ABSTRACT: Pendulum tuned mass dampers (PTMDs) are common auxiliary damping devices used to attenuate excessive motion in tall structures. The concept of effective or equivalent damping is commonly referenced when quantifying tuned mass damper (TMD) performance. Effective damping refers to determining the damping in a single-degree-of-freedom (SDOF) oscillator operating at the same natural frequency that would produce an equal mean squared displacement response as the combined main and auxiliary system. Despite its simplicity, effective damping introduced by a TMD has experienced relatively little use in describing the performance of in-service TMDs, since its theoretical computation is based on the displacement response of the structure, which is seldom measured. Instead, acceleration response measurements are taken from which displacements need to be inferred. The proposed method applies the extended Kalman filter for combined state and parameter estimation for the purpose of estimating the effective damping introduced by a PTMD. The acceleration response measurements are fitted to the response of a SDOF system, where the unknown modal damping is appended to the state vector and estimated. The assumption of known bare structure natural frequency is subsequently relaxed and the natural frequency of the structure without the PTMD is estimated alongside the effective damping. The algorithm is first demonstrated using a numerical example and compared to the theoretical calculation. The methodology is also shown using full-scale acceleration response measurements collected from a structure equipped with a PTMD. The results of the study demonstrate the approach is an accurate and reliable means of quantifying the performance of in-service PTMDs without a direct measure of the displacement response of the attenuated structure and a priori knowledge of the underlying structure’s natural frequency.

KEY WORDS: Effective damping, Equivalent damping, Tuned mass dampers, Extended Kalman filter

1 INTRODUCTION

Tuned mass dampers (TMDs) are devices, used in structures susceptible to vibrations, which introduce supplemental damping to attenuate the responses. TMDs consist of a small inertial mass with stiffness and damping elements. The stiffness element is generally a spring used to adjust the frequency characteristics of the device to closely match the main structure frequency for the mode of vibration to be controlled; alternatively, a suspended mass is used where frequency adjustment is performed by changing the suspension length, known as a pendulum tuned mass damper (PTMD). TMDs are designed to minimize the root mean square (RMS) displacement or acceleration response of the structure, or to maximize the effective or equivalent damping introduced. The latter term refers to the level of modal damping in a single-degree-of-freedom (SDOF) oscillator operating at the same frequency as the controlled mode of vibration (generally the mode with the largest contribution to the overall response) that would produce the same RMS displacement response as the attenuated system. Theoretical equations relating the various main and auxiliary system parameters (frequency ratio, mass ratio, main mass damping ratio, and auxiliary mass damping ratio) to the effective damping [17, 11, 16] are commonly used in predicting the performance of a TMD during the design phase; however, the measure has experienced relatively little use in describing the performance of in-service TMDs. This is because the computation is based on the measured RMS displacement response of the main structure, which is generally impractical to measure directly. This is particularly true for flexible tall structures, which are most susceptible to wind-induced vibrations and commonly employ TMDs. The acceleration responses are usually measured, from which the displacements need to be inferred. The noisy nature of acceleration response measurements preclude direct integration for the displacement response, and initial and final at-rest assumptions to correct accelerometer bias are generally not appropriate for wind-excited structures. The issue is of particular significance when it must be demonstrated that a prescribed level of effective damping has been achieved.

The current work proposes using a concept known as state estimation to determine the effective damping introduced by the PTMD. PTMDs are considered exclusively hereafter, but the concepts can be readily applied to conventional TMDs. State estimation is performed by modeling a process in order to provide an estimate of an internal (generally unobserved) state given the measurement of the actual system. A mathematical model of a known physical system is developed to provide the related extended Kalman filter (EKF). Estimating the effective damping is a combined state and parameter estimation problem, which is inherently nonlinear, and requires the related extended Kalman filter (EKF).

The paper is organized as follows. First, a brief theoretical background of PTMDs and effective damping is introduced.
Second, the concept of combined state and parameter estimation using the EKF is presented. Third, the effective damping for a simple numerical model with known natural frequency is estimated and compared with the theoretical calculation. Finally, the effective damping for a simple numerical model with known natural frequency is estimated and compared with the theoretical calculation. Finally, the effective damping of a full-scale structure is estimated using acceleration response measurements; in this instance, the requirement of a known frequency for the controlled mode of vibration for the bare structure is relaxed.

The findings demonstrate that the proposed methodology is accurate and reliable for estimating the effective damping introduced by a PTMD on an in-service structure.

