A spatial windowing technique to account for finite dimensions in 2.5D elastodynamic transmission and radiation problems

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ABSTRACT: The dynamic interaction between a layered halfspace and quasi translationally invariant structures such as roads, railway tracks, tunnels, dams, and lifelines can be modelled using a computationally efficient 2.5D approach, assuming invariance of the geometry in the longitudinal direction. This assumption is not always fulfilled in practice, however. Even for elongated structures, full 3D computations may still be required for an accurate solution of the dynamic soil–structure interaction problem. This paper presents a spatial windowing technique for elastodynamic transmission and radiation problems that allows accounting for the finite length of a structure, still maintaining the computational efficiency of a 2.5D formulation. The proposed technique accounts for the diffraction occurring at the structure’s edges, but not for its modal behaviour resulting from reflections of waves at its boundaries. Numerical examples of a barrier for vibration transmission and a surface foundation on the soil are discussed to demonstrate the accuracy and applicability of the proposed methodology. Full 3D calculations are performed to provide a rigorous validation for each of these examples. It is demonstrated that the proposed technique is appropriate as long as the response is not dominated by the resonant behaviour of individual modes of the structure.

KEY WORDS: Dynamic soil–structure interaction; elastodynamic wave propagation; 2.5D modelling; FE–BE models.

1 INTRODUCTION

The numerical solution of three–dimensional (3D) dynamic soil–structure interaction (SSI) problems is a challenging task, in particular for structures with large dimensions. In order to obtain a substantial reduction of the computational effort, the geometry of the problem is in some cases assumed to be invariant in the longitudinal direction. This allows for the application of an efficient two-and-a-half-dimensional (2.5D) approach, where a Fourier transform of the longitudinal coordinate allows representing the 3D response on a 2D mesh. The assumption seems to be valid for roads, railway tracks, tunnels, dams, vibration isolation screens, and lifelines [1].

Many applications of the 2.5D concept can be found in the literature. Gavrić [2] uses 2.5D finite elements (FE) to model thin–walled waveguides, while Stamos and Beskos [3] consider 2.5D boundary elements (BE) to model the seismic response of long lined tunnels embedded in a halfspace. In 2.5D BE formulations, analytical full space Green’s functions are commonly used [4]. The discretization of the free surface and the layer interfaces can be avoided, however, by employing Green’s functions for a layered halfspace [1]. Coupled FE–BE models allow to model complex geometries with the FE method and to account for the radiation of waves in domains of (semi–)infinite extent with the BE method. 2.5D coupled FE–BE formulations have been presented, among others, by Sheng et al. [5] and Lombaert et al. [6] for the prediction of railway [5] and traffic [6] induced vibrations. The efficiency of coupled FE–BE methods is strongly reduced in the case of embedded structures, however, as the Green’s functions have to be evaluated for a large number of source/receiver depths for the assembly of the BE matrices. Alternative numerical solution procedures in a 2.5D framework have therefore been formulated as well, such as a 2.5D finite–infinite element approach proposed by Yang et al. [7] or a 2.5D perfectly matched layer (PML) technique described by François et al. [8].

The assumption of longitudinal invariance adopted in 2.5D models is not always fulfilled, however. For example, the length of a vibration isolation screen in the soil is in practice limited and comparable to the wavelength in the soil in the frequency range of interest. Rigorously accounting for the finite length requires the solution of a full 3D dynamic SSI problem, which is computationally very demanding. The development of adequate numerical methods such as the fast multipole BE method [9] or BE methods based on hierarchical matrices (H–matrices) [10] enables an efficient solution of such large scale problems, but the associated computation times remain relatively high.

In this paper, a spatial windowing technique for elastodynamic transmission and radiation problems is presented that allows accounting for the finite length of a structure, still maintaining the computational efficiency of a 2.5D formulation. The spatial windowing technique has been proposed by Villot et al. [11] to include the effect of diffraction associated with the finite size of plane structures on sound transmission and radiation. The basic idea of this approach is to apply a spatial rectangular baffle to the structural wavefield of an infinite structure; the windowed wavefield is subsequently employed to compute the radiated wavefield in the wavenumber domain. As a result, only a limited part of the infinite structure contributes to the sound radiation. This technique is mainly used in vibro–acoustic applications, e.g. for the calculation of the transmission loss of sandwich composite panels [12] or for the investigation of the vibro–acoustic response of orthogonally
stiffened plates [13]. Spatial windowing is not well suited for acoustic applications at low frequencies (i.e. when individual modes of the structure dominate the response), as it is unable to account for reflected waves at the boundaries to reproduce the resonant behaviour of the modes [13]. At higher frequencies, however, the response shifts from the resonant to the non-resonant mass–law regime and application of spatial windowing results in good agreement with experiments [11].

