An innovative approach to model dissipation mechanism in coupled shear walls

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ABSTRACT: The dynamic response of tall buildings plays a significant role in determining both design wind loading and seismic behavior. The degree of energy dissipation, or damping, that a building can provide directly affects the resonant response and thus the effective design loading. In this paper, Replacement Beam Method as a Sandwich beam is presented for coupled shear walls and a one-dimensional finite element method (FEM) has been adopted for lateral analyses of building systems. In this method in addition to structural elements stiffnesses, the damping mechanism has been defined as Distributed Internal Viscous Damping (DIVD) model along shear walls and connecting beams, while acting as bending and shearing mechanisms. Then, an equivalent continuum model of coupled walls, the so called Generalized Sandwich Beam (GSB) model along shear walls and connecting beams, while acting as bending and shearing mechanisms. A reference example has been analyzed numerically to investigate the performance of proposed damping models in basic dynamic characteristics of coupled shear walls such as damped eigenproblems, damping ratio, and near-resonance transversally response. The results have shown the proper efficiency of such damping models in addition to simple application in analysis and primary design of tall building coupled shear wall systems.

KEY WORDS: Tall building; Couple shear wall; Damping; Replacement beam method; DIVD

1 INTRODUCTION

With respect to some up-to-date technical literature [1-3], simple continuum models called Replacement Beams can often provide a useful initial tool as an approximately equivalent model to capture basic features of dynamic response of a wide class of regular buildings, especially for tall and super-tall ones. This approach basically consists in the replacement of the complex building (3D) structure by a continuous equivalent (1D) beam. Depending on the structural characteristics of the building, this replacement beam is characterized by a proper kinematical model and equivalent stiffness parameters, which properly represent the real stiffness of the system as a whole. Up to now, the research works carried out on the replacement beams approach are mostly focusing on the equivalent stiffnesses of the tall building systems, but there is no specific work on the equivalent damping properties assigned to the replacement beams. However in this study it is tried to develop a proper damping model in continuum replacement models and also convertible into the structural and additional damping systems in tall building structures especially coupled shear walls.

Indeed, considering the significant influence of damping on vibration characteristics of continuous beams, wide and deep investigations on the effects of internal and external, distributed or lumped dissipation sources have been conducted. An exhaustive overview on the most used damping models can be found in literature [4-8]. Conceiving damping properties as a mechanism of vibration reduction or suppression, many studies on the characterization of the vibrational behavior of externally damped beam structures have been proposed throughout the past decades.

More recently many authors dedicated their researches on the vibration of elastic systems with viscoelastic damping. Kocatürk and Şimşek [9] studied the dynamic response of eccentrically prestressed viscoelastic Timoshenko beams under a moving harmonic load, characterized by a Kelvin-Voigt damping model. The results indicated that the eccentricity of the compressive load, the shear deformation, the excitation frequency of the moving harmonic load and the internal damping significantly affect the dynamic response of the beam. Kayacik et al. [10] proposed a wide dissertation on the damped natural frequencies of a cantilever beam with Kelvin-Voigt damping and a piezo-patch actuator and sensor bonded onto it, evaluating the effect induced by their locations on the damped fundamental frequencies of the system.

Also more recently, the effect of the location of damped segments on the vibrations of beams with partially distributed internal viscous damping (DIVD) was investigated by Tsai et al. [11]. The vibration equations of a Timoshenko beam with DIVD subjected to transverse loading were derived and the transfer matrix method (TMM) has been used to determine the frequency equations and to study the vibration characteristics. To demonstrate feature of DIVD effects, various damping and restraining conditions have been taken into account. The influence of the damping, length and location of damped segments on the vibration of beams with DIVD has been investigated and discussed. Wei-Ren Chen [12] extended this latter research considering the vibration of an axially loaded
Timoshenko beam with locally distributed Kelvin–Voigt damping and different boundary conditions. Using a finite-elements approach, the quadratic eigenvalue problem of a damped system was formulated to evaluate the eigenfrequencies of the damped Timoshenko beams. The effects of damping amount, lengths and locations of the damped segment, axial load and restraint types on the damped natural frequency of beams have been investigated and discussed.

