Tunnel-soil-pile interaction in the prediction of vibration from underground railways: validation of the sub-models

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ABSTRACT: This paper proposes a method for calculating the vibration levels from an underground tunnel, adjacent to a piled-foundation, embedded in a homogeneous half-space on the basis of strong coupling. The method relies on superposing the vibration field generated by the tunnel with that generated by the piled-foundation. The soil in this paper is modelled utilising the boundary element method, while the tunnel is modelled using thin-shell theory and the piled-foundation is modelled by adopting the elastic bar and Euler beam theories. Only the results of the sub-models (tunnel and piled-foundation) are presented herein and compared with previous work in the literature. The current tunnel model is contrasted to the well-known PiP model whereas the piled-foundation model is validated against a previous boundary element model. The comparisons reveal good agreement between the results of the current model and those of the previous models. The robustness of the current model has been highlighted by examining the responses of the tunnel at points on the free surface when it is subject to a point harmonic load at its invert. The responses of the piled-foundation to horizontal and vertical point loads on the pile-head are also investigated, in addition to the displacement field on the free surface due to a vertical point load.

KEY WORDS: Ground-borne vibration; Boundary element method; Soil-structure interaction, Tunnel; Piled-foundation

1 INTRODUCTION

Underground railway noise and vibration can be a major source of disturbance to occupants in close proximity. Vibration is generated at the wheel-rail interface, due to wheels and track irregularities, and propagates through soil to nearby buildings. Whilst these vibrations may not induce structural damage, their effects can impair human comfort and activity leading to long-term implications [1, 2] or can cause malfunctioning of sensitive equipment.

The problem of ground-borne vibration has caught the attention of researchers during the past decades. To better understand the transmission of vibration from underground railways, different numerical simulation techniques have been exploited. These techniques are essentially aimed at identifying ways to tackle unacceptable levels of vibration from existing as well as future railway lines. In the literature, there exist a number of models to calculate vibration from railways that are based on space discretisation and superposition of elastic waves.

Models based on space discretisation employ boundary element (BE) and finite element (FE) methods to simulate the dynamic soil-tunnel interaction, where the FE method is used to model the tunnel’s wall, and the surrounding soil is simulated by the BE method. In the last decade, these methods were often coupled together to provide more rigorous, efficient computation. This was achieved by assuming homogeneity in the track direction allowing for the implementation of a two-and-a-half-dimensional (2.5D) or wavenumber FE-BE model [3], or by incorporating periodicity of the tunnel and soil with the Floquet transform [4, 5]. The periodicity approach was also utilised within the context of a BE method to model soil-piled foundations dynamic interaction [6].

Models based on superposition of elastic waves, on the other hand, are deemed to provide computationally efficient tools. A model that is particularly popular is the pipe-in-pipe (PiP) model, which is a semi-analytical three-dimensional (3D) model accounting for the dynamic soil-tunnel interaction [7, 8]. The main model accounts for a tunnel embedded in a full-space by using the elastic wave equations for two concentric pipes with infinite length. The PiP model has been also augmented to consider a tunnel embedded in a half-space or a multi-layered half-space [9].

Despite the research effort devoted to the topic of underground railway vibration, simplifying assumptions remain necessary in all numerical models due primarily to computational limitations. A common simplifying assumption is to neglect the interaction between neighbouring structures. It must be mentioned, however, that there exist in the literature a few studies investigating the dynamic interaction between neighbouring tunnels [10, 11], and between an underground tunnel and strip-foundations [12] or piled-foundations [13, 14]. In the studies of tunnel/piled-foundations interaction [13, 14], a sub-domain modelling approach was adopted in which the displacements and tractions generated by the tunnel’s vibration were used as input variables for the piled-foundations. Put differently, the presence of piles in the soil was neglected when calculating vibration field due to the movement of a train in a tunnel. This approach results in a weak coupling, and it thereby does not predict accurately the behaviour of the coupled system.

This paper reports on a novel technique for modelling the dynamic interaction of a fully coupled underground tunnel and a piled-foundation embedded in a homogeneous half-space. The surrounding soil is modelled using the BE method adopting half-space Green’s functions, whereas the thin-shell
theory is used to simulate the tunnel’s wall and the piled-foundation is modelled using the bar formulation for axial deformation and beam formulation for bending. The technique achieves the strong coupling by superposing the vibration field generated by the tunnel with that generated by the piled-foundation.

