Output-only measurement-based parameter identification of dynamic systems subjected to random load processes

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ABSTRACT: In this paper a new output-only measurement based method is proposed which allows identifying the modal parameters of structures subjected to natural loads such as wind, ocean waves, traffic or human walk. In contrast to the existing output-only identification techniques which model the unmeasured load as white noise process, statistical information about the dynamic excitation, e.g. obtained by measurements of the wind fluctuations in the vicinity of the structure, are taken into account which improve the identification results as well as allow identifying the unmeasured load process exciting the structure. The identification problem is solved on basis of a recently developed method called H-fractional spectral moment (H-FSM) decomposition of the transfer function which allows representing Gaussian random processes with known power spectral density (PSD) function as output of a linear fractional differential equation with white noise input. In the present work the efficiency and accuracy of this method is improved by the use of an alternative fractional operator. Based on the H-FSM decomposition a state space representation of arbitrarily correlated Gaussian processes is proposed which neither requires the factorization of the PSD function nor any optimization procedure. Combined with the state space model of the structure, it leads to an overall model with white noise input, which can be efficiently combined with any state-space model-based parameter identification algorithms such as the (weighted) extended Kalman filter algorithm used here. The method is applied for the identification of the stiffness and damping parameters of a three story shear building subjected to wind turbulences with von Kármán velocity PDF function.

KEY WORDS: Extended Kalman Filter; Parameter identification; Load identification; Fractional spectral moments; Digital filter; Turbulence spectra; Time series models; Stationary Gaussian random process.

1 INTRODUCTION

1.1 Motivation

Forced vibration tests on structures of civil engineering interest are expensive and time consuming as they are performed using impact hammers or heavy shakers, needed to excite the modes of interest with sufficient energy. Moreover, they often require a temporary out of service state of the structure which causes increments of costs. Conversely, ambient vibration tests (AvT) can be conducted continuously in time measuring the structural response for large time intervals using the excitation of both natural and/or service loads such as wind, traffic, seismic ground motion or human walk. Such loads are caused by the superposition of multiple inputs and thus lead to a broad-band excitation of a significant number of vibration modes.

In recent years, AvT gained great attention in civil engineering in the scope of modal parameter identification, model updating as well as damage detection and health monitoring. A detailed literature review can be found in [1].

In case that the unmeasured system’s excitation can be modeled as a stochastic white noise process, various experimental modal identification methods for output-only measurements are available. Whiteness implies that the process is uncorrelated and its power spectral density (PSD) function is constant over all frequencies. From a physical point of view, the white noise process cannot exist in nature as the constant PSD leads to a process with infinite variance corresponding to an unbounded, infinitely fast varying signal. However, the white noise assumption is justified, if the PSD function of the input process is flat within the system’s bandpass, i.e. the frequency range in which the system is vibrating predominately.

In case of non-white excitations, the parameter identification problem is more complex and classical ambient vibration identification methods lead to poor identification results. In this case, the parameter identification problem to be solved consists of two parts, namely: i) the digital simulation of the random load process; and ii) the estimation of the structural response to the random load using output-only model identification techniques. In case that both parts are handled individually, numerous methods for the system identification as well as for the simulation of stochastic processes are available, but for the solution of the combined problem few techniques appeared in literature.

This leads to the motivation to address the identification problem of structures subjected to arbitrarily correlated load processes. Similar to the classical ambient vibration identification techniques, the proposed method is based on output-only measurements of the system response, while the actual load process exciting the structure remains unmeasured. Though, in order to include the load process in the identification algorithm, it is assumed that information about the statistics of the process are available, e.g. from additional measurements of the wind velocity fluctuations in the vicinity of the structure.
1.2 Parameter identification under correlated loads

The solution of the identification problem under correlated loads is based on a concept found in [2] which allows introducing colored processes in the Kalman filter algorithm by state space augmentation: It is based on the spectral factorization theorem which allows modeling a wide sense stationary random process with given rational PSD function as an output of a linear system, a so-called shaping filter, with white noise input. This system can then be added to the original system by augmenting the state space representation leading to an overall linear system driven by white noise once again to which standard tools as the Kalman filter based on linear system theory for response analysis, optimization, and design of active control devices can be applied.

