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Pedestrian-structure interaction in the vertical direction: coupled oscillator-force model for vibration serviceability assessment

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ABSTRACT: Despite a lot of effort has gone into research on human-induced vibrations of footbridges in the last decade, there is still a lack of reliable models and adequate design guideline pertinent to dynamic loading due to multiple pedestrians. There are three key issues that a new generation of models should urgently address: (i) interaction between pedestrians and the structure they occupy and dynamically excite; (ii) pedestrian intelligent behaviour; (iii) inter-subject and intra-subject variability of pedestrian walking loads. This paper presents a model of pedestrian-structure dynamic interaction in the vertical direction which addresses the first two issues. The model comprises three sub-models: (1) a model of a footbridge featuring a SDOF system having the dynamic properties of an empty structure, (2) a microscopic model of multiple pedestrian traffic that simulates position and velocity of each individual pedestrian in space and time, and (3) a model of individual pedestrian actuator featuring a periodic force model coupled with a spring-mass-damper oscillator which move together along the structure. The proposed model is applied to a lively footbridge with known modal properties and results are compared to the measured vibration response due to a light pedestrian traffic.

KEY WORDS: human-induced vertical vibrations; pedestrian-structure interaction; footbridges.

1 INTRODUCTION

In the last fifteen years, excessive vibrations of footbridges and floors induced by pedestrians walking have become one of the leading research topic in structural dynamics. Despite considerable advances have been made in the experimental characterisation and numerical modelling of individual pedestrian loading [1]-[4] there is still a lack of reliable models and adequate design guideline relevant to dynamic loading due to multiple pedestrians.

There are three key features of vertical dynamic loading induced by crowds of pedestrians that a new generation of models should urgently address:

(i) *interaction between pedestrians and structure*. Pedestrians are a complex and sensitive dynamic system which motion and the corresponding forces are likely to be influenced by perceptible vibrations of the supporting structure. Although the phenomenon has intensively been studied in the lateral direction (for a review, see [1][5][6]) after the infamous lateral instability problem of the London Millennium Bridge [7], studies of pedestrian-structure interaction in the vertical direction are very rare and limited (e.g., [8][9]).

(ii) *pedestrian intelligent behaviour*. Pedestrians are active agents, able to take decisions and modify their gait while interacting with the other pedestrians and the surrounding environment. The interaction becomes stronger with increasing pedestrian density, particularly for crowds.

(iii) *inter-subject and intra-subject variability of pedestrian walking loads*. It is now widely accepted that not only different people exert different walking loads, but the same person shows a significant variability of footfalls on a step-by-step basis [10]. Neglecting these variations can lead to errors in predicted dynamic response as high as 40 % [2].

The vertical footfalls measured on a rigid surface proved to be inadequate for predicting perceptible structural vibrations that are high enough to affect the gait and therefore alter footfall forces. Caprani et al. [11] focused on a single pedestrian only and proposed a model featuring a single degree of freedom spring-mass-damper oscillator driven externally by a periodic force, which represents footfalls as generated on a rigid surface. The combined force-oscillator system moves along the footbridge at constant velocity. The role of the oscillator was to account for the pedestrian-structure interaction by altering effective dynamic properties of the occupied structure. Numerical simulations of a virtual footbridge showed that the response due to the combined force-oscillator model was closer to what was observed on actual bridges. More recently, Bocian et al. [12] took a different approach and proposed a pedestrian crowd model featuring each individual as a simple inverted pendulum oscillating independently in the vertical direction. They demonstrated that the presence of the crowd increased both the effective damping and mass of the footbridge.

Several mathematical models of crowd intelligent behaviour have been proposed by applied mathematicians and transportation engineers since the early sixties in the fields of urbanism and public safety. These models can be roughly divided into two main categories: (1) macroscopic models (e.g., [13][14][15]), in which the flow of pedestrians is viewed as a continuous flow of a homogeneous fluid, and (2) microscopic models (e.g., [16][17][18]), which describe the position and velocity of each pedestrian in time under the influence of the surrounding people. In general, macroscopic model is preferable when high pedestrian densities are involved, while microscopic model can be more effective in cases of low density pedestrian traffic. So far, both kind of models have been used to simulate pedestrian crowds on laterally vibrating footbridges (e.g., [19][20]), but none of them has been applied to the case of vertical vibrations.