This consideration prompted the authors to investigate the application of such DIVD model in the replacement beam models (i.e. Sandwich Beam) of coupled shear walls. In the present study vibration equations of a laterally loaded sandwich beam with DIVD are derived through Hamilton’s principle by taking into account two damping models: the Kelvin–Voigt model, composed by shear and bending components in both walls and connecting beams (i.e. with independent $c_s$ and $c_b$ coefficients), and the classical external linear viscous damping model (i.e. $c$) only in walls which introduces dissipative forces linearly proportional to the beam vibration velocity.

By means of a proper space discretization based on Finite Element, the partial differential equations (PDE) of motion are discretized and the time controlled problem is then solved through classical Newmark’s method for a specific loading choice. At the same time a direct solution of the quadratic eigenvalue problem leads to the determination of eigenfrequencies and decay characterization of the damped systems. Also an equivalent continuum model of coupled walls, the so called Generalized Sandwich Beam (GSB), has been provided capturing both elastic and dissipative performances by adopting strain and dissipation energy balance of the set of connecting beams, respectively.

The developments herein introduced with respect to most recent previous works [11-13] consists of evaluating the effects of different damping mechanism combinations ($c_s$ and $c_b$, and $c$), in order to observe the respective and relative damping effects. Considering a reference coupled shear walls system, the influence of DIVD has been evaluated in both real sandwich beam and GSB replacement models.

A further insight improvement of the analysis is finally represented through the GSB and by addressing the problem of the optimal distribution length of non-uniformly partial DIVD such to minimize beam dynamic response (i.e. tip displacement) and control its vibration due to a lateral sinusoidal dynamic load with a near-resonance frequency. It is finally worth noticing that this modeling approach has been primarily conceived as a first step in dealing with practical structural problems such as approximate dynamic modeling of high-rise and tall buildings by means of the replacement beam analysis, where the (continuous) DIVD density reflects the shear wall system dissipation and the smeared equivalent of the (discrete) presence of external damping devices.

2 DAMPED REPLACEMENT BEAM MODELS

2.1 The Real Sandwich Beam (RSB)

In this study, a cantilevered Real Sandwich Beam (RSB) consisting of two main Timoshenko beams connected by a certain number of connecting beams subjected to a homogeneously, sinusoidal in time, distributed transverse load is considered (see Fig.1). The connecting beams are also assumed as Timoshenko beams without axial deformation. In this model, all the beams are able to contain the DIVD model.

![Real sandwich beam (RSB) with DIVD and distributed external force $f(x,t)$](image1)

The transverse equations of motion of such a beam are readily obtained by applying Hamilton’s Principle to the Lagrangian $L$ and the Rayleigh dissipation function $R$ of the system obtained by assuming the kinematic model and the constitutive law of the classical Timoshenko’s theory:

$$\delta \int_0^L \left( L - R \right) \, dt = \delta \int_0^L \left( T - V + W_f - R \right) \, dt = 0 \quad (1)$$

Where Total Kinetic Energy $T$, the Potential Energy $V$, and the Work $W_f$ produced by the external transversal load are respectively expressed as:

$$T = \frac{1}{2} \int_0^L \left[ A_1 (\ddot{u})^2 + I_1 (\dot{\theta})^2 + A_2 (\ddot{\psi})^2 + I_2 (\dot{\phi})^2 \right] \, dx \quad (2)$$

$$V = \frac{1}{2} \int_0^L \left[ E_1 A_1 (\ddot{u}' - \dot{\theta})^2 + E_2 A_2 (\ddot{\psi}' - \dot{\phi})^2 + G A_1 (u'' - \ddot{\psi}) \dot{\psi}' \right] \, dx + \frac{1}{2} \int_0^L \left[ G A_1 (u'' - \ddot{\psi}) \dot{\psi}' \right] \, dx$$

$$W_f = \int_0^L f(x,t) u \, dx \quad (4)$$

Where $A_1$ and $A_2$ are cross-section area of two main Timoshenko beams, and $I_1$ and $I_2$ are moment of inertia of two beams. The Kinetic Energy due to connecting beams is neglected in this study. Young’s elastic modulus $E$, the shear elastic modulus $G$ and the shear correction factor $K$ are also defined.

Also parameters $\nu$ and $\varphi$ defined in Eq.3 refer to lateral and rotational deflections in connecting beams. $n$ and $l_b$ are the number and length of connecting beams.