Though, difficulties arise if the PSD function is of non-rational form, as in this case, the spectral factorization is difficult and in general not possible in analytic form. A detailed literature review on the state-of-the-art of classical approaches for the digital simulation of stationary Gaussian random processes with target PSD function including the spectral representation methods, parametric time series models such as ARMA-based approaches and the spectral factorization theorem as well as a critical discussion with respect to computational efficiency, applicability and restrictions of these methods can be found in [1].

In this paper, the linear filter problem is solved on basis of a modification of the recently developed method which allows representing PSD and autocorrelation (AC) function in closed form by means of a generalized Taylor expansion using fractional spectral moments (FSMs) [3], [4]. Based on this concept, a state space representation of arbitrarily correlated load processes is derived in analytic form which neither requires the factorization of the PSD nor any optimization procedure and which can be easily combined with common state space model based system identification methods such as the Kalman filter algorithm used here. Moreover, it shall be highlighted, that the method is i) applicable to a wide range of Gaussian processes of both, short and long memory; ii) it allows the simultaneous estimation of the structural parameters and the unmeasured load process; and iii) due to its analytic form its implementation is straightforward.

2 DIGITAL SIMULATION OF GAUSSIAN RANDOM PROCESSES

The focus of this paper lies on the stochastic excitation by wind turblences. A large number of actual measured data indicates that the dynamic wind fluctuations can be modeled as stationary Gaussian random process neither with fixed real part $\rho$ chosen such that the integral converges, that is with the real part $\gamma_0 < Re \gamma < \gamma_1$. In some cases it cannot be calculated in analytical form, but as the Gamma function $\Gamma(\gamma)$ decays exponentially fast in vertical strips, i.e. for $Im \gamma \rightarrow \infty$, depending on the decay of $\Pi_H(\gamma)$, the integral might be truncated along the imaginary axis with constant real part $\rho$. Defining $\gamma_k = \rho + i\Delta \eta$, the integral is calculated up to a certain value $\Delta \eta = \pm m \Delta \eta$ discretizing the interval into $2m + 1$ small increments $\Delta \eta$ yielding the approximation

$$ F(t) \approx \frac{\Delta \eta}{4m} \sum_{k=-m}^{m} \Pi_H(-\gamma_k)(1^{1-\gamma_k}W)(t) $$

Hence, the main difficulty in the simulation of the process lies in the efficient calculation of the Riesz fractional integral $(1^{1-\gamma_k}W)(t)$ of the Gaussian white noise process $W(t)$.

Assuming that the latter is discretized on a finite interval $[0, n\tau]$, where $j, n \in \mathbb{N}, \tau > 0$ and zero elsewhere the Riesz fractional integral operator can be approximated in term of the Grünewald-Letnikov (GL) series yielding [8]

$$ (1^{1-\gamma_k}W)(\tau) = \sum_{j=0}^{n-\gamma_k} \frac{\Gamma(j+\gamma_k)}{\Gamma(\gamma_k)} W(\tau - j\tau) + \sum_{j=-\gamma_k}^{n-\gamma_k} \frac{\Gamma(\gamma_k-j)}{\Gamma(\gamma_k)} W(\tau + j\tau) $$

which can be sought as generalization of the backward difference operator to complex orders, where the coefficients
are given analytically by
\[
\alpha_k(y) = \frac{(-1)^k y^{1+y} f(1-y)}{2 \cos(\pi y/2) f(1+k+y) f(1-y-k)} \quad k \in \mathbb{N}
\] (8)

It must be stressed that the first sum in eq.(7) includes the weighted sequence of past white noises up to the actual time \(j \tau\), while the second sum represents the weighted sequence of future white noises. The dependence on the future, that is the non-causality of the generated process, is caused by disregarding the imaginary part of the transfer function in eq. (2). However, due to the linearity of the underlying differential equation and the statistical independence of the Gaussian white noise process, the output remains a strict stationary Gaussian process.