The layout of the paper is as follows. Section 2 describes the modelling strategy for the coupled system and its components, along with the formulations used to perform the calculations. Section 3 provides the models’ parameters and briefly describes the previous models in the literature used to validate the current models. Section 4 presents the modelling parameters and compares the results with previous work in the literature. Finally, the findings are highlighted together with work to follow in section 5.

2 MODELLING STRATEGY

The model presented here aims to study the influence of an existing piled-foundation on the vibration field generated by an underground tunnel. The proposed system comprises a horizontal tunnel embedded at a depth \(d\) from the free surface and a vertical piled-foundation, which is at a horizontal distance \(h\) from the tunnel’s centreline, see Figure 1. The proposed system is modelled by first simulating the vertical and horizontal cavities and then coupling them to the tunnel’s wall and piled-foundation.

![Figure 1. The layout of the model for a coupled tunnel-piled foundation system.](image)

The superposition method is used to solve the system in Figure 1, by breaking it into two sub-models. First, a model of a horizontal cavity embedded in a half-space, without a vertical cavity. Second, a model of a vertical cavity embedded in a half-space, without a horizontal cavity. This is illustrated in Figure 2, where the dashed lines in sub-model 1 represent the position of the vertical cavity and the dashed lines in sub-model 2 refer to the position of the horizontal cavity from the original problem in Figure 1. These are shown to aid in explaining the method.

![Figure 2. The superposition method used for solving the coupled system, showing the sub-models.](image)

The superposition method used in this work is similar to that applied by Kuo et al. [11] to model a twin-tunnel system embedded in a full-space from the solution of a single tunnel. In this paper, the displacement field, for each sub-model, due to forces acting on a single cavity, be they horizontal or vertical, can be calculated at its interface and also at the interface of the virtual cavity. The vibration response of the coupled system can be written as the superposition of the two displacement fields \((U^h, U^v)\). One displacement field is the result of forces acting on a single cavity, whereas the other displacement field is the result of the interaction between the two cavities. Likewise, the total forces applied to the coupled system \((F^h, F^v)\) are equal to the summation of the forces acting on one cavity with those representing the motion induced by the neighbouring cavity. This is illustrated by the following equations,

\[
U^h = U^h_1 + U^h_2 \\
F^h = F^h_1 + F^h_2 \\
U^v = U^v_1 + U^v_2 \\
F^v = F^v_1 + F^v_2
\]

in which the subscripts refer to the sub-model and superscripts refer to the orientation of the cavity, where \(h\) denotes horizontal and \(v\) represents vertical.

From sub-model 1 in Figure 2, relationships between the forces \((F^h_1)\) in the horizontal cavity and its displacements \((U^h_1)\) as well as the displacements \((U^v_1)\) in the virtual vertical cavity read,

\[
U^h_1 = H^{hh}_1 F^h_1 \\
U^v_1 = H^{hv}_1 F^h_1 \\
Q^v_1 = G^{vh}_1 F^h_1
\]

where \(H^{hh}_1\) is the frequency response function (FRF) matrix between the horizontal cavity’s forces and displacements, and \(H^{hv}_1\) and \(G^{vh}_1\) are the FRF matrices between the horizontal cavity’s forces and virtual vertical cavity’s displacements and tractions respectively. The forces \((F^h_1)\) are obtained by integrating the tractions \((Q^v_1)\).

Similarly for sub-model 2 in Figure 2, relationships between the forces on the interface of the virtual cavity \((F^v_2)\) and its displacements \((U^v_2)\) in addition to the displacements \((U^h_2)\) and tractions \((Q^h_2)\) on the virtual horizontal cavity yield,

\[
U^h_2 = H^{vh}_2 F^v_2 \\
U^v_2 = H^{hv}_2 F^v_2 \\
Q^h_2 = G^{hv}_2 F^v_2
\]

in which \(H^{vh}_2\) is the FRF matrix between the vertical cavity’s forces and displacements, and \(H^{hv}_2\) and \(G^{hv}_2\) are the FRF matrices between the vertical cavity’s forces and the virtual horizontal cavity’s displacements and tractions respectively. The forces \((F^v_2)\) are obtained by integrating the tractions \((Q^h_2)\).