The aim of this paper is to investigate whether the spatial windowing technique is suited to account for a structure’s finite length in 2.5D dynamic SSI problems, using a coupled FE–BE method. Its application to dynamic SSI problems fundamentally differs from acoustic problems, however, as the resonant behaviour of individual modes is strongly affected by the dynamic interaction between the structure and the soil. The text is organized as follows. Section 2 briefly summarizes the governing equations of 3D and 2.5D coupled FE–BE methods. The spatial windowing technique is subsequently introduced in section 3. Numerical examples are considered in sections 4 and 5 to investigate the applicability of the proposed approach. The example in section 4 involves a barrier for vibration transmission in a homogeneous halfspace, which is a structure with a finite length that is relatively large compared to the other dimensions. Application of the 2.5D approach hence seems to be appropriate for this case. In section 5, the validity of the spatial windowing technique is further explored by considering a square surface foundation on the soil, which is a structure that can not at all be regarded as invariant. The importance of dynamic SSI is assessed by comparing a foundation on a horizontally layered halfspace to a foundation on a single layer on bedrock. A rigorous validation of the spatial windowing methodology is provided for each of these examples through full 3D computations based on an efficient coupled FE–BE method. Concluding remarks regarding the suitability of the proposed technique are summarized in section 6. This paper provides a summary of the most important results on spatial windowing; a more elaborate discussion with additional numerical examples is given by Coulier et al. [14].

2 COUPLED FE–BE METHODS FOR DYNAMIC SSI

Dynamic SSI problems can be solved by means of a subdomain formulation [15], allowing for the application of different numerical techniques for the soil and the structure. In this paper, finite elements are used to model the structural domain \( \Omega_b \), while boundary elements on the soil–structure interface \( \Sigma \) are employed to model wave propagation in the surrounding soil domain \( \Omega_s \) (figure 1a). Continuity of displacements and equilibrium of stresses are enforced on the interface \( \Sigma \). In the following, it is assumed that tractions \( \hat{t}_{\sigma}^{b}(\omega) \) are imposed on the boundary \( \Gamma_{\sigma} \) of \( \Omega_b \), while an incident wavefield \( \hat{u}_{i}(\omega) \) is present in the soil domain \( \Omega_s \). A hat above a variable denotes its representation in the frequency domain.

2.1 3D coupled FE–BE method

If a structure with an arbitrary geometry is considered, the rigorous solution of a full 3D dynamic SSI problem is required. A weak variational formulation of the equilibrium of the structure \( \Omega_b \) results in the following coupled FE–BE equation [16]:

\[
\begin{bmatrix}
K_b + C_b - \omega^2 M_b + \hat{K}_b^{s}(\omega) \\
\end{bmatrix} \hat{u}_b(\omega) = \hat{L}_b(\omega) + \hat{F}_{\sigma}(\omega)
\]

where \( \hat{u}_b(\omega) \) collects the nodal degrees of freedom of \( \Omega_b \), while \( K_b, C_b, \) and \( M_b \) are the finite element stiffness, damping, and mass matrices. \( \hat{K}_b^{s}(\omega) \) is the dynamic soil stiffness matrix and is calculated by means of a 3D BE method. The force vector \( \hat{F}_{\sigma}(\omega) \) results from tractions \( \hat{t}_{\sigma}^{b}(\omega) \) imposed on the boundary \( \Gamma_{\sigma} \), whereas \( \hat{F}_{\sigma}(\omega) \) denotes dynamic SSI forces at the soil–structure interface \( \Sigma \) associated with the incident wavefield \( \hat{u}_{i}(\omega) \) [15]. Solving equation (1) provides the structural response \( \hat{u}_b(\omega) \), which corresponds to the soil displacement vector \( \hat{u}_b(\omega) \) on the soil–structure interface \( \Sigma \) due to continuity. The BE equations allow to retrieve the soil tractions \( \hat{L}_b(\omega) \) on \( \Sigma \):

