Evaluation of displacement demand for unreinforced masonry buildings by equivalent SDOF systems

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ABSTRACT: The determination of the displacement demand for masonry buildings subjected to seismic action is a key issue in performance-based assessment and design of these structures. A technique for the definition of single degree of freedom (SDOF) nonlinear systems representing the global behaviour of multi degree of freedom (MDOF) structural models has been developed. The definition of SDOF systems is based on the dynamic equivalence of the elastic properties (vibration period and viscous damping) and on the comparability with nonlinear hysteretic behaviour obtained by cyclic pushover analysis on MDOF models. Both SDOF and MDOF systems are based on a nonlinear macro-element model able to reproduce the in-plane shear and flexural hysteretic behaviour of pier and spandrel elements. The comparison of the results in terms of maximum displacement obtained for the SDOF system and for the MDOF system demonstrates the feasibility and reliability of the proposed approach. The comparisons on several building prototypes have been carried out based on the results of dynamic analyses performed with a large database of natural records covering a wide range of magnitude, distance and local soil conditions. The SDOF system was used to verify and propose a corrected relation between strength reduction factor, ductility and period (R-µ-T relation) by means of a parametric study. This resulted in a proposed simplified formulation for determining the inelastic displacement of a masonry structure starting from an idealized push-over and an elastic spectrum.

KEY WORDS: SDOF; Displacement Demand; Masonry, R-µ-T relation.

1 INTRODUCTION

The determination of the displacement demand is an essential step in the performance based assessment of structures; at the moment an easy and reliable method to compute it is missing, at least for masonry structures. This is due basically to two reasons. First of all the difficulty to represent the hysteretic behaviour of masonry structures by mean of simplified hysteretic shapes and, second, the very short structural period that does not allow the use of the “equal displacement rule”. Nonlinear dynamic analysis can be used to simulate the seismic response of a masonry structure if a refined model able to reproduce the main failure modes and the hysteretic decay is available. In many cases, the global seismic response of these structures can be modelled by means of an equivalent frame technique which also allows for an easy macroscopic interpretation of the damage pattern [1].

The goal of this work is to develop a single degree of freedom (SDOF) model able to interpret in a synthetic but reliable way the dynamic response of a masonry structure subjected to seismic action. The low computational effort involved with this type of simplified analyses facilitates the use of nonlinear dynamic calculations in parametric studies. The SDOF dynamic analyses can be used to study the correlation between the displacement demand and various seismic intensity measures (e.g. [2]), to perform simplified incremental dynamic analyses (e.g. [3]), to calculate state-dependent fragility curves (e.g. [4, 5]) or to create simplified methods that can estimate the maximum displacement demands on masonry structures (similar to what was done for general structures in [6]). Due to the lack of studies in this last field, in this work the SDOF system was used to verify and propose a corrected relationship between strength reduction factor, ductility and period (R-µ-T relation) by means of a parametric study. This resulted in a simplified formulation for determining the inelastic displacement of a masonry structure starting from an idealized push-over and an elastic spectrum.

2 SIMPLIFIED SDOF MODEL

2.1 Concept of the model

In order to perform simplified nonlinear dynamic analyses, a single degree of freedom model (SDOF) was created. This model is able to synthetically interpret the seismic response of a multi degree of freedom (MDOF) model representing a complete masonry building (if the MDOF is governed by a single dominating mode of deformation, typical in regular buildings). It consists of two macro-elements in parallel characterised by nonlinear behaviours typical of masonry panels. The two elements are connected by a top rigid link. Such approach arises from the aim of to completely decoupling the shear and flexural/rocking mechanisms (or behaviours) consequently facilitating an independent calibration of the parameters. The variables that govern the model are the geometry of the elements, their axial compression, the mechanical characteristics (related to the flexural behaviour in one and shear behaviour in the other), and the inertial mass. The single degree of freedom is the top horizontal displacement of the two elements where the mass is concentrated; the rotation of the top edge of the elements is restrained. Figure 1 reports a simplified illustration of the
SDOF model. The model was created with the TREMURI computer program [7, 8], a nonlinear analysis program capable of performing monotonic and cyclic pushover analyses and time-history analyses of masonry buildings.