The FRF matrices \((H^{hh}, H^{hv}, H^{vh} and H^{hh})\) are obtained by applying a unit force at each degree of freedom (DoF) of the cavity \((F^h)\) and calculating the displacements at all DoFs of the cavity \((U^h)\) as well as the virtual cavity \((U^v)\). The FRF matrices \((G^{vh} and G^{hv})\) are acquired from the displacements at the DoFs of the virtual cavity as follows:
• calculate strains from displacements
• calculate stresses from strains using constitutive relationships
• calculate tractions from stresses using Cauchy’s formula

Now, there are 10xDoFs unknowns and 10xDoFs equations presented in (1)-(3), which could be combined together to solve the coupled system in Figure 1. The solution of the equations yields the FRF matrix of the coupled horizontal and vertical cavities system. This FRF matrix can then be coupled with matrices of a tunnel and a piled-foundation assuming compatibility and equilibrium at the interface. In the following sub-sections, the formulation adopted for modelling the soil, the tunnel and the piled-foundation are presented.

2.1 The soil model

The BE method is used to model the soil, where Green’s functions for a homogeneous half-space soil are used as fundamental solutions in the formulation. The half-space Green’s functions are calculated with the aid of the ElastoDynamics Toolbox (EDT) [15]. The EDT is based on the direct stiffness method and the thin layer method in order to model wave propagation in layered media.

The BE model consists of a total of \(N_s\) constant elements in which tractions and displacements are assumed to be uniform and equal to the values at their central nodes. For each of the \(N_s\) nodes of the BE mesh, there are three values of displacement and traction. These variables are related by,

\[
Hu = Gp, \quad (4)
\]

where \(H\) and \(G\) are \(3N_s \times 3N_s\) matrices describing the behaviour of the soil in terms of its density \((\rho)\), shear modulus \((\mu)\), Poisson’s ratio \((\nu)\), damping ratio \((\eta)\), shear wave speed \((C_s)\), pressure wave speed \((C_p)\) and frequency of interest \((f)\). The \(3N_s \times 1\) \(u\) and \(p\) vectors are assembled from the complex displacement and traction amplitudes of each node as follows,

\[
u = [u_1, u_2, u_3, \ldots, u_N]^T, \quad p = [p_1, p_2, p_3, \ldots, p_N]^T, \quad (5)
\]

where \(u\) and \(p\) are the displacement and traction vectors of node \(j\) respectively. Equation (4) can be rearranged as,

\[
u = H^{-1}Gp, \quad u = H_p, \quad (6)
\]

where \(H\) is the soil’s FRF matrix relating displacements and tractions at the frequency of interest. This FRF matrix is coupled with the FRF matrices of the tunnel and piled-foundation in order to scrutinise the soil-structure interaction.

For the scenario of the embedded tunnel (horizontal cavity), it is assumed that the system is invariant in the longitudinal direction allowing for the use of 2.5D approach. In this approach, the calculations are performed in the wavenumber domain \((k)\), allowing for representing the 3D response of the structure and the radiated wave field on a two-dimensional (2D) mesh. Hence, a FRF matrix for a 2D cavity is calculated at the frequency of interest and coupled to the FRF matrix of the tunnel’s wall. After that, the coupled response is transformed back to the space domain by means of inverse Fourier transform, as

\[
u(y, \omega) = \frac{1}{2\pi} \int \nu(k, \omega) \exp(ik_y y) dk_y, \quad (7)
\]

where the 2.5D Green’s functions of EDT are used for the fundamental solution of the BE formulation.

For the scenario of the piled-foundation (vertical cavity), a 3D BE mesh is utilised using 3D Green’s functions for the fundamental solution. In this case, a FRF matrix describing the 3D behaviour of the vertical cavity at the frequency of interest is obtained and is coupled with the FRF matrix of the piled-foundation in order to calculate the response of the coupled system. It has been made sure throughout the BE analysis that there are more than six constant elements per wavelength to satisfy Dominguez [16] recommendations.