The damping model herein assumed is such to introduce retarding and dissipative forces arising from damping effects during the motion. The DIVD is originated from the Kelvin-Voigt strain velocity damping, where shear and bending dissipative additional stresses, $\tau^d_x$ and $\sigma^d_x$, linearly proportional to the strain velocity through internal damping coefficients, have to be introduced leading to the following stress-strain relations for left and right main beams (walls) and also connecting beams, respectively:
\[
\begin{align*}
\left( \sigma_x \right)_i &= \left( \sigma_x' \right)_i + \left( \sigma_x'' \right)_i = (Ee_x)_i + c_{b1} \dot{e}_x_i \\
\left( \tau_y \right)_j &= \left( \tau_y' \right)_j + \left( \tau_y'' \right)_j = (Gf_y)_j + c_{b2} \dot{f}_y_j \\
\left( \sigma_x \right)_j &= \left( \sigma_x' \right)_j + \left( \sigma_x'' \right)_j = (Ee_x)_j + c_{b3} \dot{e}_x_j \\
\left( \tau_y \right)_j &= \left( \tau_y' \right)_j + \left( \tau_y'' \right)_j = (Gf_y)_j + c_{b4} \dot{f}_y_j
\end{align*}
\]  
(5)

A further contribution is represented by the classical dissipative force directly proportional to the transverse velocity \( \dot{u}(x,t) \) of the beam, through the external linear viscous damping coefficient \( c \) only in walls. Therefore, the Rayleigh dissipation function can be obtained assuming the subsequent expression:

\[
R = \left\{ \frac{1}{2} \int_{0}^{L} \left[ c_{b1} \ddot{u}^2 + c_{b2} \ddot{f}_y^2 + c_{b3} \ddot{e}_x^2 + c_{b4} \ddot{f}_y^2 \right] dx \right\} + \sum_{\ell} \left[ \int_{0}^{L} \left[ c_{l1} \ddot{\varphi}^2 + c_{l2} \ddot{\psi}^2 \right] dx \right]
\]
(6)

It is worth noticing that the relationship between the viscous damping coefficients \( c_{b1} \) and \( c_{b2} \) can be assumed analogously to that between Young’s elastic modulus \( E \) and shear elastic modulus \( G \) for an isotropic material. Thus, in view of the stress-strain relations of Eq.5 the bending and shear damping coefficients are assumed to be proportional to Young and Shear elastic modules by two independent amplification factors for both main beams and connecting beams, as Eq.7 shows [11-12]:

\[
\begin{align*}
\eta_{b1} &= \eta_{b1} E; c_{b1} = \eta_{b1} G \\
\eta_{b2} &= \eta_{b2} E; c_{b2} = \eta_{b2} G \\
\eta_{b3} &= \eta_{b3} E; c_{b3} = \eta_{b3} G
\end{align*}
\]
(7)

Substituting all the required terms (Eq.2, Eq.3, Eq.4, and Eq.6) in Hamilton’s Principle (Eq.1), PDEs of motion of the real sandwich beam system under investigation can be straightforwardly obtained.

2.2 A Six Degrees of Freedom FE System for RSB

In order to investigate the dynamic response and damping effects of the sandwich beam with DIVD model combined with the classical external viscous damping contribution, the classical FEM approach is adopted in reducing the PDEs of motion into linear second-order ordinary differential equations with time as independent variable. A simple low order beam FE model is therefore introduced for two main beams with linear variation in displacement \( u(x) \) and rotations \( \theta(x) \) and \( \psi(x) \) resulting in six degrees of freedom (DOFs) (Fig.2).

Dimensionless coordinate \( \zeta \) and nodal displacements vector of the \( \ell \)th generic FE are:

\[
\zeta = \frac{2x}{L} \quad \ell \quad U_{x} = [u_{x}, \theta_{x}, \psi_{x}, u_{x}, \theta_{x}, \psi_{x}]^T
\]
(8)

The generalized displacement vector \( s_{x}(x,t) \) of the \( \ell \)th FE can be expressed as:

\[
s_{x}(x,t) = [u \quad \theta \quad \psi]^T = N(x)U_{x}(t)
\]
(9)