![Figure 1](image1.png)

**Figure 1. Scheme of the simplified SDOF model obtained assembling two pier elements, one governed by flexural and one by shear behaviour.**

2.2 Reference experimental campaign

An experimental campaign was used to calibrate the MDOF three-dimensional numerical model, it consisted of a cyclic quasi-static test on an unreinforced masonry structure. The full-scale bricks structure was tested at the laboratory of Department of Structural Mechanics of University of Pavia in 1994 [9].

![Figure 2](image2.png)

**Figure 2. Plan and views of the building specimen used to calibrate the MDOF three-dimensional numerical model [9].**

It is important to notice that façade D is decoupled from the walls A, C and that the pushing forces were equal at the two floor levels.

Thanks to the above mentioned test it was possible to determine the parameters of the MDOF model that allowed a fair simulation of the experimental results. In particular the calibrated masonry mechanical properties are: elastic modulus $E$, shear modulus $G$, compression strength $f_m$, the cohesion $c$ and friction coefficient $\mu_f$. Values are reported in Table 1.

![Table 1](image3.png)

**Table 1. Calibrated masonry mechanical model.**

<table>
<thead>
<tr>
<th></th>
<th>$E$ [MPa]</th>
<th>$G$ [MPa]</th>
<th>$f_m$ [MPa]</th>
<th>$c$ [MPa]</th>
<th>$\mu_f$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3000</td>
<td>500</td>
<td>2.8</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The so calibrated macro-element model was used to create two larger symmetrical models that represent realistic two story buildings with rigid floors. One is characterized by a flexural dominated cyclic behaviour and the other by a shear one.

In order to create a model that exhibits a shear related failure it was chosen to create a squat pier by removing one of the two doors in wall D in one of the models.

The parameters related to the flexural dominated structure ($F$) are indicated with superscript "f" while those related to the shear dominated structure ($S$) with "s". The total masses of the structures are $M_F=224.5$ t and $M_S=228.2$ t, the first modal elastic periods are 0.21 s and 0.17 s and the participating masses for the first vibration mode are 185 t (82.4%) and 188 t (82.3%) respectively. Figure 3 plots the nonlinear pushover analyses used to define an equivalent bilinear capacity curve. These analyses are run considering a limit shear drift equal to 5%, i.e. the lateral strength and stiffness of elements exceeding such drift value is set to zero.

![Figure 3](image4.png)

**Figure 3. F Building (top) and S Building (bottom) pushover curve and equivalent bilinearization.**

The equivalent bilinear curve is defined according to the recommendations reported in the Commentary to the Italian Building Code [10, 11].

The modal participation factors is equal to $\Gamma_f = \Gamma_s =1.2$ in both cases and the masses of the equivalent SDOF are
The periods of vibration are \( T = 0.21 \text{ s} \) and \( T = 0.17 \text{ s} \), respectively.

\[ T = \frac{2\pi}{\sqrt{\frac{M_s \cdot d_M}{F_{My}}} \cdot \frac{1}{T_M}} \]  

(1)

The periods of vibration are \( T = 0.21 \text{ s} \) and \( T = 0.17 \text{ s} \), respectively.

2.3 Calibration of the model

The calibration procedure is described in [12-14]. The first calibration to be performed involves the use of nonlinear static analysis. The goal of this calibration is to obtain a SDOF model with a cyclic behaviour similar to the global one of the reference MDOF model.

It is useful to repeat more than one cycle for each displacement level, the macro-element implemented in the TREMURI model is able to take in account mechanical property degradation. The modal participation factor \( (\Gamma) \) was tested and considered reliable to compare force and displacement of the two systems. A SDOF cyclic pushover is run at the same maximum displacements of the one run for the MDOF. In particular the pushover run for the MDOF model was adaptive, with a force distribution related to the current deformed shape [15]. The parameters that characterize the system were calibrated comparing the static responses of the two analyses. In particular, the variables that have been checked are: the maximum base shear, the initial stiffness, the unloading and reloading stiffness, the shear deformation and stiffness and the area of the hysteresis loops. The properties to be calibrated on the SDOF are: the geometry of the elements, their mechanical characteristics and the load axial forces.