Inter-subject variability has been tackled by probabilistic force models (e.g., [21]), where key parameters of pedestrian loading (e.g. dynamic load factor, pedestrian weight, step frequency, step length, walking velocity) is described using a probability density function. A review of the basic statistics of these parameters can be found in [1]. On the other hand, intra-subject variability has never been reported in great detail.

This paper presents a new model of vertical pedestrian crowd excitation, which addresses two out of three key features of pedestrian dynamic excitation listed above, i.e. human-structure interaction and intelligent pedestrian behaviour. The modelling approach is motivated by the model proposed by Caprani et al. [11], with the following two advances: (1) it describes more realistic loading case scenario of a multi-person traffic rather than an individual pedestrian, and (2) the velocity of pedestrians is not time-invariant, but is determined using a crowd microscopic model. Broadly speaking, the model can be applied to any kind of lively structure with any kind of pedestrian traffic. For example, complex two-dimensional spaces such as floors and footbridges with obstacles along the walking path, as well as different crowd densities. In this study, the model will be verified against measured vibration response of a footbridge under light pedestrian traffic.

2 DESCRIPTION OF THE MODEL

Modelling multiple-pedestrian traffic and dynamic interaction of each pedestrian with the supporting structure is elaborated in the following subsections and its general framework is sketched in Figure 1.

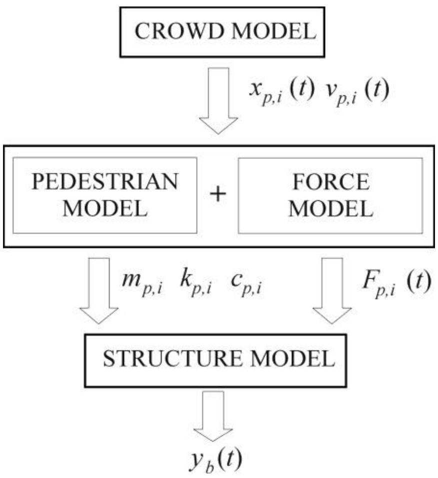

Figure 1: Framework of the pedestrian-structure interaction model

2.1 *Microscopic crowd model*

The crowd dynamics is described via a microscopic crowd model introduced in [22]. Such model describes the evolution of a given crowd in a two dimensional space over time and accounts for the following key features of the pedestrian behaviour [23]:

- pedestrians are able to interact with others within so called a *sensory region* [24], i.e. the space around a pedestrian limited by their vision and proximity of the surrounding people;
- pedestrians are *anisotropic* agents, i.e. they are not equally affected by different external stimuli (e.g. sound) coming from different directions (e.g. ahead and behind);
- walking velocity is affected by the surrounding environment and the interaction with the other pedestrians.

For a crowd of N persons, the model defines the velocity of each individual. Specifically, the velocity of the i-th pedestrian ($i=1,2,...,N$), who occupies the position $\mathbf{x}_{p,i}=\{x_{p,,i}, z_{p,,i}\}$, is given by two contributions:

- a desired velocity $\mathbf{v}_{d,i}$, which is the velocity she would keep in the absence of others;

- a social velocity $\mathbf{v}_{s,i}$, which accounts for her interaction with others.

Hence, the velocity of the i-th pedestrian reads as

$$\dot{\mathbf{x}}_{p,i} = \mathbf{v}_i = \mathbf{v}_{d,i} + \sum_{\substack{j=1 \\ j\neq 1}}^{N} \mathbf{v}_{s,i}\left(\mathbf{x}_{p,i}, \mathbf{x}_{p,j}\right), \tag{1}$$

where

$$\mathbf{v}_{s,i} = \left[-c\frac{\mathbf{x}_{p,i}-\mathbf{x}_{p,j}}{\left|\mathbf{x}_{p,i}-\mathbf{x}_{p,j}\right|^2} \right] \cdot h\left(\mathbf{x}_{p,i}, \mathbf{x}_{p,j}\right), \tag{2}$$