2 THEORETICAL COMPUTATION OF EFFECTIVE DAMPING

The concept of effective or equivalent damping introduced by a TMD was first proposed by Vickery [17] and McNamara [12]. The method seeks to match the mean square displacement response of the TMD-attenuated system with a SDOF oscillator operating at the same frequency of the controlled mode. The damping in the SDOF system is the effective damping of the attenuated structure. Equations relating the parameters of the main and auxiliary systems of the TMD-equipped structure to the effective damping have been developed for main mass excited [12] and base excited structures [8] with conventional translational TMDs and PTMDs. PTMDs are considered herein, but the concepts can be easily applied to conventional TMDs.

The equations of motion for a PTMD attenuated structure (Figure 1) expressed in modal coordinates are as follows:

\[
M_r \ddot{y}(t) + C_r \dot{y}(t) + K_r y(t) + m_a \ddot{\theta} = F_r(t)
\]

\[
m_a L^2 \ddot{\theta}(t) + c_a h^2 \dot{\theta}(t) + \left( m_a g L + k_a h^2 \right) \dot{\theta}(t) + m_a L \ddot{y}(t) = 0
\]

where \( M_r, C_r, K_r, \) and \( F_r(t) \) are the modal mass, damping, stiffness, and force and \( \ddot{y}(t) \) is the modal coordinate. \( \dot{\theta}(t) \) is the rotation of the auxiliary mass, \( m_a \), with auxiliary damping, \( c_a \), and stiffness, \( k_a \). \( L \) is the length of the pendulum and \( h \) is the length to the attachment of the auxiliary spring and damping. The function \( F(t) = e^{i\omega t} \) is selected in order to compute the complex frequency response functions, \( H_y(i\omega) \) and \( H_\theta(i\omega) \), where \( \omega \) is the forcing frequency. The responses are then

\[
y(t) = H_y(i\omega) \frac{F_r(t)}{K_r}
\]

\[
\theta(t) = H_\theta(i\omega) \frac{F_r(t)}{K_r L}
\]

Computing the first and second derivative of the responses in Equations (2) and substituting into Equations (1) gives

\[
\begin{bmatrix}
-\omega^2 \left( M_r + m_a \right) + i \omega C_r + K_r \\
-\omega^2 m_a L
\end{bmatrix}
\begin{bmatrix}
-\omega^2 m_a L \\
-\omega^2 m_a L^2 + i \omega c_a h^2 + k_a h^2
\end{bmatrix}
\begin{bmatrix}
H_y / K_r \\
H_\theta / K_r L
\end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

where \( \omega_n = \sqrt{\frac{K_r}{M_r}} \) is the circular natural frequency.

Equation (3) is solved simultaneously for the complex frequency response functions, replacing some of the terms with their non-dimensional counterparts. The non-dimensional forcing frequency is

\[
\phi = \frac{\omega}{\omega_n}
\]
\[ H_y(i\omega) = \frac{-\phi^2 + 2i\phi f_r\zeta_a + f_r^2}{\phi^4 - 2i\phi^3 A - \phi^2 B + 2i\phi C + f_r^2} \] 
\[ H_\theta(i\omega) = \frac{\phi^2}{\phi^4 - 2i\phi^3 A - \phi^2 B + 2i\phi C + f_r^2} \]
where
\[ A = \zeta + (1 + \mu) f_r\zeta_a \]
\[ B = 1 + (1 + \mu) f_r^2 + 4f_r\zeta_a \]
\[ C = f_r(f_r\zeta_a + \zeta_A) \]
\[ D = \zeta + (1 + \mu) f_r\zeta_a \]

For the attenuation system with modal damping, \( \zeta_e \), the mean square responses are [8]
\[ E[y(t)^2] = \frac{\omega^2 S_0}{2K^2 \zeta_e} \] 
(7)

For a SDOF oscillator operating at the same frequency as the attenuated system with modal damping, \( \zeta_e \), the mean square response is [6]
\[ E[y(t)^2] = \frac{\omega^2 S_0}{2K^2 \zeta_e} \] 
(8)

Equating the Equations (8) and (9) for the main structure modal responses, the theoretical value for the effective damping for the combined main and auxiliary system is
\[ \zeta_e = \frac{C(BD - C) - f_r^2D^2}{f_r^2(BD - C + 2A(2\zeta_e^2 - 1)) + C} \] 
(9)

where \( A, B, C, \) and \( D \) are defined in Equation (7).