Figure 4 shows the comparison between the MDOF and SDOF hysteresis cycles in terms of base shear/displacement. It is possible to notice how the SDOF models are able to interpret the static behaviour of the MDOF systems. The shear dominated MDOF model has a non-symmetrical force/deformation relationship due to the geometry of the walls D. This obviously could not be simulated by the simplified SDOF models.

In order to create a dynamically calibrated model there is the need to assign a mass \( M_s \) and the Rayleigh damping parameter \( \alpha_s \). The mass \( M_s \) is assigned using the equation of the modal analysis:

\[ M_s = \phi^T M \phi \]  

(4)

where the vector \( \phi \) is the mode shape normalized for \( d_{max}=1 \) and \( M \) is the mass matrix of the MDOF system.

In this way the mass of the SDOF dynamic system is chosen considering the first modal elastic vibration mode. It will represent better the behaviour of structures with a high first mode participation factor.

Concerning the Rayleigh damping to assign to the model, Eq. 5 gives the damping matrix for a general MDOF system according to Rayleigh hypothesis:

\[ C = \alpha_M M + \beta_M K \]  

(5)

where \( \alpha_M \) and \( \beta_M \) are constants with units of \( s^{-1} \) and \( s \), respectively, and \( K \) is the linear stiffness matrix of the structure when the initial tangent stiffness is used. Thus, \( C \) consists of a mass-proportional term and a stiffness-proportional term. The MDOF models have Rayleigh damping parameters \( \alpha_M = 0.43 \) and \( \beta_M = 7.8 \times 10^{-4} \), which gives to the models a nearly constant damping of 2% between the first elastic period and the secant period at collapse (minimum value 1.8%). The damping coefficient of the simplified system is not necessarily equal to the ones used in the MDOF analyses. This could be due to the fact that the SDOF model is not able to account for the contribution of any higher modes of vibration.

The Rayleigh parameters of the two SDOF models were calculated by mean of 10 nonlinear dynamic analyses run on each MDOF model. In the SDOF system the viscous damping matrix \( C \) is a 1-by-1 (scalar). Note that the stiffness-proportional part of \( C \) does not contribute to the total damping force in SDOF systems. The SDOF damping parameters that are able to best replicate the nonlinear analyses output are \( \alpha_F = 1.87 \) and \( \alpha_S = 4.19 \) (which means \( \xi_F = 3\% \) at elastic period for \( F \) structure and \( \xi_S = 5\% \) at elastic period for \( S \) structure).

The slight increase of damping is likely due to the contribution of higher modes that are not taken in account by the SDOF model and the choice to use the elastic modal participation factor \( \Gamma \).

2.4 Validation of the method

In order to validate the method a large database of natural accelerograms was used. Nonlinear dynamic analyses were run using both models (complete and simplified). Maximum displacements obtained by means of two models were plotted.
as a function of different earthquake intensity measures. The equivalent frame modelling has computational times lower than finite element models, but the analyses still take more than one hour to be completed (standard structures run on a normal PC). Conversely, the analyses for the SDOF equivalent models take only a few seconds to be completed.

The database used is SIMBAD and it was obtained by assembling records according to the criteria described in [16]. The multiple nonlinear dynamic analyses allowed the comparison of displacement demands of the two systems. Figure 5 shows the correlation between the top displacements of the SDOF and MDOF systems. The graphs do not consider displacements higher than the collapse point \( d_{MC} = 2.5 \text{ cm} \) and \( d_{sMC} = 1.3 \text{ cm} \). Lines for mean and ± mean one standard deviation are annotated on the graphs. In particular, all the Figures below refer to top displacements of the building. It is noticeable how the SDOF model is able to interpret the response of the MDOF one. The correlation coefficients between the vectors of maximum MDOF and SDOF displacements are 0.97 and 0.98 respectively for the flexural and the shear dominated models.

**Figure 5. Correlation between top displacements obtained with the flexural (first) and shear (second) dominated SDOF and MDOF models.**

The use of un-scaled natural accelerograms allows for the correlation of the structural responses with different spectral intensity measures [12, 13, 14].