and c is a positive free parameter that weights the interactions. Function h accounts for the interactions within the sensory region, which is modelled as a circular sector of radius R and angular semi-amplitude $0<\alpha<\pi/2$ with respect to the direction of $\mathbf{v}_{d,i}$ (Figure 2):

$$h\left(\mathbf{x}_{p,i}, \mathbf{x}_{p,j}\right) = \begin{cases} 1 & if\ \mathbf{x}_{p,i}-\mathbf{x}_{p,j} < R\ \&\ \dfrac{(\mathbf{x}_{p,i}-\mathbf{x}_{p,j})\cdot v_{d,i}}{\left|\mathbf{x}_{p,i}-\mathbf{x}_{p,j}\right|\cdot\left|v_{d,i}\right|} > \cos\alpha \\ 0 & otherwise \end{cases} \tag{3}$$

In other words, h equals 1 when $\mathbf{x}_{p,j}$ is in the sensory region of $\mathbf{x}_{p,i}$ and 0 otherwise.

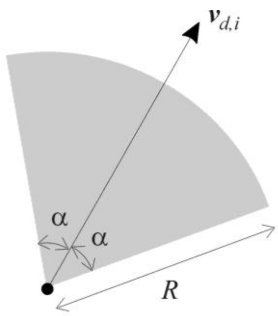

Figure 2: Sensory region

Pedestrians are kept within the boundaries of the sensory domain by directing the vector of desired velocity inwards the domain. In cases of very long walking spaces illustrated in Figure 3 (i.e. when L>>B), the direction of the desired velocity is chosen to have an angle $\theta(z)$ with the longitudinal axis x, where $\theta(z) = 2\theta_0\, z/B$. More details can be found elsewhere [22] .

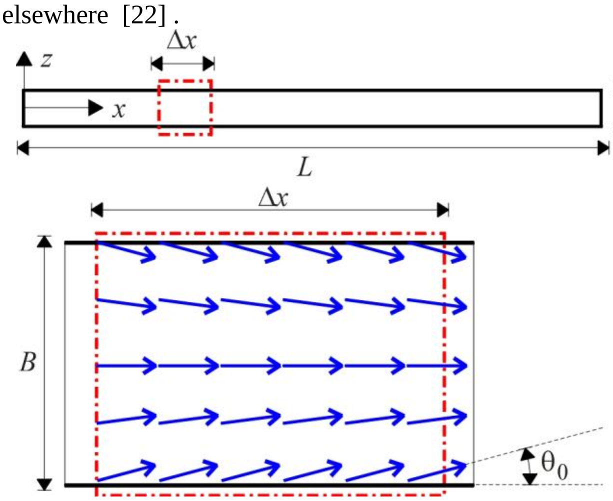

Figure 3: Direction of $\mathbf{v}_{d,i}$ when L>>B

The magnitude of the desired velocity is assumed equal to the free velocity, i.e., the walking velocity in an unconstrained walking regime (see [25] for a review of mean values proposed by several authors).

2.2 Modelling pedestrian excitation

Each pedestrian is modelled as a spring-mass-damper oscillator (SMDO) coupled with a driving force which move together on a structure at the velocity $v_{p,,i}$ and exerts the force $F_{p,,i}$ (Figure 4). With respect to the model proposed in [11], the velocity of each pedestrian is not constant in time, but is a result of the crowd microscopic model described in 2.1.

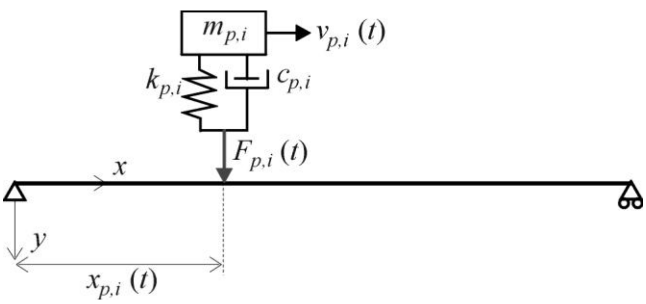

Figure 4: The pedestrian model travelling on a bridge structure.