3 EXTENDED KALMAN FILTER FOR COMBINED STATE AND PARAMETER ESTIMATION

State estimation requires a known mathematical model to represent a physical system, and seeks to estimate unknown internal states by relating them to measured outputs. The Kalman filter addresses the problem of estimating the state \( \mathbf{x}_k \) based on knowledge of the linear discrete-time process given by
\[ \mathbf{x}_k = \mathbf{A}_{k-1} \mathbf{x}_{k-1} + \mathbf{G}_{k-1} \mathbf{u}_{k-1} + \mathbf{w}_{k-1} \] 
(10)
with noisy measurement \( \mathbf{z}_k \) given by
\[ \mathbf{z}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{v}_k \] 
(11)

where \( \mathbf{A}_k, \mathbf{G}_k, \) and \( \mathbf{C}_k \) are the system, input, and measurement matrices, \( \mathbf{u}_k \) is the known input, and \( \mathbf{w}_k \) and \( \mathbf{v}_k \) are the unknown process and measurement noises with covariance \( \mathbf{Q}_k \) and \( \mathbf{R}_k \), respectively. When the model parameters are unknown (as is the case when the effective damping parameter is to be estimated), the conventional approach is to append the unknown parameters to the state vector with constant transitions, and perform traditional state estimation. However, the system of equations become nonlinear (despite the fact that underlying system is linear) due to the presence of states (appended parameters) in the system and measurement matrices, producing a product of states within the transition and measurement equations. The EKF is applied for nonlinear state estimation. The EKF is a Kalman filter that linearizes about the current mean and covariance.

For wind-excited structures, the input is generally unknown. For the case of acceleration response measurements of wind-excited structures, there is also a direct feed-through of the unknown input in the measurement equation. For the case of unknown feed-through stochastic disturbance noise, \( \mathbf{d}_k \) with covariance \( \mathbf{S}_k \), the transition of the states is governed by the following nonlinear difference equation:
\[ \mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{d}_{k-1}) + \mathbf{w}_{k-1} \] 
(12)

with noisy measurement given by
\[ \mathbf{z}_k = h(\mathbf{x}_k, \mathbf{d}_k) + \mathbf{v}_k \] 
(13)

For the sake of brevity, the reader is referred elsewhere for a detailed derivation of the EKF for the above system [14]; a summary is provided next. For each time step, the filter equations are
\[ \mathbf{J}_k = \mathbf{E}_{k-1} \mathbf{S}_{k-1} \mathbf{F}_k^T (\mathbf{F}_k \mathbf{S}_{k-1} \mathbf{F}_k^T + \mathbf{R}_k)^{-1} \]
\[ \hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{0}) + \mathbf{J}_{k-1} \left[ \mathbf{z}_{k-1} - h(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{0}) \right] \] 
(14)
\[ \mathbf{P}_{k|k-1} = (\mathbf{A}_{k-1} - \mathbf{J}_{k-1} \mathbf{C}_{k-1}) \mathbf{P}_{k-1|k-1}(\mathbf{A}_{k-1} - \mathbf{J}_{k-1} \mathbf{C}_{k-1})^T + \mathbf{E}_{k-1} \mathbf{S}_{k-1} \mathbf{E}_{k-1}^T + \mathbf{Q}_{k-1} - \mathbf{J}_{k-1} \mathbf{F}_{k-1} \mathbf{S}_{k-1} \mathbf{F}_{k-1}^T \]
\[ \mathbf{J}_k \] is a one-step predictor gain matrix, \( \hat{\mathbf{x}}_{k|k-1} \) is the \( a \ priori \) (when all measurements up to \( k - 1 \) are available) estimate of the state \( \mathbf{x}_k \), and
are the Jacobian matrices of partial derivatives of $f(x, 0)$, $f(x, d)$, $h(h, 0)$, and $h(x, d)$ with respect to $x$ and $d$, evaluated at the current or previous a priori state estimate. $P_{x|k-1}$ is the a priori state estimate error covariance. After a new measurement is taken, the measurement update equations are

$$K_k = P_{x|k-1}C_k^T(C_kP_{x|k-1}C_k^T + F_kS_kF_k^T + R_k)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k[z_k - h(\hat{x}_{k|k-1}, 0)]$$

$$P_{x|k} = (I - K_kC_k)P_{x|k-1}$$

$K_k$ is the Kalman gain, $\hat{x}_{k|k}$ is the a posteriori (when all measurements up to and including $k$ are available) state estimate and $P_{x|k}$ is the corresponding state estimate error covariance. An updated estimate of the measurement is given by [7]

$$\hat{z}_k = h(\hat{x}_{k|k}) + F_kS_kF_k^T(F_kS_kF_k^T + R_k)^{-1}[z_k - h(\hat{x}_{k|k}, 0)]$$