Eq.(7) can be efficiently calculated in matrix form by
\[
Z(y) = A(y)W \quad \text{where} \quad [4]
\]
\[
Z(y) = \begin{bmatrix} (I^TW)(0) \\ (I^TW)(\tau) \\ \vdots \\ (I^TW)(nt) \end{bmatrix}, \quad W = \begin{bmatrix} G(0) \\ G(\tau) \\ \vdots \\ G(nt) \end{bmatrix},
\]
\[
A(y) = \begin{bmatrix} 2\alpha_0(y) & \alpha_1(y) & \cdots & \alpha_n(y) \\ \alpha_1(y) & 2\alpha_0(y) & \cdots & \vdots \\ \vdots & \vdots & \ddots & \alpha_1(y) \\ \alpha_n(y) & \cdots & \alpha_1(y) & 2\alpha_0(y) \end{bmatrix}
\] (9)

The white noise process \([W(t)]\) discretized in the interval \([0, j \tau]\) is described by the realizations of a zero-mean Gaussian random process \(G(0), G(\tau), \ldots, G(nt)\) with standard deviation \(\sqrt{\tau \sigma}\). Finally, the vector of the colored load process \(F = [F(0), F(\tau), \ldots, F(nt)]^T\) is obtained by
\[
F = \frac{\Delta}{4\pi} \sum_{k=-m}^{m} \Pi_H(-y_k)Z(1-y_k) = \sum_{k=-m}^{m} h(y_k)W \quad \text{(10)}
\]
where \(h(y_k) = \Delta [/(4\pi)] \Pi_H(-y_k) A(1-y_k)\) denotes the matrix transfer function.

2.1 Applying the short memory principle

It must be noted that the evaluation of the fractional integral approximation in eq.(7) requires at each time step the re-calculation and summation of every previous time point and thus becomes increasingly cumbersome for large times \(t = nt \gg 0\) where a significant numbers of computations and memory storage is needed. However, Podlubny [5] observed, that the GL coefficients \(\alpha_k(y)\) defined in eq.(8) decay with increasing value \(k\) and can be set to zero for \(k > p\). Instead of taking into account the complete process’s history \(M\). The effect of the chosen sampling interval \(\tau\) and the considered memory \(M\) on the accuracy of the simulation is investigated on the example of the exponentially correlated random process with the AC function \(R(t) = \sigma^2 \exp(-a|t|)\) choosing \(a = 0.2\) and \(\sigma = 100\) [N]. As a measure of accuracy, the mean square error (MSE) between the sample AC function of the generated time series and the analytic function \(R(t)\) is calculated over a finite length \(t = [0, 50]\) [s] where the AC function drops below a value of \(R(50) = 0.0045\) and thus can be considered to be zero.

In order to investigate the truncation and discretization error 5000 samples of fixed length \(T = 200\) [s] each are generated as weighted sum of \(p\) past and future Gaussian white noises by means of eq.(10) and the sample AC function is calculated. Then two tests are conducted:

i) Test 1: The discretization error is investigated by keeping the considered memory of the process constant setting \(M = 25\) [s] and varying the sampling interval \(\tau\) between 0.025 and 0.5 [s].

ii) Test 2: The truncation error is investigated by varying the memory \(M\) between 2.5 and 25 [s], while the sampling interval is kept constant setting \(\tau = 0.025\) [s] in order to keep the discretization error small.

In both cases the number of coefficients is set \(p = M/\tau\).

Figure 1 (top) illustrates that with increasing sampling interval \(\tau\), the variance of the process is over-estimated while the tail of the AC function is approximated in all cases with comparable accuracy up to a lag of 24 [s]. From Figure 1 (bottom) it is evident that a too short length \(M\) mainly causes the AC function to decrease much faster than the target function and leads to small errors in the peak value. It can be concluded that the sampling interval mainly influences the scaling, that is the variance of the generated process, while the choice of the considered memory \(M\) affects the range, in which the AC function is approximated well.

In order to reduce the discretization error in [1] the use of the centered GL operator introduced in [6] is proposed which represents a generalization of the centered difference operator to complex orders.
The approximations follow the form of eq.(7), but with different coefficients given by

\[ a_{c,k}(y) = r^{-y/2} \prod_{i=1}^{k} (1-y/2i) \quad k \in \mathbb{N}. \tag{11} \]

Once again the two tests are conducted: Figure 2 (top) illustrates the accuracy of the centered GL form is almost independent from the discretization width \( \tau \) while the error introduced by the truncation of the series is comparable to the one of the classical GL representation as illustrated by Figure 2 (bottom). Though, as the required \( p = M/\tau \) number of coefficient increases inverse proportionally to the discretization step width; the new representation leads to a significant reduction of the model order \( p \) [1].