3 SIMPLIFIED PREDICTION OF THE INELASTIC DISPLACEMENT DEMAND

3.1 Models combination

Starting from the two calibrated structures (Section 2), five others with intermediate behaviours (mixed shear and flexural response) were created. This allowed to have a bunch of seven structures, to represent the nonlinear dynamic behaviour of a variety of masonry buildings. The idea was to combine in different ways the two structures in order to create more than one system with an intermediate behaviour. The easiest way to obtain these models is to parallelize the two calibrated systems weighting them to obtain different configurations. In particular 7 systems were created, from the shear dominated one (\( T = 0.165 \text{ s} \)) to the flexural dominated one (\( T = 0.212 \text{ s} \)).

Figure 6 shows a schematic representation of one of the models created while Figure 7 plots in one graph the cyclic push overs of all the models in terms of equivalent acceleration and displacement. It could be noticed that the more flexural the response is, the thinner the hysteretic loops are (solid black line).

**Figure 6. Scheme of one of the combined systems (20% shear)**

**Figure 7. Direct comparison of the cyclic pushovers of the 7 SDOF systems.**

The Jacobsen [17] method was used in order to characterize the 7 SDOF structures by comparing their cyclic dissipated energy at ultimate ductility. \( \zeta_{hyst} \) varies almost linearly from a value of 19.9 % for the more shear governed structure to a value of 13.8% for the flexure dominated one.
3.2 Displacement prediction according N2 method

Nonlinear time history analyses were performed on each structure. The significant amount of dynamic analyses allowed to verify the reliability of the method for the assessment of existing structures code proposed by the Italian and European codes [10, 18]. The method was proposed by Fajfar [19, 20] based on an extensive study on inelastic spectra calculate by means of elasto-plastic equivalent oscillators. The particular dynamic behaviour of masonry structures (short structural period and complex hysteretic loops) deserves a dedicated study, considering also the great diffusion of masonry structures in the Italian historical centres.

A visual interpretation of the N2 method uses an Acceleration-Displacement Response Spectrum (ADRS) format, in which spectral accelerations are plotted against spectral displacements, with the periods represented by radial lines (from the origin of the axes). In particular, the close form solution (Eq. 6) could be used if the capacity spectrum is approximated with a standard shape (e.g. Newmark-Hall type). For this reason each spectrum of the database was approximated by a standard shape code spectrum [10] selecting the best $F_0$, $T_C$ ($F_0$ is a factor that quantify the maximum spectral amplification, $T_C$ is the corner period between the constant acceleration branch and the constant velocity branch of the response spectrum). The regression was obtained applying the least squares method in a period range between 0 and 4 seconds on each acceleration spectra with a damping coefficient of 5%.

All the parameters to use the N2 method were calculated in terms of capacity of the structure ($d_y$, $F_y$) and in terms of spectral characteristics. With these data it is possible to compute the displacement demand for $T<T_C$ (rigid structures):

$$S_d = \frac{S_{de}}{R} \left\{ (R-1) \frac{T}{T_C} + 1 \right\}$$

where $S_d$ is the inelastic displacement, $S_{de}$ is the fitted spectral displacement at $T$ (5% damping) and $R=m'S_y/F_y$.

The predicted displacement (or ductility demand) was compared with the same quantity calculated by the nonlinear 7 SDOF systems created in this work.

From the direct comparison of the results it was possible to notice that the method underestimates the ductility demand for ductilities higher than approximately 3. It should be considered that the life safety limit state is reached for ductilities higher than 4 for all the structures. This underestimation is more evident in the flexural dominated structures (higher $T$, lower $\zeta_{\text{hyst}}$). These results are illustrated in Figure 8 that shows the ductility demand predicted by the N2 method and plotted against the calculated by of nonlinear time history analyses of equivalent SDOF systems one (for the flexural structure).

In particular, the graph plots the predicted ductility demand vs. calculated by inelastic SDOF ductility demand. The ductility demand values were divided in intervals and the distributions of N2 predicted ductility demand were studied.