\[
\hat{L}_b(\omega) = \hat{U}^{-1}(\omega) \left( \hat{T}(\omega) + \hat{I} \right) \hat{u}_b(\omega)
\]

where \( \hat{U}(\omega) \) and \( \hat{T}(\omega) \) are BE matrices, requiring integration of the Green’s displacements and tractions, respectively. The displacements \( \hat{u}_b(\omega) \) and tractions \( \hat{L}_b(\omega) \) on \( \Sigma \) are subsequently used to evaluate the radiated wavefield in the soil \( \hat{u}_s(\omega) \) through the discretized boundary integral equation:

\[
\hat{u}_s(\omega) = \hat{U}_s(\omega) \hat{L}_b(\omega) - \hat{T}_s(\omega) \hat{u}_b(\omega)
\]

where \( \hat{U}_s(\omega) \) and \( \hat{T}_s(\omega) \) are BE transfer matrices.

3D FE–BE models can be used to solve dynamic SSI problems of any size as long as the proper computational resources are available. The fully populated unsymmetric matrices \( \hat{U}(\omega) \) and \( \hat{T}(\omega) \) arising from classical BE formulations
lead to stringent memory and CPU requirements, however, restricting the applicability of the method to problems of moderate size. These drawbacks can be circumvented through the application of fast BE methods [9], [10]. In this paper, a fast BE method based on \( \mathcal{K} \)-matrices [17] is employed for the solution of 3D problems; the reader is referred to the literature [10], [18] for a comprehensive overview of this methodology. The application of \( \mathcal{K} \)-matrices renders the conventional FE–BE coupling strategy of equation (1) less efficient, however, as it requires the assembly of a dynamic soil stiffness matrix [19]. An alternative iterative algorithm is therefore employed, in which the governing equations of the FE and BE subdomain are solved separately, while the boundary conditions at the soil–structure interface are updated until convergence is achieved. A detailed description of this coupling approach can be found in [19].

Although the application of fast BE methods allows increasing the problem size considerably compared to classical BE formulations, the solution of large scale problems remains computationally very demanding. Additional assumptions can be made to simplify the problem, as will be discussed in the next subsection.

### 2.5D coupled FE–BE method

In the case of structures with a longitudinally invariant geometry (figure 1b), the longitudinal coordinate \( y \) can be transformed to the wavenumber \( k_y \) by means of a forward Fourier transform \( \mathcal{F} \left[ f(y), k_y \right] = \int_{-\infty}^{\infty} f(y) \exp(ik_y y) \, dy \), resulting in a computationally efficient 2.5D solution procedure in the frequency–wavenumber domain. As the 3D response can hence be represented on a 2D mesh [1], a substantial reduction of the number of degrees of freedom (and the associated matrix dimensions) is achieved. The governing equations are briefly summarized in this subsection; an extensive discussion of the 2.5D coupled FE–BE methodology can be found in [1], [20].

The dynamic equilibrium equation of the coupled FE–BE system reads as follows in the frequency–wavenumber domain [1]:

\[
\begin{align*}
\left[ \mathcal{K}_b(k_y, \omega) + C_b - \omega^2 M_b + \mathcal{K}_s(k_y, \omega) \right] \mathbf{u}(k_y, \omega) = \mathbf{b}(k_y, \omega) &= \mathbf{b}_0(k_y, \omega) + \mathbf{b}_0(k_y, \omega) \\
\end{align*}
\]