2.2 The tunnel model

Cylindrical thin-shell theory is used to model the tunnel, which is assumed to be invariant in the longitudinal direction, allowing for the formulation of the equations of motion in the wavenumber-frequency domain. In linear vibration theory, if the applied loading comprises harmonic traction components in space and time, the equations of motion are satisfied by similarly harmonic displacement components. Hence, the modal displacements at the mean radius \(a\) of the cylinder due to applied forces in the radial, tangential and longitudinal directions read,

\[
\begin{pmatrix}
\tilde{U}_r \\
\tilde{U}_\theta \\
\tilde{U}_z
\end{pmatrix} = -\frac{1}{Eh} A^{-1} \begin{pmatrix}
\tilde{Q}_r \\
\tilde{Q}_\theta \\
\tilde{Q}_z
\end{pmatrix}, \quad (8)
\]

where \(E\) is the Young’s modulus, \(h\) is the thickness, and the coefficients of matrix \(A\) can be found in [11] for symmetric and antisymmetric cases. The radial \((\tilde{U}_r)\) and longitudinal \((\tilde{U}_z)\) displacements are associated with \(\cos\theta\), while the tangential \((\tilde{U}_\theta)\) displacement is associated with \(\sin\theta\).

Since, the tunnel in this paper is subject to a point harmonic load at its invert, it is necessary to decompose the point load into its space-harmonic components before finding the corresponding displacements. The variation of the load can be written as a linear combination of the ring modes \((n)\) by means of a Fourier series, as

\[
\tilde{U}(\theta) = \frac{1}{2\pi a} \sum_{m} b_m \cos m\theta, \quad (9)
\]

on the interval \(-\pi < \theta < \pi\), where \(\delta(\theta)\) is the Dirac-delta function. Using the representation of the point load in (9), the input force can be written in a form that suits the thin-shell formulation in order to calculate the response of symmetric and antisymmetric force for a given ring mode \((n)\).

In order to couple the tunnel to the soil’s cavity, the FRF matrix of the tunnel is needed. This is formulated by using the dynamic stiffness approach, in which the shell is divided into a number of nodes. Each node has three DoFs representing the longitudinal, tangential and radial directions (see Figure 3). With the aid of the thin-shell formulation in (8) and the representation of point load in (9), a FRF matrix relating the response at each DoF to the applied force reads,

\[
U = H_F, \quad (10)
\]

where the size of matrix \(H\) is \(3N_s \times 3N_s\).
and the coefficients $\alpha, \beta$ are found from the boundary conditions.

The general solution of the pile to a unit harmonic force or moment with angular frequency $\omega$ applied in/around the transverse directions $(x, y)$ at node $j$ reads

$$u_j(z, \omega) = u_j^l(z, \omega) = A_j^l \cos \alpha z + B_j^l \sin \alpha z$$

for $0 \leq z \leq z^l$, (13)

$$u_j(z, \omega) = u_j^r(z, \omega) = A_j^r \cos \alpha z + B_j^r \sin \alpha z$$

for $z^l \leq z \leq L_p$.

where $\alpha = \omega \sqrt{\frac{\rho_p}{E_p}}$, the superscripts $I$ and $II$ indicate the sections above and beneath the node $j$ and coefficients $A_j^I, B_j^I, A_j^{II}, B_j^{II}$ are found from the boundary conditions.

The general solution of the pile to a unit harmonic torque with angular frequency $\omega$ applied around the longitudinal direction $(z)$ is,

$$\theta_j(z, \omega) = \theta_j^l(z, \omega) = A_j^l \cos \lambda z + B_j^l \sin \lambda z$$

for $0 \leq z \leq z^l$, (15)

$$\theta_j(z, \omega) = \theta_j^r(z, \omega) = A_j^r \cos \lambda z + B_j^r \sin \lambda z$$

for $z^l \leq z \leq L_p$, in which $\lambda = \omega \sqrt{\frac{\rho_p}{G_p}}$ and coefficients $A_j^I, B_j^I, A_j^{II}, B_j^{II}$ are found from the boundary conditions.