3 AMBIENT PARAMETER IDENTIFICATION USING THE KALMAN FILTER

Ambient identification algorithms aim to provide robust estimates of structural features which are indirectly observed through output-only measurements. These might either be the system’s states (e.g. displacements/accelerations) or parameters of the model (e.g. stiffness, damping coefficients).

The Kalman Filter is an optimal recursive procedure which provides an estimate of the desired variables such that the error is minimized statistically in the mean square sense and can be applied in case of Gaussian white noises for the estimation of state variables.

Let \( \mathbf{x} \in \mathbb{R}^n \) be the system’s state vector, \( \mathbf{z} \in \mathbb{R}^m \), \( \mathbf{w} \in \mathbb{R}^u \) and \( \mathbf{v} \in \mathbb{R}^m \) two independent vectors of Gaussian zero mean white noises with covariance matrices \( Q \in \mathbb{R}^{uxn} \), \( R \in \mathbb{R}^{mxxm} \), representing uncertainties in the model and measurements, respectively, \( T \in \mathbb{R}^{mxxn} \), \( S \in \mathbb{R}^{mxxu} \), \( H \in \mathbb{R}^{mxxn} \) system’s matrices, and \( \mathbf{z}_{k+1} \) the searched vector of estimates at time step \( k+1 \). Assuming a linear model for the system’s dynamics under additive noise in state space form

\[ \mathbf{x}_{k+1} = T\mathbf{x}_k + \mathbf{w}_k; \tag{12} \]

and a linear model with additive noise for the measurements

\[ \mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k \tag{13} \]

The Kalman filter provides recursively an estimate of the desired variables in such a manner that the error is minimized statistically by combining i) prior knowledge about the system and measuring device dynamics, ii) statistical information about both, the measurement errors and the process noise and iii) any available information about initial values of the variables of interest. The optimal posterior estimate \( \mathbf{\hat{x}}_{k+1} \) is obtained by minimizing the error \( \mathbf{\hat{e}}_{k+1} = \mathbf{x}_{k+1} - \mathbf{\hat{x}}_{k+1} \) in the mean square sense, i.e. \( \Sigma_{k+1} = \mathbb{E}[\mathbf{e}_{k+1}\mathbf{e}_{k+1}^T] \rightarrow \text{min}. \) by the following prediction - correction procedure (e.g. [2])

**Prediction (Time – Update)**

\[ \mathbf{\hat{x}}_{k+1 | k} = T\mathbf{x}_k \]

\[ \Sigma_{k+1 | k} = T\Sigma_k T^T + SQS^T \tag{14a} \]

**Correction (Measurement – Update):**

\[ \mathbf{K}_{k+1} = \Sigma_{k+1 | k} T^T (T\Sigma_k T^T + R)^{-1} \]

\[ \mathbf{\hat{x}}_{k+1} = \mathbf{\hat{x}}_{k+1 | k} + \mathbf{K}_{k+1} (\mathbf{z}_{k+1} - H \mathbf{\hat{x}}_{k+1 | k}) \]

\[ \Sigma_{k+1} = \Sigma_{k+1 | k} - \mathbf{K}_{k+1} T \Sigma_{k+1 | k} T^T \tag{14b} \]

The residual \( d_{k+1} = \mathbf{z}_{k+1} - \mathbf{\hat{x}}_{k+1} \) between the actual measurement \( \mathbf{z}_{k+1} \) and the predicted one \( \mathbf{\hat{x}}_{k+1} = H\mathbf{\hat{x}}_{k+1 | k} \) by means of eq.(14) can be interpreted as the part of the measurement that contains new information about the state and thus is sometimes denoted as innovation. It can be shown that under optimal conditions, the innovation is a zero mean Gaussian process with covariance matrix \( T\Sigma_k T^T + R \) and will be used in section 3.3 to check the consistency of the filter.