where a tilde above a variable denotes its representation in the frequency–wavenumber domain. This equilibrium equation is similar to the 3D coupled FE–BE equation (1), except that the stiffness matrices, the displacement vector, and the load vectors become wavenumber dependent. Solving equation (4) provides the structural response \( \mathbf{u}_s(k_y, \omega) \), corresponding to the soil displacements \( \mathbf{u}_s(k_y, \omega) \) on the soil–structure interface \( \Sigma \). The BE equations allow to retrieve the soil tractions \( \mathbf{t}_s(k_y, \omega) = \mathbf{U}^{-1}(k_y, \omega) \left( \mathbf{T}(k_y, \omega) + \mathbf{l} \right) \mathbf{u}_s(k_y, \omega) \), where \( \mathbf{U}(k_y, \omega) \) and \( \mathbf{T}(k_y, \omega) \) are wavenumber dependent BE matrices. The representation theorem expressed in the frequency–wavenumber domain finally allows for the computation of the radiated wavefield in the soil \( \mathbf{u}_s(k_y, \omega) \) [11]. The latter corresponds to the discretized boundary integral equation (3), where each variable should be replaced by its wavenumber dependent counterpart:

\[
\mathbf{u}_s(k_y, \omega) = \mathbf{U}_s(k_y, \omega) \mathbf{u}_s(k_y, \omega) - \mathbf{T}_s(k_y, \omega) \mathbf{u}_s(k_y, \omega) \quad (5)
\]

The response in the frequency–spatial domain can be found by means of an inverse Fourier transform \( \mathcal{F}^{-1} \left[ f(k_y, \omega) \right] = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(k_y) \exp(-ik_y y) \, dk_y \) from the wavenumber \( k_y \) to the longitudinal coordinate \( y \), using an efficient Filon quadrature scheme [21].

### 3. 2.5D COUPLED FE–BE METHOD WITH SPATIAL WINDOWING

The spatial windowing technique has been presented by Villot et al. [11] to account for the finite size of a plane structure in sound transmission and radiation calculations. This section describes how this technique can be incorporated in the 2.5D coupled FE–BE method to account for the finite length of a structure in dynamic SSI problems.

Consider a plane wave with a constant longitudinal wavenumber \( k_{y0} \) travelling along an infinite structure. The displacement field in the spatial domain yields:

\[
\hat{u}(y, \omega) = \frac{1}{2\pi} \hat{u}_0(\omega) \exp(-ik_{y0}y) \quad (6)
\]

while the wavenumber spectrum corresponds to a Dirac delta function at \( k_y = k_{y0} \) (figure 2a):

\[
\hat{u}(k_y, \omega) = \int_{-\infty}^{+\infty} \frac{1}{2\pi} \hat{u}_0(\omega) \exp(-ik_{y0}y) \exp(ik_y y) \, dy = \hat{u}_0(\omega) \delta(k_y - k_{y0}) \quad (7)
\]

A structure with a finite length \( L_y \), situated between \( y_1 \) and \( y_2 = y_1 + L_y \), is only able to contribute to the radiation of waves into the soil domain \( \Omega_s \) from \( y_1 \) to \( y_2 \). The wavenumber spectrum of the displacement field is consequently determined by applying a forward Fourier transform to equation (6), restricting the integration in equation (7) to \( y \in [y_1, y_2] \):

\[
\hat{u}_{sw}(k_y, \omega) = \int_{y_1}^{y_2} \frac{1}{2\pi} \hat{u}_0(\omega) \exp(-ik_{y0}y) \exp(ik_y y) \, dy = \frac{1}{2\pi} \hat{u}_0(\omega) \exp[i(k_y - k_{y0})y_1] (1 - \exp[-i(k_y - k_{y0})L_y]) \quad (9)
\]

\[
\text{with } \lim_{k_y \rightarrow k_{y0}} \hat{u}_{sw}(k_y, \omega) = \frac{1}{2\pi} \hat{u}_0(\omega)L_y. \quad \text{The subscript ‘sw’ refers to a spatially windowed quantity. Equation (10) reveals that spatial windowing results in a distribution of the energy over the entire wavenumber range [11], while it was originally concentrated at } k_y = k_{y0}. \quad \text{This is illustrated in figure 2.}
\]

Application of the spatial windowing technique in the framework of the 2.5D FE–BE methodology outlined in subsection 2.2 implies that the contribution of each wavenumber component of the displacement vector \( \mathbf{u}_s(k_y, \omega) \) is distributed over the entire wavenumber domain according to equation (10). The spatially windowed displacement vector \( \mathbf{u}_{sw}(k_y, \omega) \) can hence be expressed as:

\[
\mathbf{u}_{sw}(k_y, \omega) = \mathbf{u}_s(k_y, \omega) * \left[ \frac{1}{2\pi} \frac{1}{ik_y} \exp(ik_y y_1) (1 - \exp[-ik_y L_y]) \right] \quad (11)
\]
for a trench depth greater than about 0.60 times the Rayleigh wavelength in the soil [22]. Trenches are assumed to be infinitely long in most of the numerical studies reported in the literature [24], [25]; this assumption is not fulfilled in practice, however. It is shown next how the spatial windowing technique allows accounting for the finite length of the trench.