Likewise the tunnel, the FRF matrix of the pile’s centroid $(H_p)$ can be assembled which has a size of $6N_p \times 6N_p$. This matrix is then transformed to give the FRF matrix $(H_{P})$ of the pile’s nodes around the circumference as,

$$H_p = T_p^T H_p T_p$$.

(16)

where the matrix $H_p$ has a size of $3N_p \times 3N_p$ and the size of the transformation matrix $T_{p2}$ is $6N_p \times 3N_p$.

To this end, the systems in (6) and (16) can be coupled in the same way as in (12).

3 MODEL PARAMETERS AND COMPARISONS

In this paper, only the results of the uncoupled system, i.e. sub-models 1 and 2 in Figure 2, are presented. The model of the tunnel is validated against the PiP model [9], which carries out the computations assuming that the tunnel’s near field displacement is not influenced by the free surface. The PiP simulates the vibration of a tunnel embedded in a half-space in three steps. First, the model calculates the displacement at the tunnel-soil interface using a model of a tunnel embedded in a full-space. It then calculates equivalent internal forces in a model of a full-space, without a tunnel, that produce the same displacements at the tunnel-soil interface as computed in the first step. Finally, the PiP considers a half-space model, without a tunnel, and multiplies its Green’s functions by the equivalent forces in the second step.

For comparisons, a scenario of a tunnel embedded at a depth of 5m and subject to a harmonic point load at its invert is considered. The responses are calculated in the frequency
range 1-80Hz. The number of elements in the BE mesh, which is 40 constant node-collocated elements of equal size, conforms to the number of elements in the tunnel. The parameters used in modelling are summarised in Table 1.

Table 1. Tunnel and soil parameters used for calculating the results of current model and PiP model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tunnel</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>(a)</td>
<td>2.75m</td>
</tr>
<tr>
<td>Density</td>
<td>(\rho)</td>
<td>2500kg/m³</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>(E)</td>
<td>50GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>(\nu)</td>
<td>0.3</td>
</tr>
<tr>
<td>Damping loss factor</td>
<td>(\eta)</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Soil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>(r)</td>
<td>3m</td>
</tr>
<tr>
<td>Density</td>
<td>(\rho)</td>
<td>1800kg/m³</td>
</tr>
<tr>
<td>Shear wave speed</td>
<td>(C_s)</td>
<td>200m/s</td>
</tr>
<tr>
<td>Pressure wave speed</td>
<td>(C_p)</td>
<td>400m/s</td>
</tr>
<tr>
<td>Damping loss factor</td>
<td>(\eta)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The model of the piled-foundation (sub-model 2) is contrasted to the model of Talbot and Hunt [6], which utilised the BE method for the soil applying the fundamental solution of full-space Green’s functions. This has led to the discretization of the free surface in order to account for the half-space behaviour. Another feature of Talbot and Hunt’s model is that it represented the circular cross-section of the pile by four elements in order to ease the discretization of the free surface using rectangular elements. The current model has 16 constant node-collocated elements of equal size, which conform to the number of elements in the tunnel. The tunnel wall could also be modelled as a continuum (i.e. thick-wall theory) instead of the BE method for the soil applying the fundamental solution of full-space Green’s functions. This led to the discretization of the free surface in order to account for the half-space behaviour. Another feature of Talbot and Hunt’s model is that it represented the circular cross-section of the pile by four elements in order to ease the discretization of the free surface using rectangular elements. The current model has 16 constant node-collocated elements of equal size, which conform to the number of elements in the tunnel. The number of elements in the BE mesh is equal to that in the pile’s model.

The responses of the pile due to axial and transverse loading are considered in this paper. These are presented in the form of flexibility coefficients \(F_{ij} = I_{ij} + \lambda_{ij}\), which are obtained by normalising the driving-point FRFs to their static values. Table 2 gives the parameters used in modelling the piled-foundation.