In this study, the force is modelled as a single sine function, representing the first harmonic of the walking load:

$$F_{p,i}(t) = \alpha_{p,i} m_{p,i} g \sin\left(2\pi f_{p,i} t + \theta_{p,i}\right) \tag{4}$$

where the dynamic load factor $\alpha_{p,i}$ is expressed as a function of the walking frequency $f_{p,,i}$ [10]:

$$\alpha_{p,i} = -0.2649 f_{p,i}^3 + 1.3206 f_{p,i}^2 - 1.7597 f_{p,i} + 0.7613 \tag{5}$$

and the phase angle $\theta_{p,i}$ is modelled as a random variable uniformly distributed on the interval [0, 2π].

The walking frequency of each pedestrian is determined from the results of the crowd simulation ($x_{p,,i}(t)$ and $v_{p,,i}(t)$), as the mean walking frequency during the footbridge crossing.

2.3 Modelling pedestrian -structure interaction

The interaction between the pedestrians and the structure is described by a dynamic system that couples a 1DOF system, representing the structural vibration mode of interest, to N SMDOs representing the walking pedestrians. In the modal domain, the dynamics of the coupled system can be written in matrix form as:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F} \tag{6}$$

where the mass, damping and stiffness matrices are:

$$\mathbf{M} = \begin{bmatrix} m_b & 0 & \cdots & 0 \\ 0 & m_{p,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{p,N} \end{bmatrix} \tag{7}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & 0 & \cdots & c_{NN} \end{bmatrix} \tag{8}$$

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1N} \\ k_{21} & k_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & 0 & \cdots & k_{NN} \end{bmatrix} \tag{9}$$

with $c_{11} = c_b + \sum_{i=1}^{N} c_{p,i}\Phi^2\left(x_{p,i}\right)$ and $k_{11} = k_b + \sum_{i=1}^{N} k_{p,i}\Phi^2\left(x_{p,i}\right)$; $c_{ii} = c_{p,i}$ and $k_{ii} = k_{p,i}$, for i=2,3,…N; $c_{ij} = c_{ji} = -c_{p,i}\Phi\left(x_{p,i}\right)$ and $k_{ij} = k_{ji} = -k_{p,i}\Phi\left(x_{p,i}\right)$, for i,j=2,3,…N.

The displacement and force vectors are:

$$\mathbf{y} = \begin{bmatrix} y_b \\ y_{p,1} \\ \vdots \\ y_{p,N} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \sum_{i=1}^{N} F_{p,i}\Phi\left(x_{p,i}\right) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{10}$$

with i=1,2,...,N. m_b, c_b and k_b are the modal mass, damping and stiffness of the footbridge; $y_{p,i}(t)$ and $y_b(t)$ are the displacement response of the i-th pedestrian and at the antinode of the footbridge, respectively, while ϕ is the unity-normalised mode shape.

3 APPLICATION

3.1 Description of the benchmark footbridge

The model is tested on a steel box girder footbridge over the river Morača in Podgorica (Montenegro, Figure 5). The footbridge has been the object of an experimental campaign in 2004, extensively described in [26].

Figure 6 summarises the modal properties of the fundamental mode of vibration.

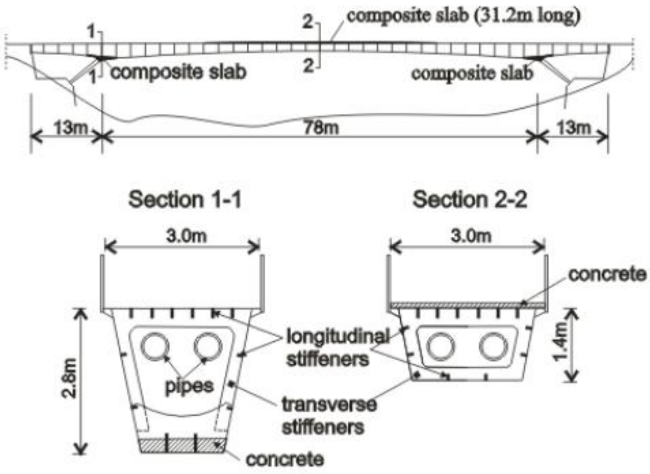

Figure 5: Arrangement of the Podgorica footbridge [26]

