Numerical simulation of an overhead power line section under wind excitation using wind tunnel test results and in-situ measured data

Dominik Stengel¹, Milad Mehdianpour¹, Mathias Clobes², Klaus Thiele²
¹BAM Federal Institute for Materials Research and Testing, Unter den Eichen 87, 12205 Berlin, Germany
²Institute of Steel Structures, Technische Universität Carolo Wilhelmina, Beethovenstraße 51, 38106 Braunschweig, Germany
email: dominik.stengel@bam.de, milad.mehdianpour@bam.de, m.clobes@is.tu-braunschweig.de, k.thiele@is.tu-braunschweig.de

ABSTRACT: Overhead transmission lines are very sensitive structures in regards to wind action. The cables, spanning over a few hundred meters contribute in particular to the overall action on the suspension towers. These slender structures incorporate both structural nonlinearities from the large deformation of the cables and aerodynamic nonlinearities which need to be accounted for when it is to estimate the system response to strong wind events. In this work, a finite element procedure is presented to model an existing power line section using nonlinear cable elements. The wind force is assumed quasi-steady with force coefficients determined in wind tunnel test on a conductor section. Further, aerodynamic damping is incorporated by considering the relative velocity between cable nodes and oncoming wind flow. The results are compared with on-site measurements of the cable’s support reaction. The results show a significant effect of damping since almost no resonant amplification is visible both in observation and simulation. In addition, wind tunnel tests approved aerodynamic damping to be large for the system of sagging cables, but nonlinear in its nature. It is concluded, that the dynamic response of overhead transmission line cables has to be modeled with care, considering all sources of nonlinearities. That is of particular interest in case of random excitation such as wind because the peak response depends on the probability distribution of the system’s response.

KEY WORDS: Overhead transmission lines; Cable dynamics; Aerodynamic damping; Wind tunnel; In-situ measurements.

1 INTRODUCTION

Spanning over several hundreds of meters, conductors of overhead transmission lines contribute significantly to the overall wind load acting on the supporting suspension towers [1]. Cables of such length own a specific nonlinear characteristic due to the large deformations which need to be accounted for in case of an adequate modelling. Same applies for the equation of motion which includes aerodynamic nonlinearities.

Early works [2] gave a preliminary insight into the structural characteristics of overhead transmission line cables more than into the details of the wind excitation. Those works were soon been followed by simulations [3] highlighting the importance of incorporating nonlinear effects and aerodynamic damping in regards to support reaction of suspended cables. But even in recent work, that aspect is not always included [4]. Later studies on wind tunnel models [5] stress again the need of detailed analysis when it comes to in-depth investigations of such a complex issue as the random wind excitation of nonlinear structures.

In the following, a procedure for modelling overhead transmission line cables will be presented and validated. The importance of aerodynamic damping included in the nonlinear equation of motion will be emphasized and supported by wind tunnel tests. Results of simulation shall be compared with observations on an existing test line.

2 PRESENTATION OF TEST LINE

2.1 Cable configuration

The detailed presentation of the test line [6] shall be summarized here with the main parameters, necessary for the later model of the line, see Table 1. The conductors are quad bundles of Al/St 265/35 wires.

Table 1. Quad bundle configuration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus</td>
<td>74000 N/mm²</td>
</tr>
<tr>
<td>Diameter</td>
<td>4.224 mm</td>
</tr>
<tr>
<td>Cross-section</td>
<td>4.0998 mm²</td>
</tr>
<tr>
<td>Weight</td>
<td>4.297.7 kg/m</td>
</tr>
</tbody>
</table>

The line section includes three spans with different span lengths $l_s$ supported by two suspension towers T1 and T2 in between and two dead-end towers WA at both ends, see Table 2. A common parameter to classify sagging cables is Irvine’s parameter $\lambda^2$ which also considers the stressed cable length $l$ and the horizontal tensile force $H$ and is also given in Table 2.

$$\lambda^2 = \left(\frac{\mu \cdot g \cdot l_s}{H} \right)^2 \cdot \frac{l_s}{H \cdot \frac{F}{A}}$$

(1)
Table 2. Test line presentation.

<table>
<thead>
<tr>
<th>Field number</th>
<th>Span length $l_e$ (m)</th>
<th>Height difference $\Delta h$ (m)</th>
<th>Stressed length $l$ (m)</th>
<th>Irvine’s parameter $\lambda^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>389.5</td>
<td>-10.3</td>
<td>391.0</td>
<td>135</td>
</tr>
<tr>
<td>2</td>
<td>406.5</td>
<td>17.8</td>
<td>408.4</td>
<td>147</td>
</tr>
<tr>
<td>3</td>
<td>439.0</td>
<td>-8.9</td>
<td>441.0</td>
<td>171</td>
</tr>
</tbody>
</table>

Figure 1. Elevation of test section.