Figure 2. FE model for the proposed sandwich beam with six DOFs

Where \( N(x) \) represents the shape functions matrix shape functions matrix containing linear function interpolations. In this study, in order to accurately take into consideration the connecting beams effects, each FE is assumed between two subsequent connecting beams elevation. It should be mentioned that because of connecting beams connection to shear walls, the displacement \( \nu(y) \) and rotation \( \phi(y) \) along connecting beams are attained as functions of the main beams rotations \( \theta(x) \) and \( \psi(x) \) at the connection positions (see Appendix). Assuming however the connecting beams end rotations equal to \( \theta \) and \( \psi \) and also a flexural deflection of connecting beams, following expressions as end boundary conditions of connecting beams can be defined:

\[
\nu\left( \frac{L}{2} \right) = \frac{B_{1}}{2} \theta; \phi\left( \frac{L}{2} \right) = \psi
\]
(10)

By applying Lagrange’s equation to any DOF \((i=1,\ldots,6)\) of the sandwich beam FE, the resulting system of equations of motion of assumes the subsequent expression:

\[
M_{\ell} \ddot{U}_{x} + C_{\ell} \dot{U}_{x} + K_{\ell} U_{x} = Q_{x}
\]
(11)

\( M_{\ell}, \quad C_{\ell}, \quad K_{\ell} \) represent respectively the mass, damping and stiffness matrices of the \( \ell \)th generic FE and \( Q \), the generalized forces vector. Considering the expressions of the Kinematic and Potential energy (Eqs.2-3), of the Rayleigh function (Eq.6) and of the Work due to the external forces (Eq.4), the above matrices can be readily evaluated (see Appendix).

It should be noted that the damping contribution of connecting beams \((C_{1}, C_{2}, \text{ and } C_{3} \text{ in Appendix})\) is obtained by expanding the second term in Rayleigh Function (Eq.6) with respect to connectivity of connecting beams to the walls. By simply assembling and imposing the appropriate boundary conditions, the global system of equations of motion of the sandwich cantilever beam is thus determined, assuming the classical form:

\[
M \ddot{U} + CU + KU = Q
\]
(12)

In order to allow simple assemblage of the matrices and to solve the dynamic system of equations of motion, a FEM-based code has been implemented in Matlab. Newmark’s step-by-step method of direct integration over discrete time steps [14] has been preferred to other algorithms (e.g. non-classical modal analysis which would have provided complex conjugate pairs of eigenvalues and eigenvectors) due to its higher flexibility in dealing with the different damping and loading situations of the (non-classical) problem at hand.

2.3 Damped Eigenfrequency Analysis

The system of equations of motion of the damped freely vibrating sandwich cantilever beam assumes the following form:
\[
M \ddot{U} + C \dot{U} + KU = 0 \quad (13)
\]

Accordingly, introducing \( U = u e^{i \omega t} \) into Eq.13 yields the Quadratic Eigenvalues Problem (QEP):
\[
\left[ \omega^2 M + \omega C + K \right] \phi = 0 \quad (14)
\]

The solution of Eq.14 provides the eigenvalues \( \omega_i \) of the QEP and the associated eigenvectors \( \phi \) which occur in 2n complex conjugate pairs for a non-classical n-DOFs system, such as the cantilever beam under analysis. The QEP requires to identify eigenvalues \( \omega_i \) and associated non-zero eigenvectors \( \phi_i \) (\( i=1,\ldots,2n \)), satisfying the subsequent associated characteristic equation:
\[
\det \left[ \omega^2 M + \omega C + K \right] = 0 \quad (15)
\]

Considering the first eigenvalue \( \omega_1 \), which is associated to the first mode of vibration, by adopting a notation common to the one used for a viscous-damped SDOF system [14], the Real and Imaginary part of \( \omega_1 \) can be identified by the subsequent relation:
\[
\omega_1 = \begin{cases} \text{Real} (\omega_1) + i \text{Im} (\omega_1) = \sqrt{\text{Real}^2 (\omega_1) + \text{Im}^2 (\omega_1)} e^{i \zeta_1} \\ \text{Im} \omega_1,1 + i \omega_{d,1} \sqrt{1 - \zeta_1^2} = -\omega_{d,1} \zeta_1 + i \omega_{d,1} \end{cases} \quad (16)
\]