Table 2. Piled-foundation and soil parameters used for calculating the results of the current model and Talbot and Hunt’s model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Piled-foundation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>(L_p)</td>
<td>7.5m</td>
</tr>
<tr>
<td>Radius</td>
<td>(r_p)</td>
<td>0.35m</td>
</tr>
<tr>
<td>Density</td>
<td>(\rho_p)</td>
<td>1687kg/m³</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>(E_p)</td>
<td>25GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>(\nu)</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Soil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>(r)</td>
<td>0.35m</td>
</tr>
<tr>
<td>Density</td>
<td>(\rho)</td>
<td>2250kg/m³</td>
</tr>
<tr>
<td>Shear wave speed</td>
<td>(C_s)</td>
<td>200m/s</td>
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<tr>
<td>Pressure wave speed</td>
<td>(C_p)</td>
<td>490m/s</td>
</tr>
<tr>
<td>Damping loss factor</td>
<td>(\eta)</td>
<td>0.03</td>
</tr>
</tbody>
</table>

4 Results and Discussion

In this section, the results of the current model are presented and compared against previous models in the literature.

4.1 The tunnel model results

The first set of the results considers the vertical displacements on the free surface \((z = 0)\) at the points \((20m, 0m, 0m)\) and \((20m, 20m, 0m)\). Such results are particularly important for practicing engineers to assess underground railway and design mitigation.

Figure 5 depicts the modulus of the vertical displacement \((U_z)\) at the point \((20m, 0m, 0m)\), calculated with the current model and the PiP model. Some differences between both solutions can be seen in the frequency range of 1-20Hz and also in the range of 60-80Hz. However, at mid-range frequencies (25-55Hz) the models compare reasonably well.

The vertical displacement \((U_z)\) modulus computed by the two models at the point \((20m, 20m, 0m)\) on the free surface is presented in Figure 6. The models show at some frequencies reasonable agreement. However, differences of about 10dB are observed around the frequency range 20-30Hz.

The reason of these discrepancies in Figure 5 and Figure 6 could be attributed to a number of reasons. One is that the PiP model simulates the tunnel using continuum theory, which is more robust than the thin-shell theory adopted in the current model. Another reason that could cause such differences is the reflections of waves from the free surface. The PiP model does not include a free surface. A third could be due to the discretization rules followed in developing the BE model and also in the wavenumber sampling.

Despite the differences between both models in Figure 5 and Figure 6, the predictions of the current model are promising and could be further improved following more investigations. It is essential to ensure that the current model of the tunnel provides adequate predictions for the vibration levels before proceeding to the next steps of coupling the tunnel to the piled-foundation. The tunnel wall could also be modelled as a continuum (i.e. thick-wall theory) instead of using the thin-shell theory. However, this is unlikely to make a difference over the frequency range under consideration.

![Figure 5. Modulus of the vertical displacement at the point (20m, 0m, 0m) on the free surface computed by the current model and the PiP model.](image-url)
The second set of results deals with displacement fields on the free surface at frequencies 10Hz and 50Hz. These are only presented for the current model. Waves generated at the tunnel due to the harmonic load at its invert propagate through the soil and result in Rayleigh waves at the surface of the half-space.

In Figure 7, the horizontal displacement field is illustrated for both frequencies. It can be observed that the displacements through the zero x-axis equal zero due to symmetry. The displacements at other symmetry points are equal in magnitude and opposite in direction.

Figure 8 shows the longitudinal displacement field for both frequencies. All displacements through the plane of zero y-axis equal zero due to symmetry. Along the horizontal axis, the displacements are equal in magnitude and in the same direction, whereas along the longitudinal axis the displacements are equal in magnitude and opposite in direction.

The vertical displacement field is shown in Figure 9. It can be seen that the wavefronts on the surface of the half-space are not cylindrical due to the nature of the source and the dynamic interaction between the soil and the tunnel. In this figure the displacements at the symmetry points are equal in magnitude and opposite in direction.

Based on the parameters provided in Table 1, the Rayleigh wave speed is about 190m/s. This results in a Rayleigh wavelength for the frequency 10Hz of about 19m and for the frequency 50Hz of approximately 3.8m. Indeed, these values are calculated based on the single source of a point harmonic load, which is not the scenario for the results presented in Figure 7 - Figure 9.

It can also be discerned from Figure 7 - Figure 9 that the magnitude of the longitudinal displacement (Figure 8) is less than that of the transverse displacements.