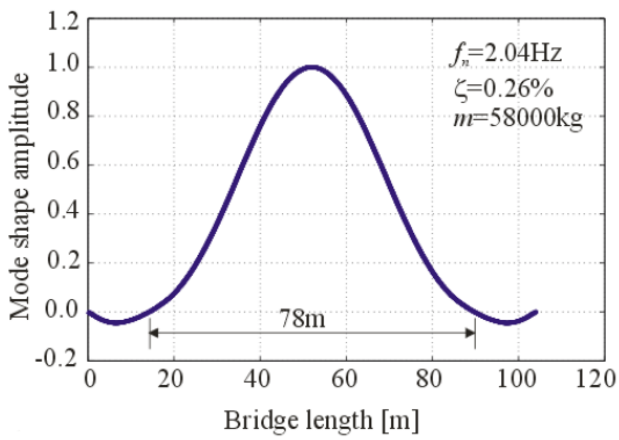

Figure 6: Fundamental mode of vibration from an updated model [26]

3.2 Description of pedestrian traffic and SMDOs properties

The pedestrian traffic data measured during Test 3 are used as input data for the computational simulations (Table 1).

Table 1. Pedestrian traffic data in Test 3 [26].

Parameter	
Mean number of people	26
Walking speed (mean, std) [m/s]	1.38, 0.19
Step frequency (mean, std) [Hz]	1.86, 0.18
Step length (mean, std) [m]	0.74, 0.08

The actual pedestrian traffic recorded on the bridge was light, so that only weak interactions among pedestrians are expected. This means that the velocities recorded on the footbridge can be considered as desired velocities. Hence, the magnitudes of the pedestrian desired velocities $v_{d,i}$ are randomly generated using a Gaussian distribution whose parameters are reported in Table 1 .

The dynamic properties of each SMDO are summarised in Table 2. Pedestrian masses are randomly assigned from a Gaussian distribution [26]. On the other hand, due to the lack of more accurate statistical descriptions, $c_{p,i}$ and $k_{p,i}$ are randomly assigned from uniform distributions. The values of human damping and stiffness reported in the literature depend on the kind of activity (e.g., walking, hopping, running) and bio-features. The whole range of possible values is from 1000 to 100000 N/m for stiffness and from 0 to 1000 Ns/m for damping [27]. In this work, the range of stiffness values proposed in [28] are adopted, while the damping of a walking person could be reasonably set in the range 0-400 Ns/m.

Table 2. Pedestrian dynamic properties.

Parameter	
Mass $m_{p,i}$ [kg] (mean, std)	75, 15
Damping $c_{p,i}$ [Ns/m] (min, max)	0, 400
Stiffness $k_{p,i}$ [N/m] (min, max)	2000, 13000

3.3 Simulations and results

Simulations of pedestrian traffic are carried out on a spatial domain having the dimensions of the Podgorica footbridge, that is, length L = 104 m and width B = 3 m. Pedestrians' positions are randomly initialised in a virtual inlet. Pedestrian flow was modelled in a steady-state regime, i.e. every time one pedestrian exited the footbridge, another pedestrian was generated in the footbridge inlet. The following values of the traffic parameters were used: $\alpha = \pi/2$ rad; R = 2 m; $\theta_0 = \pi/60$ rad; $c = 0.001\ L\ v_{\text{mean}}$, where v_{mean} is the mean value of the walking speed reported in Table 1. The simulation time equals the duration of the experimental tests, that is, 44 minutes.

In total 30 crowd simulations were performed in order to provide statistical reliability. As an example, Figure 7 plots pedestrian positions (red circles) and velocities (blue arrows) at t=180 s for one of the 30 simulations. Mean and standard deviation (std) value of the main walking parameters are averaged across 30 simulations and reported in Table 3. The walking speeds, which are directly obtained from the simulations, were slightly lower than the ones measured during the experimental test (see Table 1). This result was expected, since the interaction among the pedestrians have the effect of reducing the magnitude of the desired velocities.