Consider a halfspace characterized by a shear wave velocity $C_s = 200\, \text{m/s}$, a dilatational wave velocity $C_p = 400\, \text{m/s}$, a density $\rho = 2000\, \text{kg/m}^3$, and material damping ratios $\beta_p = \beta_s = 0.025$ in deviatoric and volumetric deformation. A trench with a width $w = 2\, \text{m}$, a depth $d = 2\, \text{m}$, and a length $L_y$ is situated at a distance $\mathcal{D} = 4\, \text{m}$ from the $y$-axis and is positioned symmetrically with respect to the $x$-axis, i.e. $y_1 = -L_y/2$ and $y_2 = L_y/2$ (figure 3a). The numerical analysis is performed for trenches with a length of 15 m and 60 m.

The dimensionless trench depth $\bar{d}$ is defined as $d/\lambda_R(f)$, where $\lambda_R(f)$ is the frequency dependent Rayleigh wavelength in the soil; a value $\bar{d} = 0.60$ is obtained at 56Hz. In order to facilitate physical interpretation, an incident wavefield is generated by the application of a unit vertical harmonic point load at the origin of the coordinate system, rather than considering a train passage.

The spatial windowing technique outlined in section 3 is used to calculate the wavefield in the soil, accounting for the presence of the open trench with length $L_y$. The interface $\Sigma$ of the trench is modelled with 30 2.5D boundary elements; the element dimensions are limited in order to ensure that 10 elements per shear wavelength $\lambda_s = C_s/f$ are used at a frequency of 100Hz. Finite elements are not required, as no infill material is considered. The 2.5D computations with spatial windowing are compared to 3D $\mathcal{H}$-BE calculations, where four-node quadrilateral boundary elements are employed to discretize the interface $\Sigma$. The discretization of the free surface is avoided in both approaches by employing Green’s functions for a halfspace in the 2.5D BE and 3D $\mathcal{H}$-BE formulations [1], [17].

The vibration reduction efficiency of a trench is characterized through the vertical insertion loss $\hat{IL}_z(x, \omega)$:

$$\hat{IL}_z(x, \omega) = 20\log_{10}\left|\frac{u_{\text{ref}}^z(x, \omega)}{u_r(x, \omega)}\right|$$

which compares the vertical displacement $u_{\text{ref}}^z(x, \omega)$ in the reference case (without a trench) to the vertical displacement $u_r(x, \omega)$ in case a trench is included; positive values of the insertion loss indicate a reduction of the free field vibrations.
Figures 4–6 show the insertion loss $\hat{IL}_z(x, \omega)$ for a trench with a length of 15 m and 60 m at 15 Hz, 30 Hz, and 60 Hz, respectively. The dimensionless trench depth equals $\overline{y} = 0.16$ at 15 Hz, $\overline{y} = 0.32$ at 30 Hz, and $\overline{y} = 0.64$ at 60 Hz. The insertion loss remains rather limited at 15 Hz, as a significant part of the energy still passes underneath the trench. The penetration depth of the Rayleigh waves decreases at higher frequencies, causing reflection of the waves by the trench and resulting in insertion losses up to 10 dB and more at 30 Hz and 60 Hz. Extending the length of the trench leads to an enlargement of the area where vibration levels are effectively reduced. The results are furthermore compared to rigorous 3D $\mathcal{H}$-BE calculations, and an almost perfect agreement between the spatially windowed 2.5D and the 3D computations is observed for all trench lengths and at all frequencies under concern. The correspondence is not only apparent at the surface of the halfspace, but also at depth. These figures indicate that the proposed spatial windowing technique is particularly well suited for the case under concern. A trench of limited length is only able to reflect that part of the wavefield that impinges on the trench, which is clearly visible for a trench of 15 m (figures 4–6a). Furthermore, diffraction around the edges of a finite trench leads to a decreased efficiency in part of the shadow zone. Both phenomena are accounted for in the spatial windowing technique.