3.1 Load identification by state space augmentation

In case the input noise process \( \mathbf{w}_k \) in eq.(12) is not white, i.e. if the structure is excited by a wind load process \( \{F(t)\} \) with known PSD, the Kalman filter can be applied by a procedure found in Lewis. The concept is that the state space model in eq.(12) is augmented by a set of linear filter equations in the form

\[ y_k = A\mathbf{y}_k + B\mathbf{w}_k \]

\[ F_k = C\mathbf{y}_k \tag{15} \]
with additive white noise as input and the sought univariate load process as output. Defining the augmented vector \( \mathbf{x}_{a,k} = [x_k, y_k]^T \) and introducing the linear model (15) in eq.(12) yields
\[
\mathbf{x}_{a,k+1} = \mathbf{T}_a \mathbf{x}_{a,k} + \mathbf{S}_a \mathbf{w}_k; \\
\mathbf{z}_k = \mathbf{H}_a \mathbf{x}_{a,k} + \mathbf{v}_k
\]
with
\[
\mathbf{T}_a = \begin{bmatrix} T & \mathbf{SC} \\ 0 & \mathbf{A} \end{bmatrix}; \quad \mathbf{S}_a = \begin{bmatrix} \mathbf{0} \\ \mathbf{B} \end{bmatrix}; \quad \mathbf{H}_a = \begin{bmatrix} \mathbf{H} \end{bmatrix}
\]
which is once again a linear system driven by white noise to which the Kalman filter equations (14) can be applied in order to estimate the states of the original system as well as the load process exciting the system.

### 3.2 Generalized state space representation of colored random processes

As mentioned previously finding a linear model of the process with arbitrarily PSD function in the form eq.(15) is difficult as the spectral factorization problem can be solved analytically, in general, just in the rational case.

Based on the result given in eq.(10), in the following, a general state space representation for colored load processes is developed. It must be stressed that it is valid for arbitrary correlated Gaussian processes and can be given directly once the H-FSMs in eq.(3) have been calculated.

Using the considerations in section 2.2 one steady state realization \( F_j = F(\mathcal{T}) \), \( \mathcal{T} = [p + 1, n - p] \) of the discrete load process \( F(t) \), is given by
\[
F_j = \frac{\Delta n}{4\pi} \sum_{k=-m}^{m} \mathcal{H}(-\gamma_k) \begin{bmatrix} \alpha_{c,p}(1 - \gamma_k) \\ \alpha_{c,p-1}(1 - \gamma_k) \\ \vdots \\ \alpha_{c,p-1}(1 - \gamma_k) \\ \alpha_{c,p}(1 - \gamma_k) \end{bmatrix}^T \begin{bmatrix} G_{j-p} \\ G_{j-p+1} \\ \vdots \\ G_{j-p} \\ G_{j-p+1} \end{bmatrix}
\]
which reduces to
\[
F_j = \begin{bmatrix} \beta_p \\ \beta_{p-1} \\ \vdots \\ \beta_p \\ \beta_{p-1} \\ \beta_p \end{bmatrix}^T \begin{bmatrix} G_{j-p} \\ G_{j-p+1} \\ \vdots \\ G_{j-p} \\ G_{j-p+1} \end{bmatrix} = \mathbf{b} \mathbf{W}_j
\]

It must be stressed that the result in eq.(19) coincides with a (non-causal) moving average (MA) representation of the process. Though, in contrast to classical approaches where the coefficients of the MA models are calculated by solving a non-linear optimization problem, it shall be highlighted that here the coefficients are given analytically.

Noting that a MA representation is obtained, it is now straightforward to define a state space representation in the form eq.(15) defining
\[
\mathbf{y}_k = \begin{bmatrix} G_{k-p} & G_{k-p+1} & \ldots & G_k & \ldots & G_{k+p-1} & G_{k+p} \end{bmatrix}^T
\]
as state vector and by substituting the system matrices \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{C} \)
\[
\mathbf{A} = \begin{bmatrix} \mathbf{0}_{p \times 1} & \mathbf{I}_{p \times p} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{p \times 1} \\ 1 \end{bmatrix} \\
\mathbf{C} = \left[ \beta_p, \beta_{p-1}, \ldots, 2\beta_0, \beta_{p-1}, \beta_p \right]
\]
where \( \mathbf{I}_{n \times m} \), \( \mathbf{0}_{n \times m} \) are the identity and zero matrix, respectively. That is, while the state equation in eq.(15) leads to a forward shift of the white noise process, the measurement equation generates the process with target PSD by weighting the updated noise sequence by the time invariant coefficient vector \( \mathbf{C} \).

