited to those cases where a suitable characterization of the source strength (force density, reference vibration level) and the vibration transmission (line source transfer mobility, attenuation law) is available. The combination of empirical and numerical methods in a hybrid or semi-empirical prediction procedure allows alleviating some of these limitations.

Combining a numerical prediction of the source strength with an experimental assessment of the local transfer of vibration seems particularly appealing. The numerically predicted source strength allows new train types, track conditions or even track retrofitting to be considered, while experimental transfer functions allow accurately accounting for the local characteristics of vibration transmission. When cast in the general framework of the Detailed Vibration Assessment of the U.S. Department of Transportation, such a hybrid prediction $L_{\text{HYB}}(x')$ of the vibration velocity level would be obtained as follows:

$$L_{\text{HYB}}(x') = L_{\text{F}}(x',x) + TM_{\text{L}}(x',x')$$ (12)

where $TM_{\text{L}}(x',x')$ is the experimental line source transfer mobility characterizing the propagation path and $L_{\text{F}}(x',x)$ is the predicted force density level. The force density level can be computed indirectly by simulation of the Detailed Vibration Assessment Procedure:

$$L_{\text{F}}(x',x) = L_{\text{F}}^\text{NUM}(x,x') - TM_{\text{L}}^\text{NUM}(x,x')$$ (13)

where the evaluation of the vibration velocity level $L_v^{\text{NUM}}(x')$ requires the prediction of the free field response during a train pass-by according to equation (2) or (4). The line source transfer mobility $TM_{\text{L}}^\text{NUM}(x,x')$ is calculated from the free field response to impacts applied on the track or adjacent to the track, depending on how the experimental line source transfer mobility in equation (9) has been determined. As an alternative to the indirect determination of the force density in equation (13) by numerical simulation of the experiments, analytical expressions of these terms can be derived [63]. In both cases, however, the application of the hybrid procedure requires as a minimum the identification of the dynamic soil characteristics from in situ geophysical tests. Even when the line source transfer mobility is characterized experimentally, these parameters are important for an accurate prediction of the force density.

By introducing the predicted force density level in equation (13) in the hybrid prediction according to equation (9), the following expression is obtained:

$$L_{v_{\text{HYB}}}(x') = L_v^{\text{NUM}}(x') - TM_{L}^{\text{NUM}}(x,x') + TM_{L}^{\text{EXP}}(x,x')$$ (14)

which shows that this form of hybrid prediction corresponds to applying a correction $\Delta TM_{\text{L}}(x,x')$ to the numerically predicted vibration velocity level based on the experimental line source transfer mobility.

4.2 Case study: site at Lincent

The hybrid procedure is now illustrated for the site at Lincent. The vibration velocity level is predicted according to equation (12) based on a numerical source strength and experimental line source transfer mobility [8]. The experimental line source transfer mobility is calculated based on impacts applied on the track, a similarly computed line source transfer mobility has been used for the indirect calculation of the source strength according to equation (13).

![Figure 7](image_url)

Figure 7. Vibration velocity level at (a) 6 m, (b) 12 m, (c) 24 m, and (d) 48 m for an IC train (198 km/h): measured result (black line), numerical prediction (dark grey line), and hybrid prediction, indirectly predicted force density (light grey line).

Figure 7 compares the resulting hybrid and numerical predictions to the measured vibration velocity level for an InterCity train passing by at a speed of 198 km/h. Both predictions are relatively accurate; deviations with the measured vibration velocity are mostly limited to $\pm 6$ dB with a maximum of 10 dB at frequencies below 16 Hz. Differences between both kinds of predictions and the measured response are of the same order of magnitude as those observed between individual passages of the same train type and those for different measurement lines at the same site. The purely numerical approach therefore provides satisfactory results and a significant improvement cannot reasonably be expected from the hybrid approach. The success of the numerical approach is largely thanks to the fact that this site was selected for model validation because of its regular topography (reasonably flat terrain, straight track, ...) and the extensive in situ test campaign for estimation of the model parameters. The hybrid approach will probably be more accurate than a purely numerical approach at sites with a less regular geometry or when less extensive tests are performed. In this case, a substantial improvement is expected from the correction $\Delta TM_{\text{L}}(x,x')$ applied to the numerical prediction of the vibration velocity level $L_v^{\text{NUM}}(x')$ in equation (14).
4.3 Discussion
Hybrid prediction methods seem particularly appealing in the case of new tracks, new rolling stock or modifications to existing tracks. Numerical models can here be used to assess the influence of rolling stock characteristics and track design on ground vibration, avoiding the need for in situ tests possibly requiring the construction of test tracks. In the case of new tracks, a numerically predicted source strength can be introduced in equation (7) and combined with an experimental transfer function or attenuation law that inherently takes into account vibration transmission. An additional correction may be needed when new lines are planned and the transfer functions are determined before the track is constructed. If modifications are made to the rolling stock or track structure, numerical modelling can be used to evaluate the change in source strength and vibration transfer in equation (7), so as to correct the existing vibration level. Accurately predicting relative levels of vibration due to changes in track or vehicle parameters is expected to be much more reliable than making predictions in absolute terms [44]. Hybrid prediction methods may therefore allow the accuracy of numerical models to be improved by an adequate characterization of the vibration transfer, while at the same time providing the flexibility of numerical models to assess a wide range of rolling stock and track parameters.

5 CONCLUSION
Numerical methods for the prediction of railway-induced ground vibration have allowed for a better understanding of the governing physical phenomena but need to be applied with care in practical design situations because of the simplifying assumptions involved (type of excitation, horizontal soil stratification, free field conditions). More research is needed to quantify prediction uncertainties as these are essential for robust predictions in engineering practice and design. Empirical methods allow the particularities of vibration transmission at a given site to be taken into account, but for new-build or modified situations their application is limited to cases where prior experience is available. The limits of empirical prediction methods may be clarified by numerical simulations of the proposed procedures. Hybrid models, combining results from numerical and empirical predictions, may be developed to overcome limitations of both methods.

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REFERENCES
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