The 2.5D calculations based on the assumption of longitudinal invariance (i.e. without spatial windowing) are shown in figures 4–6d. A comparison of figures 4–6a–b and 4–6c clearly indicates that accounting for the finite length of the trench is important to correctly assess the vibration reduction efficiency. A trench of limited length is only able to reflect that part of the wavefield that impinges on the trench, which is clearly visible for a trench of 15 m (figures 4–6a). Furthermore, diffraction around the edges of a finite trench leads to a decreased efficiency in part of the shadow zone. Both phenomena are accounted for in the spatial windowing technique.

The computational effort for the 2.5D computations (with or without spatial windowing) is considerably lower than for the full 3D calculations, as is demonstrated in tables 1 and 2. Table 1 summarizes the amount of RAM memory required for the storage of the BE matrices $\tilde{U}(k_y, \omega)$ and $\tilde{T}(k_y, \omega)$ or $\hat{U}(\omega)$ and $\hat{T}(\omega)$ in the 2.5D BE or 3D $\mathcal{H}$-BE models, respectively. The amount of RAM memory that would have been required in a classical 3D BE model without the application of $\mathcal{H}$-matrices is indicated as well. It is clearly illustrated in table 1 that the 2.5D approach results in a substantial reduction of the RAM memory. The efficiency in terms of computation time is assessed in table 2. The computation time for a 2.5D open trench with a length of $L_y = 15$ m is comparable to that of a 2.5D calculation, while it significantly exceeds the latter for $L_y = 60$ m. As the 2.5D equations are solved independently for each wavenumber $k_y$ in the frequency–wavenumber domain, the 2.5D calculations can easily be parallelized. The use of MATLAB’s Parallel Computing Toolbox [26] allows for a distributed computation on eight cores, leading to a speed–up by a factor that is slightly less than eight (due to the communication overhead). The value of 1.8 h listed in table 2 indicates the total computation time on all cores; the actual computation time is only 0.25 h. A similar parallelization can not be applied to the 3D $\mathcal{H}$-BE models, however.

5 APPLICATION OF SPATIAL WINDOWING TO SHORT STRUCTURES: SURFACE FOUNDATION

The validity of the spatial windowing technique is further explored in this section. The importance of the actual length...
of the structure, its modal behaviour, and the dynamic SSI are investigated. The structure under concern is a square surface foundation on a horizontally layered halfspace; the geometry thus strongly differs from the open trench previously discussed. In subsection 5.1, the flexibility is neglected and the foundation is modelled as a rigid body. The influence of flexible foundation modes on the accuracy of the methodology will subsequently be investigated in subsections 5.2 and 5.3.

5.1 Rigid surface foundation on a horizontally layered halfspace

The concrete foundation has a width $w = 5\, \text{m}$, a length $L_y = 5\, \text{m}$, a thickness $t = 0.25\, \text{m}$, a Young’s modulus $E = 33\, \text{GPa}$, a Poisson’s ratio $\nu = 0.20$, and a density $\rho = 2500\, \text{kg/m}^3$. A hysteretic damping ratio $\beta = 0.03$ is included through application of the correspondence principle. The foundation is loaded by a unit harmonic vertical point load at its center. While a homogeneous halfspace has been considered in section 4 to facilitate physical interpretation, the soil in reality is often stratified; a layered halfspace is therefore included in this subsection. The soil consists of two layers on a halfspace, each with a thickness of 2m. The shear wave velocity $C_s$ is equal to 150m/s in the top layer, 250m/s in the second layer, and 300m/s in the underlying halfspace. The Poisson’s ratio $\nu$ is 1/3 everywhere, resulting in dilatational wave velocities $C_p$ of 300m/s, 500m/s, and 600m/s, respectively. Material damping ratios $\beta_r = 0.025$ in deviatoric and volumetric deformation are attributed to the layers and the halfspace, while a uniform density $\rho = 1800\, \text{kg/m}^3$ is considered throughout the medium.