The algorithm is initialized as follows:

$$\hat{x}_0 = E[x_0]$$

$$P_{00} = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$$

Kalman filtering requires knowledge of the unknown stochastic disturbance noise, process noise, and measurement noise covariance matrices $S_k$, $Q_k$, and $R_k$. These statistics are generally unknown for ambient vibration measurements of wind-excited structures, and therefore must be first estimated from the measured data. A noise covariance estimation approach first proposed by Bélanger [3] and adapted by Roffel [14] for the case of the direct feed-through of an unknown disturbance noise is used in this analysis.

### 4 ESTIMATING EFFECTIVE DAMPING USING THE EXTENDED KALMAN FILTER

The effective damping of an in-service PTMD can be computed using Equation (9), provided the displacement response measurements are available. Estimating the effective damping introduced by the PTMD using the EKF is proposed to overcome the challenge of lack of available displacement response measurements of the attenuated system. In the simplest case, the natural frequency of the main structure (without the PTMD) and main mass is known, and the only unknown parameter is the effective damping.

#### 4.1 Effective Damping Estimation for a SDOF Oscillator with a PTMD

The EKF algorithm is first applied to synthetic acceleration response data generated for a SDOF main system equipped with a PTMD; the estimated effective damping is compared with the theoretical value computed using Equation (10).

Since the effective damping is estimated in the modal domain, the measurements are fit to a SDOF system expressed in modal coordinates. The modal displacement and acceleration are selected as the states of the system.

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T = \begin{bmatrix} y(t) & y(t) & \zeta_o \end{bmatrix}^T$$

Following discretization, the nonlinear transition equations are

$$x_1[k+1] = x_1[k] + x_2[k]T + w_1[k]$$

$$x_2[k+1] = -\frac{k}{m}Tn_1[k] + \left(1 - 2\sqrt{\frac{k}{m}x_3[k]}\right)x_2[k] + \frac{1}{m}Td[k] + w_2[k]$$

$$x_3[k+1] = x_3[k] + w_3[k]$$

where $w_k = [w_1[k] \ w_2[k] \ w_3[k]]^T$ are the additive process noise terms with covariance $Q_k$ and $d[k]$ is the unknown stochastic input with covariance $S_k$. The measurement equation is

$$z_i[k] = -\frac{k}{m}x_i[k] - 2\sqrt{\frac{k}{m}x_3[k]}x_3[k] + \frac{1}{m}d[k] + v[k]$$

where $v[k]$ is the additive measurement noise with covariance $R_k$.

The synthetic acceleration response data from the PTMD attenuated structure is generated using the mathematical model of the system using a white noise excitation with the following parameter values. The main and auxiliary masses are $m = 100$ kg and $m_a = 1$ kg, respectively. The main structure stiffness is $k = 1000$ N/m, resulting in a natural frequency of $f = 0.5$ Hz ($\omega_n = 3.16$ rad/s). The damping coefficient is selected based on a modal damping ratio of...
The frequency ratio is $f_r = 0.9926$, resulting in an optimal pendulum length of $L = 0.995$ based on the selection of optimal PTMD parameters available in the literature [8].

The initial estimate of the appended parameter is $\hat{\zeta}_e^0 = 0.01$. The initial state estimate error covariance is selected as 1% of the initial state estimate. One hundred realizations of a filter that was 10 seconds long were run. The average final estimate of the effective damping was $\hat{\zeta}_e^f = 0.0332$ with a coefficient of variation (COV) of $\hat{\zeta}_e = 9.55\%$. This represents a 1.31% error when compared to the theoretical value of the effective damping computed earlier. This demonstrates that the EKF algorithm is can accurately and consistently estimate the effective damping introduced by the PTMD.