### 3.3 Parameter identification under colored loads

In order to apply the method for the identification of the stiffness and damping parameters a further modification is needed. Following the approach of the extended Kalman filter (EKF), the state \( \mathbf{x}_{a,k} \) has to be extended to include the unknown modal parameters \( \mathbf{p}_k \) leading to a nonlinear system equation of the extended state \( \mathbf{x}_{ext,k} = [\mathbf{x}_{a,k}, \mathbf{p}_k]^T \) in the form
\[
\mathbf{x}_{ext,k+1} = f(\mathbf{x}_{ext,k}) + \mathbf{S}_a \mathbf{w}_k \\
\mathbf{z}_k = \mathbf{h}(\mathbf{x}_{ext,k}) + \mathbf{v}_k
\]
as the system matrices \( \mathbf{T}_{ext,k} \) and \( \mathbf{H}_{ext,k} \) depend nonlinearly on the unknown parameters \( \mathbf{p}_k \).

In case of weak nonlinearities the identification problem can be solved using the EKF which linearizes about the current state estimate by applying a first order Taylor expansion of eq.(22) near the current state estimate leading to the time variant extended system matrices
\[
\frac{\partial f(\mathbf{x}_{ext,k})}{\partial \mathbf{x}_{ext,k}} \mathbf{x}_{ext,k} + \frac{\partial \mathbf{W}_j}{\partial \mathbf{x}_{ext,k}} \mathbf{x}_{ext,k} = \mathbf{y}_k
\]
\[
\frac{\partial \mathbf{h}(\mathbf{x}_{ext,k})}{\partial \mathbf{x}_{ext,k}} \mathbf{x}_{ext,k} + \frac{\partial \mathbf{W}_j}{\partial \mathbf{x}_{ext,k}} \mathbf{x}_{ext,k} = \mathbf{v}_k
\]
to be calculated at each time step.

In [7] the stability and convergence of the EKF is investigated with respect to the initial state estimates and covariance matrices and a weighted global iteration procedure is introduced into the Kalman filter algorithm containing an objective function to estimate the stability. That is, while the iterative scheme improves the accuracy of the approach, especially if the first guess of the parameters to be identified is poor, the calculation of the objective function allows assessing the accuracy of the filter and avoids the divergence to erroneous identification results.

The algorithm can be summarized as follows:
First, the Kalman filter is initialized choosing the initial state estimate and covariance matrix \( \hat{x}_1^0, \Sigma_1^0 \) as well as defining the process noise matrix \( Q \). The statistics of the measurement noise \( R \) are assumed to be time-invariant and known. Then the EKF is run using a finite measurement record of length \( T \) [s] chosen such, that the final estimates \( \hat{x}_T^T, \Sigma_T^T \) converges.

They are used for the initialization of the next iteration loop \( j + 1 \) that is, setting \( \hat{x}_0^j = \hat{x}_1^j, \Sigma_0^j = W \cdot \Sigma_1^j \) where \( W \) is a weighting factor. In [7] it is observed that a large initial covariance is favorable in order to accelerate the extended Kalman filter’s convergence, but it also might affect the stability of the filter. Thus, an objective function \( \theta \) suggested which is calculated at the end of each iteration loop \( j \) along with the state estimate and error covariance. The iteration is repeated until the prior estimate become essentially constant, that is \( \hat{x}_0^{j+1} \approx \hat{x}_0^j \) or until the objective function \( \theta \) is minimized. The latter is given by [1]

\[
\theta^{j} = \frac{1}{m} \sum_{i=1}^{m} \sum_{k=0}^{N-1} \left( d_{i,(k+1)j+1}^f \right)^2
\]

(24)

where \( N = T/\tau \) denotes the number of sampling points of the measurement record of length \( T \), \( m \) is the dimension of the measurement vector and \( d_{i,(k+1)j+1}^f \) is the \( i \)th component of the posterior residual \( d_{i,(k+1)j+1}^f = z_{i,k+1} - T_{k} \hat{x}_{k+1|j+1} \). That is, the objective function gives the average of all measurement square errors and thus \( \theta_{\text{min}} \) indicates that the global error between each observation and corresponding estimate becomes minimal [7].