The spatial windowing technique is employed to compute the response of the foundation and the wavefield in the soil based on a 2.5D calculation. The soil–foundation interface is discretized with 30 2.5D boundary elements, while 30 $\times$ 30 square quadrilateral boundary elements are used in the 3D validation calculations. This corresponds to nine elements per shear wavelength in the top layer at 100Hz. As the rigid body translation of a longitudinally invariant structure is entirely two–dimensional and corresponds to plane strain conditions [1], the 2.5D calculation is restricted to $k_y = 0$.

Figure 7a shows the real part of the vertical displacement $\hat{u}_v(x, \omega)$ of the foundation and the soil at 100Hz. Results obtained with the 2.5D BE model (for an infinitely long rigid foundation), the 2.5D BE model with spatial windowing, and the 3D $\mathcal{H}$-BE model are compared. The 2.5D model is unable to account for variations in the longitudinal direction (as it is restricted to $k_y = 0$); the displacements consequently strongly differ from the 3D results. Application of spatial windowing distributes the energy over the entire wavenumber domain, which enables the correct representation of the variation of the wavefield in the longitudinal direction. This leads to a very good agreement with the 3D calculations. The response of the foundation is also affected, however, and does not longer correspond to a uniform vertical translation.

![Figure 7a](image)

Figure 7b. Real part of the vertical displacement $\hat{u}_v(x, \omega)$ of the foundation and the soil for a (a) rigid and (b) flexible surface foundation on a layered halfspace at 100Hz. The results are calculated by means of a 2.5D model (left), a 2.5D model with spatial windowing (middle), and a 3D model (right).

The three numerical methodologies are furthermore compared in figure 8, which shows the free field mobility along the line $y = 0\, \text{m}$ at several distances from the foundation in the frequency range between 0Hz and 100Hz. As can be expected, there is a significant deviation between the 2.5D and 3D mobilities; the assumption of longitudinal invariance generally results in an overestimation of the free field mobility, especially in the far field. A very good agreement is achieved between the 2.5D model with spatial windowing and the 3D model, although some discrepancies arise in the near field. Figures 7a and 8 illustrate the appropriateness of the proposed methodology, even if applied to a structure not resembling an invariant one. The actual dimensions of the structure are not important; spatial windowing is effective as long as the response is not dominated by the modal behaviour of the structure.

5.2 Flexible surface foundation on a horizontally layered halfspace

In order to account for the flexibility of the foundation, the structure is discretized with Kirchhoff plate elements which are coupled to the boundary elements on the soil–foundation...
methodologies yield the same result, as the wavelength in the
and phase of the vertical displacement $\hat{u}$.

This is also illustrated in figure 10, which shows the modulus
62 Hz are not apparent in figure 9 due to the strong dynamic SSI.
correspondence with the 3D results.

The deviations are more pronounced in the near field and are
the case of the rigid foundation considered in subsection 5.1.
at higher frequencies, but these are much smaller than in
the near field on the layered halfspace; it is not a natural frequency
foundation on the layered halfspace; it is not a natural frequency
and the natural modes of the foundation consequently prevail

interface. Within the frequency range of interest, the free
foundations have natural modes at 24 Hz, 35 Hz, 40 Hz, and 62 Hz;
only the modes at 40 Hz and 62 Hz can be excited by the loading
under concern, however, as the projection of the excitation force
on the other mode shapes equals zero.

Figure 7b shows the real part of the vertical displacement
$\hat{u}_z(x, \omega)$ of the foundation and the soil at 100 Hz. Results
obtained with the 2.5D FE–BE model (for an infinitely long
flexible foundation), the 2.5D FE–BE model with spatial
windowing, and the 3D FE–BE–BE model are compared. The
wavefield in the soil is strongly affected by the presence of the
foundation; a 2.5D calculation is unable to accurately represent
the wavefield obtained with a 3D calculation. Application
of spatial windowing modifies the wavefield considerably,
resulting in a much better agreement with the 3D calculations.

Figure 9 shows the free field mobility along the line $y = 0$ m
at several distances from the foundation in the frequency range
between 0 Hz and 100 Hz. Below 25 Hz, the three numerical
methodologies yield the same result, as the wavelength in the
soil remains large compared to the dimensions of the foundation.
Discrepancies between the 2.5D and 3D model are observed
at higher frequencies, but these are much smaller than in
the case of the rigid foundation considered in subsection 5.1.
The deviations are more pronounced in the near field and are
almost negligible in the far field. The mobilities obtained
after application of spatial windowing are in much better
agreement with the 3D results.