In Eq.16, \( \omega_{N,1} \) represents the natural frequency, \( \zeta_1 \) the modal damping factor and \( \omega_{d,1} \) the damped natural frequency of first eigenmode according to following expressions:
\[
\omega_{N,1} = \sqrt{\text{Real}^2 (\omega_1) + \text{Im}^2 (\omega_1)} \quad (17)
\]

It is worth noticing that \( \text{Real}(\omega_1) \) represents a parameter proportional to the decay rate of the first mode shape of free vibration of the damped system due to the damping effect, whereas \( \text{Im}(\omega_1) \) represents the frequency of such a damped free oscillation. The subsequent investigations will focus only on the first mode of vibration of the structure i.e. on the first eigenvalues \( \omega_1 \) of the QEP of Eq.15. By considering separately different damping mechanisms on the system, the modal critical damping of the single mechanism can thus be determined.

2.4 Generalized Sandwich Beam (GSB) of Coupled Shear Walls

In order to generalize the initial sandwich beam as a continuum model, the discrete set of connecting beams is replaced by an equivalent homogeneous core characterized by the elastic moduli \( E_{eq} \) and \( G_{eq} \) (see Fig.3a). The achieved continuum model can be attained, under the following assumptions:
- The wall system is in plane stress condition.
- Shear walls have rigid cross section and connecting beams are inextensible.

As suggested in [15], the elastic curtain equivalent to connecting beams is obtained by equating the stress energy of a typical connecting beam to the one of its equivalent continuum. When the equivalent elastic modulus \( E_{eq} \) of the continuum core is assumed equal to zero, the equivalent shear modulus of the continuum core \( G_{eq} \) equivalent with a typical connecting beam reads
\[
G_{eq} = \frac{1}{4} \left( \frac{1}{12E_h} + \frac{1}{GkA} \right)^{-1} \quad (18)
\]

Where \( E \) and \( G \) are elastic and shear modulus of a typical connecting beam with moment of inertia \( I_h \) and area \( A_h \). In evaluating \( G_{eq} \), the stiffness due to local deformation in between connecting beams and walls junctions is also taken into account through the relation
\[
\ell_{b,1} \simeq \frac{\ell_{b,1}}{2} + \frac{2}{3} d \quad (19)
\]

Where \( d \) is the connecting beam height. Compatibility between shear walls and continuum core (shear beam-like) rotations [Fig.3b] in correspondence to the vertical centroidal axis of walls is assumed as
\[
\rho(x) = \frac{\left( B_1 \theta(x) + B_2 \psi(x) \right)}{2 \ell_{b,1}} \quad (20)
\]

Figure 3. a) A Generalized Sandwich Beam (GSB) with equivalent continuum core and DIVD b) Compatible rotation field indicated in a typical portion of GSB

Furthermore, in order to attain the equivalent damping in core \( c_{eq} \), Dissipation Energy due to internal bending damping \( c_b \) and shearing \( c_s \) in a typical connecting beam with DIVD is equated with the dissipation due to the curtain damping. The resulted expressions associated to bending and shearing damping are presented in following equation:
\[
\left\{ \begin{array}{l}
c_{eq} = 4c_b I_h (m_1 + m_2) I_1 \left[ \frac{m_1 + m_2}{m_1 + m_2} \right]^2 + c_{eq} = \frac{4c_s A_h (m_1 + m_2) I_1}{h (B_1 + B_2)} \frac{m_1 + m_2}{m_1 + m_2} \end{array} \right\} \quad (21)
\]

Where \( m_1, m_2, m_b \), and \( m_d \) are defined in Appendix. It should be noted that wall rotations \( \theta \) and \( \psi \) can be assumed approximately the same when two walls widths ratio is between0.25 and 4. Consequently the Potential Energy and Rayleigh Dissipation Function in the proposed GSB are specified respectively with
\[
V(u,\theta,\psi) = \frac{1}{2} \int_{0}^{2\pi} \frac{E_1 (\theta')^2 + G A_1 (u' - \theta)^2}{G_{eq} (\kappa \ell_{b,1}) (u' - \rho)^2} \ \text{dx} \quad (22)
\]
\[
R(u,\theta,\psi) = \frac{1}{2} \int_{0}^{2\pi} \frac{c_{eq} A_h (u')^2 + c_{eq} A_1 (\theta')^2}{c_{eq} (\kappa \ell_{b,1}) (u' - \rho)^2} \ \text{dx} \quad (23)
\]
Note that $\dot{\rho}$ can be defined as a function of $\dot{\theta}$ and $\dot{\psi}$, and $c_{eq}$ performs internally as a shearing damping mechanism in core. Both Eq.22 and Eq.23 are accompanied with the same shape functions of $u(x), \theta(x)$ and $\psi(x)$ applied for the real sandwich beam, and accordingly the stiffness and damping matrices related to a FE of GSB are derived.