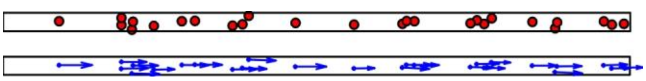

Figure 7: example of pedestrian positions and velocities

Table 3. Walking parameters from crowd simulations.

	Mean	Std
Walking speed [m/s]	1.35	0.05
Step frequency [Hz]	1.87	0.19

The results of each crowd simulation were used as input data for successive two sets of structural dynamic analysis. In the first set (A) pedestrian-structure interaction was neglected, i.e. pedestrians are modelled just as moving loads. In the second set (B) the interaction was taken into account. Figure 8 shows an example of the acceleration response obtained from one of the 30 simulations in both cases A and B.

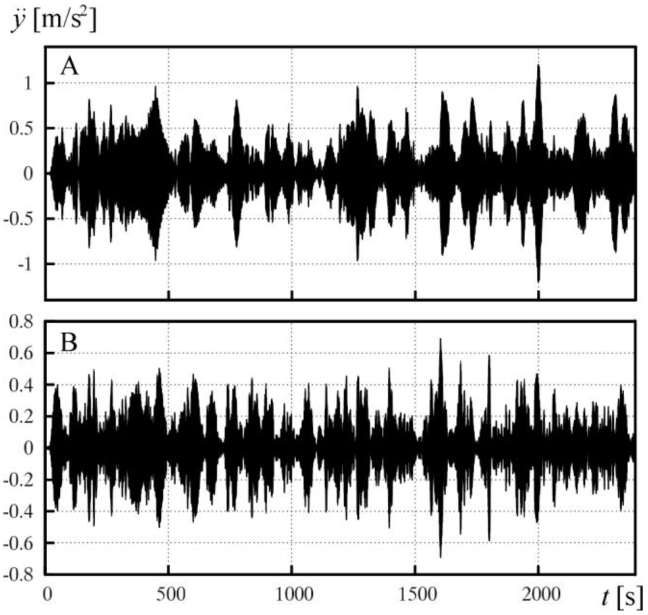

Figure 8: example of acceleration response signals

The simulation results, averaged across 30 simulations, are summarised in Table 4 and compared to experimental measurements. Four descriptors of the structural response were evaluated: the maximum absolute value and the root mean square (rms) value of the acceleration response signal, as well as the mean and std of the peak response per cycle. When pedestrians are modelled just as moving loads, all the evaluated parameters were overestimated for approximately 100%. On the other hand, when the interaction was taken into account, a very good match between experimental and simulation results was observed.

Table 4. Comparison between experimental [26] and simulation results.

	Exp.	Set A [m/s^2]	(A-Exp) /Exp [%]	Set B [m/s^2]	(B-Exp) /Exp [%]
Abs. max	0.69	1.233	+73	0.685	-0.3
Rms	0.14	0.283	+102	0.153	+7.8
Mean peak per cycle	0.17	0.333	+97	0.182	+6
Std peak per cycle	0.10	0.221	+81	0.112	+14

4 CONCLUSIONS

In this study a new coupled model to describe pedestrian-footbridge interaction in the vertical direction has been proposed. The pedestrian intelligent behaviour was taken into account through a microscopic crowd model able to describe the position and velocity of each pedestrian in time. The pedestrian-structure interaction is described by modelling each pedestrian through a combined force-oscillator model, which moves along the structure at the velocity determined through the crowd model.

The model has been tested on the Podgorica footbridge, for which experimental data of both pedestrian dynamics and structural response were available. The numerical simulations have shown that the coupled model can simulate reliably the actual experimental measurements. On the contrary, when the pedestrians were modelled just as moving loads as generated on stiff surfaces, the structural response was significantly overestimated.

The proposed model could be further improved through an experimental calibration of the dynamic parameters of the SMDOs and by substituting the periodic force model with a more realistic near-periodic representation, so to account for the natural variability between successive footfalls.

Further studies should be carried out to evaluate the model on a wider range of structures, such as different types of footbridges and floors, and for different crowd situations, such as very dense crowds.

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