4 APPLICATION TO A THREE STORY SHEAR BUILDING

In order to verify the method, the W-EKF algorithm is now used for the identification of the stiffness and damping parameters characterizing the dynamic behavior of a three story shear building depicted in Figure 3 which is excited at the top floor by wind fluctuations with the Kármán velocity PSD function.

Assuming that i) the total mass of the structure is concentrated at the floor levels, ii) the columns are axially rigid and the floor beams are infinitely rigid as compared to the columns, iii) the interstory stiffness is distributed constantly over the stories and iv) the deflection of the structure is independent of the axial forces in the columns, then the structure can be modeled as lumped three degrees of freedom system, corresponding to the horizontal displacements at the floor levels. The system’s dynamics are given by the second order stochastic differential equation in the form

\[
M \dddot{y}(t) + C \dot{y}(t) + K y(t) = F(t)
\]

(25)

where \( M, K \) and \( C \) are the time-invariant mass, stiffness and damping matrices, respectively, given by

\[
M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}; \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix};
\]

(26)

\[
C = \alpha M + \beta K
\]

(26)

assuming that the structural damping is of Rayleigh type. The vectors \( y(t), \dot{y}(t) \) and \( \ddot{y}(t) \) denote the vectors of the horizontal displacements, velocities and accelerations of the floor levels and \( F(t) \) is the unmeasured colored Gaussian wind load process with von Kármán velocity PSD function exciting the structure at the top level. For the columns of the first story a HEB 320 profile and for the upper two levels a HEB 300 profile are chosen leading to the prior stiffness and damping estimates summarized in Table 1 [1]. The true parameters are chosen arbitrarily in such a way that they deviate significantly from the prior estimates.

4.1 Initialization of the load model

Let \( \rho \) be the air density, \( C_D \) the drag coefficients, \( A \) the projection area of the structure, the PSD function of the load process acting on a rectangular cross section has the form

\[
S_p(z, \omega) = (\rho C_D A \tilde{u}_z)^2 |\chi_a(z; \omega)|^2 S_c(\omega)
\]

(27)

where \( S_c(\omega) \) denotes the PSD function of the wind velocity fluctuations related to the wind force by the aerodynamic admittance function

\[
|\chi_a(z; \omega)|^2 = \left( 1 + \left( 2 \omega \sqrt{\beta_a / \tilde{u}_z} \right)^{4/3} \right)^{-2}
\]

(28)

It is assumed that the wind velocity fluctuation can be characterized by the widely used von Kármán spectrum, i.e.

\[
S_p(\omega) = \sigma^2 \left( \frac{2L}{\pi \Delta \eta} \right)^{1/3} \left( \frac{2L}{\pi \Delta \eta} \right)^{2/3} \left( \frac{1+1.5L}{2} \right)^{2/3}
\]

(29)

where \( \sigma, L \) is the standard deviation of the fluctuating component of the wind speed at height \( z \) and the integral turbulence scale lengths, respectively, and \( \tilde{u}_z \) denotes the mean velocity is discussed.

The process is generated by means of the H-FSM decomposition introduced using eq.(10) where the coefficient are calculated according to eq.(11). Setting the sampling interval \( \tau = 0.025 \) [s] and \( \sigma = 2000 \) N the load model is parameterized choosing \( m = 50, \rho = 0.6, \Delta \eta = 0.15 \).

Table 1. True values of the parameters and prior estimates.

<table>
<thead>
<tr>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( k_3 )</th>
<th>( \dot{c}_1 )</th>
<th>( \dot{c}_2 )</th>
<th>( \dot{c}_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior estimates</td>
<td>7.2E6</td>
<td>5.9E6</td>
<td>5.9E6</td>
<td>4.1E4</td>
<td>2.9E4</td>
</tr>
<tr>
<td>True values</td>
<td>0.7( k_1 )</td>
<td>0.8( k_4 )</td>
<td>0.73( k_1 )</td>
<td>0.77( c_2 )</td>
<td>0.73( c_3 )</td>
</tr>
</tbody>
</table>
In order to investigate the simulation accuracy in dependency from the chosen model order, the number of coefficients is varied in the interval $[50, 500]$ and the sample AC function and PSD function is calculated from the generated time series and compared with the analytic ones. Figure 4 illustrates that about 300 coefficients are needed in order to approximate the tail of the AC function accurately while about 200 coefficients are required in order to obtain a good agreement with the analytic PSD function. Consequently, if a smaller number of coefficients is used, the sample PSD function becomes much broader and the variance of the process is underestimated.