2.2 Sensor equipment

There is a total number of 13 anemometers installed on the line to capture the acting wind field in its horizontal components. Vertical wind velocities are considered negligible. The system’s response is measured at the highest point of the line in means of the sway angle of the insulator chain at point T2 [6].

3 WIND TUNNEL TESTS

3.1 Force coefficients

The aerodynamic force coefficients are estimated by wind tunnel tests on a section of the quad bundle in smooth flow. The resulting drag coefficients are used in the numerical simulations.

Tests are performed in the wind tunnel at the Institute of Steel Structures of Technische Universität Braunschweig. Sections of single and bundle conductors are placed in smooth flow. In order to ensure two-dimensional circulation end plates are placed at both sides of the model. Drag forces are measured with load cells at the outside of the wind tunnel, see Figure 2.

The stationary drag coefficient is preliminarily described for a single conductor, a stranded wire of aluminum-steel (Al/St 265/35) with diameter $d = 22.4$ mm.

The drag coefficients show a clear dependency on the Reynolds’s number, see Figure 3. That range approximately corresponds to wind events with velocities of 10 to 20 m/s.

3.2 Effect of shadowing

The bundle consists of four single conductors, combined in a quadratic cross section with side length of 40 cm, see Figure 4.

Wind tunnel tests on the bundle result in a so called shadowing factor that was found to be independent on the Reynolds’s number. Only dependent on the rotation of the bundle, the leeward conductors are more or less in the shadow of the windward ones. Since this effect is almost equal for all Reynolds’s numbers the shadow factors can be plotted against the rotation of the bundle, see Figure 5.
4 FINITE ELEMENT MODEL

4.1 Cable elements

To model the conductors, a total of 20 cable elements per each span are chosen with around 20 m length each. The element formulation takes into consideration the unstressed cable length $l_0$, its weight per unit length $\mu$, its stiffness $EA$ and a catenary between the element nodes $A$ and $O$, see Figure 6.

The assumed catenary between the nodes allows for determining the nodal forces and subsequently the element stiffness, mass and internal force, $\{k\}$ and $\{q\}$ respectively [7].

The insulator chains are also modelled as vertical cables with axial rigidity $EA = 18.54 \cdot 10^8$ N.

4.2 Modelling of supports

The supports, in regards to the towers are modeled as mass-spring systems, see Figure 7. Modal stiffness $k$ and natural frequencies $f_0$ have been determined [8] and an equivalent modal mass $m$ was found using the first natural frequency in both the horizontal directions $r = x, y$, see Table 3.

$$f_{0,r} = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} \tag{2}$$

![Figure 6. Catenary cable element.](image)

![Figure 7. Towers equivalent mass-spring-system.](image)

![Table 3. Equivalent mass and stiffness of supports.](image)

<table>
<thead>
<tr>
<th>Tower and direction $r$</th>
<th>First natural frequency $f_{0,r}$ (Hz)</th>
<th>Modal stiffness $k_r$ (kN/m)</th>
<th>Modal mass $m_r$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WA, $x$</td>
<td>1.52</td>
<td>117</td>
<td>1283</td>
</tr>
<tr>
<td>WA, $y$</td>
<td>1.58</td>
<td>117</td>
<td>1187</td>
</tr>
<tr>
<td>T1, $x$</td>
<td>1.11</td>
<td>39</td>
<td>802</td>
</tr>
<tr>
<td>T1, $y$</td>
<td>1.14</td>
<td>39</td>
<td>760</td>
</tr>
<tr>
<td>T2, $x$</td>
<td>1.05</td>
<td>38</td>
<td>873</td>
</tr>
<tr>
<td>T2, $y$</td>
<td>1.07</td>
<td>38</td>
<td>841</td>
</tr>
</tbody>
</table>

4.3 System’s modal properties

The system’s modal properties are dependent on the sag of the cable which again is dependent on the cable’s tensile force $H$ and are evaluated by the undamped equation of motion with $K$ and $M$ being the stiffness and damping matrix, the natural frequencies $\omega$ and eigenvector $q$.

$$[(K) - \omega^2 [M]] \{q\} = 0 \tag{3}$$

The out-of plane modes are swaying modes which coincide with the swaying modes of the single spans. The influence of the insulator chains which theoretically act as a double pendulum is insignificant and the $n$th swaying mode can be estimated according equation (4).

$$\omega_n = \frac{n \pi}{l_s} \sqrt{\frac{H}{\mu}} \tag{4}$$

Modal analysis of the structure’s model and the theory lead to similar results of the first pendulum frequency of $\omega_1 = 0.83$ rad/s which coincides with the bandwidth of the excitation of turbulent wind with most energy content between 0 and 1 Hz.