The theoretical effective damping for the attenuated system is $\zeta_e = 0.0328$ based on Equation (10). The estimation of the effective damping is plotted with time and compared to the theoretical value in Figure 2. The methodology is now extended for uniaxial multi-degree-of-freedom systems equipped with a PTMD.

\begin{equation}
\begin{align*}
x_1[k+1] &= x_1[k] + x_2[k]T + w_1[k] \\
x_2[k+1] &= -\omega^2_{n,j}T x_1[k] + (1 - 2\omega_{n,j})x_3[k]T x_2[k] \\
&\quad + \frac{1}{M_{r,j}} \phi_j^T d_k T + w_2[k] \\
x_3[k+1] &= x_3[k] + w_3[k]
\end{align*}
\end{equation}

where $\omega_{n,j}$, $\phi_j^T$, and $M_{r,j}$ are the natural frequency, mode shape vector, and modal mass for the controlled mode, respectively. The measurement equations are

\begin{equation}
\begin{align*}
z_k &= \phi_j \left( -\omega^2_{n,j} x_1[k] - 2\omega_{n,j} x_3[k]x_2[k] \right) \\
&\quad + \phi_j \frac{1}{M_{r,j}} \phi_j^T d_k + v_k
\end{align*}
\end{equation}

The performance of the effective damping estimation for the PTMD-equipped MDOF system is demonstrated next using measurement data from a full-scale structure.

5 EFFECTIVE DAMPING ESTIMATION FROM FULL-SCALE MEASUREMENT DATA

The Apron Tower at Pearson International Airport in Toronto, Ontario, Canada is considered as the test bed for demonstrating the algorithm for estimating effective damping introduced by a PTMD (Figure 3). The tower rises 49 m above the terminal roof below. It is a steel structure, with rigid diaphragm floors and a combination of braced and moment frames to resist lateral loads. The tower consists of six main columns and rests on a series of large transfer girders at the roof level of the supporting terminal. The fundamental mode of vibration is approximately 0.67 Hz in the north-south (y-) direction.

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Effective damping estimation for a SDOF PTMD-attenuated system.}
\end{figure}

4.2 Effective Damping Estimation for a MDOF Structure With a PTMD

For a MDOF system, the added complication is the greater number of measurements and the increased number of modes in the structure without the PTMD. The strategy is to perform a modal transformation within the measurement equation in order to relate the states (modal displacements and velocities for the controlled mode of vibration) to the measurements at the degrees of freedom (DOF) using the mode shape vector. Therefore, the modal mass and mode shape vector for the controlled mode, in addition to the natural frequency, must be known a priori. For the MDOF system, the transition equations are as follows (using the same selection of states as previously):

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Apron Tower at Toronto Pearson International Airport.}
\end{figure}

Due to the structure’s inherent flexibility and susceptibility to wind loads, it is equipped with a pair of PTMDs located within the truss roof structure (Figure 4). The PTMDs were installed to reduce user discomfort due to motion during high wind events. Each mass is 25,000 kg, representing a mass ratio of $\mu = 12.4\%$.
The PTMDs are suspended by a series of cables from the structural steel above; each cable is exactly vertical when the PTMD is at rest, preventing a rocking mode and ensuring the PTMD behaves as a point mass. It is generally difficult to predict the actual frequencies of the bare structure during the design phase to the degree of accuracy necessary to tune the PTMD. Therefore, it is equipped with an adjustment mechanism to adjust the pendulum length. The measured length was 0.572 m.

Each mass is equipped with four double-acting fluid viscous dampers (two in each horizontal direction) with a peak damping force of 31.1 N and a maximum stroke of 178 mm. The damping force is velocity-squared proportional; the equivalent linear viscous damping coefficient was calculated to be $c_a = 111.3 \times 10^3$ N m/s [13].

### 5.1 Measurement Program

An extensive measurement program was conducted, where the structure was instrumented with 12 seismic accelerometers along the height of the structure. Several significant wind events were measured, during which high-fidelity measurements were obtained contain energy in several modes. The structure was instrumented with PCB Piezotronics high sensitivity seismic ceramic flexural ICP accelerometers. These sensors are ideal for low-frequency vibrations and provide strong output signal with higher signal to noise ratios. The data was measured continuously at a sampling rate of 200 Hz.

Ten sensors were installed horizontally on the third and first uppermost floors as well as the top chord of the roof truss structure. The sensors were arranged such that the lateral motion in each direction as well as the rotation about the vertical axis could be measured. The remaining sensors were installed in each horizontal direction on one of the PTMDs to gather lateral acceleration response measurements of the auxiliary mass. A total of approximately 3 hours and 20 minutes of data were collected during a significant wind event.