The N2 method resulted to be unconservative beyond a calculated ductility value of 3.5 for all the SDOF structures. This unsafe limit is more critical in the flexural dominated structure, where the method underestimated the displacement for ductility demand values higher than 2.5. The underestimation of the displacements resulted to be evident for SDOF displacements around the life safety limit state or collapse for all the structures. In particular the underestimation for ductility of 6 is higher than 30% for all the structures (40% for the flexural dominated one).

The coefficient of variation (CoV) of the results is relatively low and it is in general lower for high ductility. The average CoV of the predicted ductility is higher for the flexural dominated structure (32%) and lower for the shear one (23%). The authors calibrated the methodology by mean of inelastic spectra. In the parametric study the spectra have been obtained with the bilinear elasto-plastic model and stiffness degrading Q-model (proposed by Saitid & Sozen in [22]). In both cases 10% hardening of the slope after yielding was assumed. This value was considered to be appropriate for an equivalent SDOF system with a bilinear force-deformation envelope, representing the behaviour of a MDOF structure. The unloading stiffness coefficient in the Q-model amounted to 0.5.

Vidic et al. [6] reported: “It should be noted, however, that a hysteresis with considerably lower energy dissipation capacity (e.g. Q-hysteresis with small post-yielding and unloading stiffness) may yield results outside of the bounds set by the two chosen hysteresis. Similarly, different results can be expected in the case of a strength degrading system.” The systems considered in this study have no significant hardening behaviour and the dissipation capacity could be very low, especially in the flexural dominate structure ($\zeta_{\text{hyst}}=13.8\%$).
3.3 Displacement prediction by the corrected method

In order to overcome the problems reported in previous section a different simple relation between strength reduction factor, ductility and period-corner period ratio (R–μ–(T/Tc) relation) specific for the considered structures was derived. The results of the SDOF nonlinear dynamic analyses were used to propose idealized formulations. The final objective was to create a simple formula able to interpret the mean inelastic displacement demand of short period masonry structures.

The R–μ–T relation was plotted a posteriori, with no regression on R–μ–T data. The least-square regression was directly conducted on the displacement demands calculated by dynamic analyses and by the proposed method.

An exponential correction is proposed to the classical N2 formulation. In particular an exponent β is used.

The proposed formulation for R is:

\[ R = \beta (\mu - 1) \cdot \frac{T}{T_c} + 1 \]  

(7)

from this the ductility and displacement demands could be calculated as:

\[ \mu = (R - 1)^\beta \cdot \frac{T_c}{T} + 1 \]  

(8)

\[ S_d = \frac{S_d}{R} \left( (R - 1)^\beta \cdot \frac{T_c}{T} + 1 \right) \]  

(9)

If β is not a function of the ductility, the calculation of displacement demand will be non-iterative, preserving simplicity as well as the “code implementability” of the method.

The formulation proposed in Eq. 10 considers the fact that an almost linear relation between β and dissipated energy per cycle by the structure was found. The formula was created applying a regression method fitting directly the mean values of the ductilities predicted by the proposed formulation.

The results are well interpreted by:

\[ \beta = -9.37 \cdot \xi_{hyst} + 3.36 > 1 \]  

(10)

Figure 9 shows the relation between the parameter β and Jacobsen damping ξ_{hyst} and the regression line of the proposed formulation Eq. 10.

The exponential formulation leads to an inconsistency. If R<2, the inelastic displacement ratio C=Sd/Sde resulted to be less than 1. Basically for low strength reduction factor the inelastic displacement results to be lower than the elastic one (e.g. for T=Tc, C=0.85). To solve this problem the formula was split in two parts: for R<2, β=1 and for R≥2, β>1. In this way the final results appear to be less dispersed, while the mean value is not significantly modified.

The predicted displacement (or ductility) was compared with the same quantity calculated by the nonlinear 7 SDOF systems created in this work. Figure 10 and Figure 11 show the predicted ductility demand (by means of β-corrected method) vs. the calculated ductility (by nonlinear SDOF systems) for the shear-oriented structures and for the flexural one respectively.