4.2 The piled-foundation model results

The first part of these results is concerned with presenting the flexibility coefficients at the pile-head against non-dimensional frequencies (\( \omega = \omega r / C_s \)) range from 0 to 0.5. This is the range that was considered in previous work on modelling soil-pile interaction for seismic purposes, in which higher frequencies are not considered.
Figure 10 compares the horizontal pile-head flexibility computed by the current model with those predicted by Talbot and Hunt [6]. The models agree well for both real and imaginary parts at low non-dimensional frequencies. However, small discrepancies are observed at higher frequencies, that are believed to be due to the differences in the size of the BE mesh between the two models. Talbot and Hunt’s model used four elements in the circumference whereas the current model utilises 16 elements.

Figure 11 illustrates the vertical pile-head flexibility due to a unit harmonic axial load computed by both models. For the real part flexibility, the difference between the two models is almost constant at all non-dimensional frequencies. For the imaginary part flexibility, however, discrepancies between the models become clearer at frequencies beyond \( \alpha_0 = 0.25 \).

Figure 10. Comparison of the horizontal pile-head flexibility coefficients predicted by the current model with those predicted by Talbot’s model.

Figure 11. Comparison of the vertical pile-head flexibility coefficients predicted by the current model with those predicted by Talbot’s model.

Given the differences between the current model and Talbot and Hunt’s model in Figure 10 and Figure 11, it can be generally said that the two models are in a good agreement. In essence, the BE mesh of the current model is more adequate as it conforms to the requirements of the mesh size recommended by Dominguez [16]. The current model is also able to predict the dynamic behaviour of the piled-foundation when the pile is subject to torsion – a type of loading that is likely to occur in a fully coupled tunnel-piled-foundation system.

The second part of the piled-foundation results presents the displacement field at the free surface when a unit harmonic vertical point load is applied to the pile-head at a non-dimensional frequency \( \alpha_0 = 0.5 \). Figure 12(a) shows the vertical displacement field at the free surface, where concentric circular wavefronts are observed. This confirms the correctness of the model, as it is expected to have such wavefronts when the piled-foundation is subject to a vertical load on its head.

Figure 12(b) shows a vertical section through the free surface of the displacement field at the location of the pile centroid. The figure indicates the Rayleigh wavelength to be approximately 4.2m. This agrees well with the theoretical Rayleigh wavelength for the parameters in Table 2, which is 4.22m at \( \alpha_0 = 0.5 \). These findings confirm again the accuracy of the current model in predicting the dynamic soil-pile interaction.

Figure 12. (a) The vertical displacement field predicted by the current model when subject to a point harmonic load on the pile-head at a non-dimensional frequency \( \alpha_0 = 0.5 \), (b) vertical section through the free surface at the location of the pile centroid.
CONCLUSION AND FUTURE WORK

This paper has proposed a novel technique for modelling the dynamic interaction between an underground tunnel and piled-foundation. The method is based on superposition of vibration wavefields generated by the tunnel and piled-foundation. The soil in this work is modelled using the BE method where half-space Green’s functions are used as fundamental solutions. The tunnel is modelled using thin-shell theory, whereas the piled-foundation is modelled using an elastic bar for axial deformation and an Euler-Bernoulli beam for bending deformation.

The paper has first presented the results of the dynamic interaction between a tunnel and soil and compared them against the predictions of the PiP model. The comparisons have revealed that the results of both models agree well at most frequencies. However, some differences between the two models are observed at a number of frequencies. These differences are attributed to the reflection of free surface waves that are not adequately predicted by the PiP model, and also to the discretization rules followed in the BE model.

The paper has also presented the results of the dynamic interaction between the piled-foundation and soil. The model predictions are compared against the BE model of Talbot and Hunt [6] by means of the flexibility coefficients. The comparisons include the response of the piled-foundation to a horizontal and vertical point load on the pile-head. It is revealed that both models agree well for the horizontal loading scenario, especially at low frequencies. For the vertical loading scenario, small differences are observed, in particular at higher frequencies.

In general, the presented results have highlighted the ability of the sub-models in adequately predicting the dynamic interaction between the tunnel-soil and soil-piled-foundation. Therefore, the work can now be moved to the next step where both sub-models can be coupled together. The results will then be contrasted to the work of Hussein et al. [14], which employed the sub-modelling technique to investigate the dynamic interaction between a railway tunnel and piled-foundation on the basis of weak coupling.

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