Higher orders of pendulum frequencies are in accordance as well. Same accounts for the in-plane modes. Their theoretic investigation can be found in detail in [9] but shall not be investigated in further detail herein because they are well out of the spectrum of turbulent wind excitation.

4.4 Wind forces

Wind forces $F$ are determined using the results of the wind tunnel experiments and the relative velocity between wind flow $u$ and $w$ in horizontal and vertical direction and structure under quasi-steady assumptions.

$$F_{xi} = \frac{p}{2} c_{di} d l (u_i - \dot{u}_{xi})^2$$

$$F_{zi} = \frac{p}{2} c_{di} d l (w_i - \dot{w}_{zi})^2 \tag{5}$$

Considering the relative velocity implies the effect of aerodynamic damping resulting from the structure’s movement which causes a force acting in opposite direction.
4.5 Free oscillation

In free oscillation the effect of aerodynamic damping can best be visualized. Assuming the undamped equation of motion, the whole system is deflected laterally to the line direction from its static position and left free assuming $u = w = 0$ in Equation (5).

$$[M]\{\ddot{q}_i^k\} + [K_i^k]\{\Delta q^k\} = \{f_i^k(\dot{q}_i^k)\} - \{R_i^k\} \quad (6)$$

Figure 8. Displacement in along wind direction at the insulator at T2.

Even without the incorporation of structural damping the displacement at the insulator at suspension tower T2 shows a significant decay, see Figure 8. Particularly for the first few periods the logarithmic decrement can be estimated very large $\delta = 0.10$ which is much larger than structural damping usually assumed for stranded wires [10]. Additionally, aerodynamic damping will increase with increasing wind velocity since it is coupled by the factor $2u_i\dot{q}_ix_i$ in Equation (5).

Wind tunnel tests on that topic confirm that approach. A single conductor is mounted in the wind tunnel free to move in a pendulum fashion as illustrated in Figure 9.

From measurements of the decay process of the sway angle under different velocities of oncoming flow it can be seen that damping increases with increasing wind velocity. The increase is non-linear for reason of non-linear movement of the cable on a circular path which can be approved by numerical simulation of the tests.

4.6 Time histories of wind velocities

Besides the measured time histories of wind velocities, there are a number of nodes of the model with unknown wind velocities. Eurocode standard terrain category III is assumed together with nationally defined potential profile and reference wind speed estimated by best fit from the measurements, according Equation (7).

$$\bar{u}(z) = u_{ref} \frac{z}{10}^{0.22} \quad (7)$$

A two-dimensional wind field is generated, neglecting wind acting parallel to the line direction. Cross-correlations between longitudinal and vertical wind components are neglected as well. This simplification can be justified by a minor influence of vertical wind turbulence on the total system response and a significant reduction of effort in stating the spectral density matrix $S$ in Equation (8).

$$S(f) = \begin{bmatrix} S_{uu} & S_{uw} \\ S_{wu} & S_{ww} \end{bmatrix} \approx \begin{bmatrix} S_u & 0 \\ 0 & S_w \end{bmatrix} \quad (8)$$

For each turbulence component $k = u, w$, the spectral density matrices $S_k$ are composed by auto- and cross-spectral densities, $S_{k_ik_j}$ and $S_{k_ik_j}$ of the nodes $i$ and $j$.

$$S_k(f) = \begin{bmatrix} S_{k_ik_1} & S_{k_ik_2} & \cdots & S_{k_ik_n} \\ S_{k_2i1} & S_{k_2i2} & \cdots & S_{k_2in} \\ \vdots & \vdots & \ddots & \vdots \\ S_{kn_i1} & S_{kn_i2} & \cdots & S_{kn_in} \end{bmatrix} \quad (9)$$

The off-diagonal entries account for the spatial correlation of adjacent time histories, which is considered using Davenport’s definition for the coherence function $\gamma_{k_{ij}}$.

$$\gamma_{k_{ij}} = \exp \left( -2f \sqrt{C_{ki}^2\Delta x^2 + C_{ky}^2\Delta y^2 + C_{kz}^2\Delta z^2} \right) \frac{\bar{u}_i}{\bar{u}_j} \quad (10)$$

Spectral densities and turbulence properties were identified for the nodes of measurements [6]. Further information such as the decay coefficients $C_{kr}$ were taken from literature [11].