Graphs for all the analysed structures are presented in [14]. From the results it is possible to derive some considerations:

- There is a dispersion in the results also for μ<1 (elastic range) because the model used for the SDOF is nonlinear (elastic) in this range.
- The method is able to interpret with very good precision the mean value of the displacement demand for all the considered ductility values. No trends of under or overestimation were noticed in any of the structures considered. The mean value is well interpreted for all the ductility levels considered. An overestimation was noticed in all the structures for 1<μ<2, in particular the overestimation was 25% in the shear dominated one.
- The ductility demand values were divided in intervals and the distributions of the predicted ductility demand by the method were studied. The CoV of the results is generally higher than the CoV calculated for standard N2 method, it remains quasi constant for all the ductility levels. This is very important facilitating the introduction of partial safety factors, as example, in a code formulation. The average CoV of the predicted ductility is higher for the flexural dominated structure (45%) and lower for the shear one (27%). The correlation coefficient is 90% for the shear dominated structure and 81% for the flexural dominated one.
Ductility SDOF [-]
Ductility $\beta$-corrected method [-]

Figure 10. Predicted ductility demand vs. inelastic SDOF ductility demand (mean, in black; mean ± 1 standard deviation, in blue; shear behaviour, $\zeta_{\text{hyst}}=19.9\%$, $\beta=1.5$).

Figure 11. Predicted ductility demand vs. inelastic SDOF ductility demand (mean, in black; mean ± 1 standard deviation, in blue; flexural behaviour, $\zeta_{\text{hyst}}=13.8\%$, $\beta=2.07$).

The use of an exponential conversion factor $\beta$ is able to solve the problem of underestimation for high and overestimation for low ductilities. However this solving technique is not priceless. Also the CoV of the predicted ductility is influenced by this factor and in particular it is increased up to a value of 45% for the flexure dominated structure. Good results were also obtained using constant value of $\beta=1.8$ for all the structure (this could be used if a cyclic push over is not available):

$$S_d = \frac{S_{\infty}}{R} \left( (R-1)^{1.8} \cdot \frac{T_C}{T} + 1 \right)$$

(2)

The method was applied to a calibrated 3 floor numerical model in order to verify its reliability [14].

To facilitate the applicability of the method, the next graphs represent the reduction factor $R$ for a constant value of mean ductility (Figure 12) and the mean ductility for a constant reduction factor (Figure 13). All the graphs are plotted considering the value of $\beta=1.8$, considered suitable, on average, for all the structures.

The chart in Figure 13 is easily usable to predict the ductility demand (given a reference spectrum: $T_C$, $S_d$) of a masonry structure applying the following steps:

1. Perform a pushover (PO) of the structure to be assessed;
2. Approximate the PO curve by an equivalent bilinear one ($d_y$, $F_y$);
3. Calculate the initial period $T$, then $T/T_C$;
4. Calculate strength reduction factor $R=m^* S_{\infty} / F_y$;
5. Enter in graph knowing $T/T_C$ and $R$ in order to obtain the ductility demand (or apply the equation).

This procedure gives an estimation of the mean value of the demand with no information on the distribution around the mean value.

4 CONCLUSIONS

A simplified system to simulate the nonlinear behaviour of masonry building was studied. In particular an equivalent SDOF system was developed and calibrated statically and dynamically. It synthesized, with sufficient accuracy, the seismic response of the case study MDOF system under investigation, while minimizing the computational effort. The use of a large set of natural accelerograms allowed to correlate the structural response with different intensity measures.

The SDOF model was used to evaluate the displacement demand of a masonry structure in a reliable and fast way.
proposing a relation between strength reduction factor, ductility and period ($R_{\mu}/T_{T_c}$ relation) by means of a parametric study. This resulted in a proposed simplified formulation (function of the energy dissipated by the structure) for determining the inelastic displacement of a masonry structure starting from an idealized pushover curve and an elastic spectrum, trying to fill the gap that is still present in this field of study.

Studying the applicability of the general N2 method proposed by Fajfar [19, 20] and adopted by the Eurocode 8 and the Italian building code [10, 18], the method appeared to be non-conservative for ductility demand higher than 3 (especially for flexural-dominated structures).

A new non-iterative formulation, specific for masonry structures, appeared to be more conservative and reliable compared to the N2 method.

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