The natural frequencies of the free foundation at 40 Hz and
62 Hz are not apparent in figure 9 due to the strong dynamic SSI.
This is also illustrated in figure 10, which shows the modulus
and phase of the vertical displacement $\hat{u}_z(\omega)$ at the center of
the foundation. The peak at 20 Hz corresponds to resonance of
the foundation on the layered halfspace; it is not a natural frequency
of the foundation. The response is thus not dominated by the
modal behaviour of the foundation, explaining the suitability of
the spatial windowing technique in the case under concern.

Figure 10. (a) Modulus and (b) phase of the vertical
displacement $\hat{u}_z(\omega)$ at the center of a flexible surface
foundation on a layered halfspace. Line styles are in
correspondence with figure 8.

5.3 Flexible surface foundation on a single layer on bedrock
In order to further explore the limitations of the spatial
windowing technique, the influence of dynamic SSI on the
accuracy of the methodology is investigated in this subsection.
The layered halfspace considered in subsections 5.1 and 5.2
is replaced by a single layer on bedrock, with the same wave
velocities and material damping ratios as the top layer of the
aforementioned halfspace. The layer thickness $h$, however, is
set to 0.375 m, which results in a cut–on frequency of $C_s/(4h) =
100$ Hz; the surface waves hence remain evanescent in the whole
frequency range under concern. The density of the layer is
also artificially reduced by a factor of ten to 180 kg/m$^3$, which
consequently implies a reduction of the soil stiffness.

Figure 11 compares the modulus and phase of the vertical
displacement $\hat{u}_z(\omega)$ at the center of the foundation, calculated
with the 2.5D FE–BE model (for an infinitely long flexible
foundation), the 2.5D FE–BE model with spatial windowing,
and the 3D FE–BE–BE model. As there are no propagative
surface waves in the soil, the radiation damping is very limited,
and the natural modes of the foundation consequently prevail.

Figure 8. Free field mobility along the line $y = 0$ m for a rigid
surface foundation on a layered halfspace. The results are
obtained by means of a 2.5D model (black line), a 2.5D
model with spatial windowing (circles), and a 3D model
(grey line).

Figure 9. Free field mobility along the line $y = 0$ m for a flexible
surface foundation on a layered halfspace. Line styles are in
correspondence with figure 8.
in the response of the 3D coupled soil–foundation system. The resonance peaks near 40Hz and 62Hz can clearly be distinguished in figure 11, which is also due to the low soil stiffness. The 2.5D approach gives a reasonable correspondence with these results below 40Hz, but large discrepancies are observed at higher frequencies. Application of the spatial windowing technique does not lead to a better agreement with the 3D results, however. This example illustrates the shortcoming of the technique in case of low radiation damping in the soil, as it does not succeed to account for the dominant modal behaviour of the structure.

Figure 11. (a) Modulus and (b) phase of the vertical displacement $\hat{u}_y(\omega)$ at the center of a flexible surface foundation on a single layer on bedrock. Line styles are in correspondence with figure 8.

6 CONCLUSIONS

In this paper, a spatial windowing technique has been presented that allows accounting for the effect of finite dimensions in 2.5D models for dynamic SSI. This technique enables the application of 2.5D models even if the assumption of longitudinal invariance is not fulfilled, hence maintaining the associated computational efficiency. The method redistributes the contribution of each wavenumber component over the entire wavenumber domain and can as such be regarded as a postprocessing of the original 2.5D results. Spatial windowing only accounts for the diffraction occurring at the structure’s extremities, however, and the existence of structural modes is not considered.

Numerical examples of elongated (open trench) and short (surface foundation) structures have been discussed to investigate the applicability of the proposed technique. For each of these examples, full 3D calculations have been performed to provide a rigorous validation. It is demonstrated that the proposed technique is accurate as long as the modal behaviour of the structure does not dominate the response; the methodology is in that case even appropriate for structures which do not seem to have an invariant geometry. The modal behaviour has only a limited influence in most of the applications due to the dynamic SSI and the radiation damping in the soil. If this is not the case, however, spatial windowing reaches its limits of suitability.

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