Time histories at the nodes without observations are generated by wave superposition [12] which generates based on the given measurements further correlated stochastic time histories for the nodes $j$.

$$u_j(t) = \sum_{m=1}^{N} \sum_{n=1}^{N} H_{jm}(f_n) \sqrt{2/\Delta f} \cos(2\pi f_n t + \Theta_{jm}(f_n) + \Phi_{mn}) \quad (11)$$

In Equation (11), $H_{jm}$ are derived from the lower triangular matrix of the spectral density matrix, obtained by singular value decomposition and QR-factorization. $\Theta_{jm}$ is the phase angle between the nodes $j$ and $m$, $\Phi_{mn}$ is the phase angle of process $m$ if known or a random angle uniformly distributed in the interval $(0; 2\pi)$ for artificially generated time histories.
In Figure 10, as an example three times histories of adjacent nodes are displayed. The both outer time histories are the observed ones, while for the node in between time history is generated according to the procedure explained before.

![Figure 10. Time histories of fluctuating wind velocities of three adjacent nodes.](image)

This wind field, composed from measurements and additional artificial time histories is used to model the system’s response under quasi-steady assumption for the wind load.

5 RESULTS

5.1 Mean response

Usually, the system’s response, the wind force at the tower T2 here, is separated into mean and fluctuating part. The mean response is theoretically linked to the mean wind velocity and a constant aerodynamic force coefficient $c_T = 1.0$, a single mean wind velocity $\bar{u}$ and wind span length $L = 0.5 \left( l_x + l_y \right)$.

$$\bar{R} = \frac{D}{2} c_T d L \bar{u}^2$$  \hspace{1cm} (12)

With help of the gravity force $G = 21 \text{kN}$, one can derive the mean sway angle of the response $\bar{\phi}_R$.

Table 4 summarizes data for a specific wind event together with a comparison of measured, simulated and theoretically derived mean response with air density $\rho = 1.28 \text{kg/m}^3$ and mean wind velocity averaged over all anemometers $\bar{u} = 12.28 \text{m/s}$. Accordance is satisfying, considering the simplifications behind Equation (12).

<table>
<thead>
<tr>
<th>Method</th>
<th>$\bar{\phi}_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>10.2°</td>
</tr>
<tr>
<td>Simulation</td>
<td>11.5°</td>
</tr>
<tr>
<td>Theory</td>
<td>9.9°</td>
</tr>
</tbody>
</table>

![Figure 11. Time history of fluctuating sway angle.](image)

5.2 Fluctuating response

The zero-mean fluctuating response is regarded both in time domain and frequency domain. There is an obvious discrepancy between simulation and measurements in time domain, as it can be observed in Figure 11.

In frequency domain one can observe that in both spectra, no significant resonant amplification is visible, even though the first natural frequencies of the system are to be found in the range of wind excitation below 1 Hz.

Discrepancy between observation and simulation is found around 1 Hz where the tower frequencies are to be expected. Indeed it can be shown that the first natural frequencies of the towers decrease for the complete model. For instance, the reason is thought to lie in neglecting wind acting on the towers and resulting response of those. Since energy content is very low, that issue is not further investigated.

![Figure 12. Auto-spectral density of sway angle.](image)

The comparison of the histograms in Figure 13 on the other hand shows satisfactory agreement of simulation and observation.
6 DISCUSSION

Simulations are always subjected to a lot of uncertainties since assumptions need to be made for quantities which cannot be measured. The discrepancy of simulated and measured fluctuating response in time is assumed to be due to a lack of certainty in the time history of generated wind velocities. While mean value of response is in good agreement as well as the probability distribution, accordance in time domain is hard to achieve. It can be observed that best accordance is achieved using less elements, resulting in less assumed wind velocities but also resulting in a less accurate mechanical model of the test line. Keeping that in mind and regarding the good accordance in frequency domain and probability distribution, the agreement of simulation and measurements is considered satisfying for proving the mechanical system to be accurate.

7 CONCLUSION

In this work, a finite element model has been presented to simulate the behavior of an overhead transmission line section under turbulent wind load. Drag coefficients, dependent on the Reynolds’s number as well as the rotation of the conductor bundle are determined by wind tunnel tests and used under quasi-steady assumptions. Aerodynamic damping is included as the effect of fluid-structure interaction and shows to be significant for the system’s dynamic behavior omitting resonant amplification. Measured and generated time histories of wind velocities are used to load the model and the response is evaluated in time and frequency domain. Since wind velocities are stochastic processes, perfect agreement in time domain is hard to achieve. But frequency domain analysis as well as probability distribution of the system’s response allow for conclusion of a satisfying agreement between simulation and observation.

Next steps will be the extrapolation to higher wind speeds which are the basis for dimensioning the suspension towers against turbulent wind loading. Particularly, probability distribution is of further interest to estimate the probability of exceedance of the system’s response.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the sponsoring of this research project by the transmission system operators 50Hertz Transmission GmbH, E.on Netz GmbH and TenneT TSO GmbH.

REFERENCES