In order to approximate the load process with high accuracy, as a first step, load coefficients are chosen. Parameter identification using the $H$-fractional weighted iterated Extended Kalman filter

In the following the stiffness and damping parameters and $c_1 - c_3$, respectively, of the idealized lumped model as well as the unmeasured load process are estimated. A measurement error of 10 [%] of the undisturbed system response is assumed and the model is initialized choosing the parameterization summarized in Table 1. The $H$-fractional H-EKF is run using one measurement sample of length of 5 min. and re-initializing the filter in each iteration by the obtained estimates. The results obtained at the end of each iteration are depicted in Figure 5. It can be observed that the stiffness parameters are estimated with high accuracy leading to a relative estimation error of $<1 \%$, while the estimation of the three damping parameters leads to an error of $12.7 \%, 5.6 \%$ and $0.8 \%$, respectively.

The lower accuracy can be explained by the fact, that in the example considered here, the damping parameters have no significant effect on the modal frequencies and the observed system response. As the update of the parameters is based on the minimization of the error between the obtained measurement and the predicted system’s response, it is in general difficult to identify parameters whose estimation has almost no impact on the prediction error. Thus the obtained results can be considered to be of good accuracy.

Figure 6 shows the estimated time series (left) and - for comparison - the corresponding sample AC functions (right). It illustrates that the method succeeded in identifying the unmeasured load with high accuracy.

For the description of the load process a fairly high order model of $p = 500$ load coefficients was chosen. Of course the question arises how a lower order model affects the estimation accuracy. To this aim, the sampling interval is again set to $\tau = 0.025$ [s] and the number of coefficients is successively reduced from 500 to 50.

The filter is initialized as before and run for the different parameterizations choosing the same measurement record and loading as input. Figure 7 depicts the relative identification errors in [%] in dependence on the chosen number of coefficients. It is observed that, especially in case of the damping estimates, the filter converges to erroneous values if a too small number of coefficients ($p < 100$) is chosen.

In order to evaluate the global performance of the filter for the different parameterizations, in Figure 8 the cumulative errors obtained by summing up the relative errors of the stiffness (left) and damping estimates (right), are given. At first sight it is surprising that the estimation accuracy does...
not increase with increasing model order. Indeed, in case of the stiffness estimation a minimal error is obtained choosing an order of about $p = 200 – 300$ coefficients. It is interesting to note that the required order agrees with the one needed to approximate the PSD function accurately as shown in 4.1. The results of the load identification are illustrated in Figure 9 where the AC function of the actual load process, exciting the structure at the top floor, and the one calculated from the estimated time series by means of the H-WEKF are compared. It is encouraging to note, that the load is estimated with high accuracy even in the case $p = 50$, where the damping estimates diverge. This result is important also from a computational point of view, as it shows that the required number of coefficient is in general evidently smaller than the one needed in order to approximate the AC function with comparable accuracy.

5 CONCLUSIONS

In this paper the weighted H-fractional extended Kalman filter for the treatment of arbitrarily correlated load processes in the scope of parameter identification problems was introduced. The system’s input was represented by means of the H-fractional spectral moment (FSM) decomposition as output of a fractional differential equation with white noise as input. In contrast to other techniques, such as the spectral factorization method or ARMA models, the coefficients for the noise simulation are calculated in analytical form from the FSMs of the linear transfer function. The efficiency and accuracy of this method is improved by the use of the centered Grünwald-Letnikov operator. Furthermore, a generalized state space representation for colored processes was developed, which can be given immediately, once the H-FSMSs of the transfer function are calculated. Augmenting the state space model of the excited system by the linear model corresponding to the load process results in an overall linear system driven by white noise once again to which the (weighted) extended Kalman filter, a commonly used algorithm for recursive parameter identification, can be applied. The method is applied for the identification of the stiffness and damping parameters of a three story shear building excited at the top floor by wind fluctuations with von Kármán velocity PSD function. In contrast to existing time-domain output-only identification methods, both the unknown parameters as well as the unmeasured load process exciting the structure were estimated accurately.

REFERENCES