The EKF effective damping algorithm is applied separately in each response direction. Since the fundamental mode of vibration is known to be in the north-south (y-) direction, only the y-direction response is considered further.

### 5.2 Filter Initialization

The effective damping estimation has so far relied on *a priori* knowledge of the frequency and modal mass for the controlled mode of vibration. Alternatively, the equations of motion for a PTMD-equipped structure can be cast in such a way that the modal characteristics of the bare structure (natural frequency, damping ratio, and mode shape vectors of the structure without the PTMD) can be estimated either in advance of or simultaneously with the effective damping. A detailed treatment of this approach has been developed by Roffel [14], and is used in the present analysis for providing the current estimate of the natural frequency and mode shape vector for the effective damping estimation filter. Therefore, two simultaneous EKFs are run. The first filter estimates the bare structure natural frequency, damping ratio, and mode shape. The second filter uses the current estimate of the natural frequency and mode shape vector for the bare structure and estimates the effective damping of the combined main and auxiliary mass system.

A reasonable estimate of the modal mass for the controlled mode used throughout the filter operation, as well as the initial natural frequency and mode shape estimates, was established by developing a finite element model of the structure using software package SAP2000 [5]. The initial estimate of the natural frequency of the controlled mode is 0.656 Hz; the initial damping estimate for the bare structure was selected as 2.5%. The noise covariance matrices were established using the estimation approach mentioned earlier.

The data sets were split into 20 non-overlapping data sets each with a total length of 10 minutes. Each set was resampled at 400 Hz in order to limit the effect of the approximation introduced by the discretization approach.

### 5.3 Effective Damping Identification Results

The first mode natural frequency, averaged over the 20 realizations of the filter, is shown in Figure 5. There was reasonably fast convergence on the final estimate of $f_{n,1} = 0.676$ Hz with a high degree of confidence ($\hat{\zeta}_1 = 0.98\%$). The initial and final converged estimate of the mode shape vector for the controlled mode is shown in Figure 6. The final estimate of the bare structural damping ratio is $\hat{\zeta}_1 = 0.0118$. 

![Figure 5. Natural frequency estimate for the controlled mode of vibration.](image)
The effective damping EKF is run simultaneously with the filter estimating the bare structural modal characteristics. At each time step, \( k \), the current estimate of the circular natural frequency, \( \omega_{n,j} \), and the mode shape vector, \( \phi_j \), are used to update the known model parameters in the effective damping filter (Equations 23 and 24). The initial estimate of the effective damping is 7.5% (\( \hat{\zeta}_e,0 = 0.075 \)). The noise covariance matrices, \( S_k \) and \( R_k \), are the same used for the bare structure modal identification filter.

The performance of the effective damping estimation, averaged over the 20 realizations of the filter, is shown in Figure 7. The final converged value is \( \hat{\zeta}_e = 0.0306 \) with a COV of \( \Delta \hat{\zeta}_e = 18.1\% \). This represents an increase in damping introduced by the PTMD of \( \Delta \hat{\zeta}_e = 0.0188 \). The results are within the range of the expected performance of PTMDs [15].

![Figure 6. Initial and final estimate of the mode shape vector for the controlled mode of vibration.](image)

The variation in the final converged estimate of the effective damping across the 20 data sets is due to the fact that damping in structures is believed to be dependent on the level and nature of the excitation. Although the concept of viscous damping is independent of the amplitude of the excitation [10], it is used in this case to model an inherently more complex phenomenon. This value is expected to vary depending on the return period of the specific wind events. A more comprehensive assessment of the performance would perform a statistical analysis using various events over a period of time in order to better establish the performance of the PTMD. The proposed approach is very amenable to this, as the algorithm can easily be implemented in real time and can continuously provide an estimate on the effective damping of the system.

6 CONCLUSIONS
The EKF algorithm is an accurate and reliable means of estimating the effective damping introduced by the PTMD. When the operating frequency of the effective damping system is known, the error in the estimate was only 1.3% when compared to the theoretical value. The algorithm was extended to estimate effective damping when the modal characteristics of the underlying structure were unknown. The algorithm can be implemented online, and provide a real time measure of the effective damping for the particular loading event. The primary advantage of the approach is overcoming issues related the lack of a reliable means of measuring or inferring the displacement response of the attenuated structure.

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