3 NUMERICAL EXAMPLE INVESTIGATION

In order to verify reliability of the introduced FE-model, a set of verifications has been accomplished. Firstly applying the undamped equation of motions, the lateral deflection and eigenfrequency of the undamped sandwich cantilever beam (Fig.1) can be easily evaluated and verified. For this purpose, a coupled shear walls system [16] which is a practical sample is investigated (see Fig.4). The tip displacement, 0.0159 m with literature and 0.0161 m by present method, and first four undamped eigenfrequencies (for fundamental frequency, 13.09 Hz with literature and 13.06 Hz through present method) calculated with the FE-model exhibit good agreement with the benchmark results.

3.1 Influence of DIVD in Connecting Beams on Fundamental Eigenfrequency

In order to better understand the damped vibration characterization of the replacement sandwich beam having DIVD only along connecting beams, the trend of first eigenfrequency $\omega_1$ of the whole system is investigated with regard to internal bending $c_b$ and shear damping $c_s$. The trend of first eigenfrequency oscillating and damping parts is shown in Fig.5a to Fig.6b. It can be observed in Fig.5a that the both damping mechanisms diagrams are stable up to a damping multiplier $\eta_b = \eta_s = 0.06$. It can then be seen that from $\eta_b = 0.16$ on bending diagram, the oscillating part grows up suddenly up to $\eta_b = 1$ where the branch tends again to a stable situation. The value of oscillating part is about 65.3 Hz at the final situation of the bending branch (Fig.5a). This value is corresponded to the eigenfrequency of whole system while the DIVD has damped out the bending mechanism in the set of connecting beams. In the case of shear damping mechanism $c_s$, there is a similar behavior to $c_b$ but the dramatic growing occurs at higher damping multiplier $\eta_s$. Comparing the influence of bending and shear damping, it can be easily understood that the damping bending mechanism $c_b$ is more effective than the shear one because it influences on oscillating part with lower damping multipliers $\eta$. It means that the internal bending damping $c_b$ in connecting beams is the most prone mechanism to change dynamic performance of the whole structure. It should be noted that in practice, internal damping multipliers have small values [11]. However it is zoomed in the initial part of diagrams for bending and shear damping mechanisms (Fig.5b). It can be observed that both diagrams are coincided with no sensible variation in oscillating part for small values of damping multipliers.

Similarly, it is investigated also the trend of damping part of first eigenfrequency in Fig.6a. A single-peak diagram is obtained for both damping mechanisms. It can be seen that increasing the bending damping multiplier $\eta_b$, a maximum damping part 40 Hz is obtained for $\eta_b = 0.2$, while in case of shear mechanism the same peak value is resulted for at $\eta_s = 6$. It confirms again that the internal bending damping $c_b$ is dominant in connecting beams in comparison with the shear one.
as linear, but the bending one is dominantly more effective than the shear one. It means that the bending damping mechanism in the set of connecting beams is more prone to dissipate the lateral vibration in the whole system.

![Figure 6b](image)

**Figure 6b.** The practical trend of modal damping ratio $\xi_1$ versus damping multiplier $\eta$

### 3.2 Influence of DIVD in Main Beams (Walls) on Fundamental Eigenfrequency

In order to realize the importance of each damping mechanism in shear walls, the trend of oscillating and damping part of $\omega_1$ is plotted by real sandwich beam versus corresponding damping ratio in Fig.7 and Fig.8, respectively. Firstly concerning the oscillating part, three damping mechanisms produce the same amount up to about 5% of critical damping (Fig.7). It means that there is no difference in applying each of damping mechanisms when a damping ratio less than 0.05 is required. According to Fig.7, the curve related to internal shear damping begins from natural eigenfrequency 13.09 Hz and slightly increases up to 13.35 Hz at the critical damping. Thus the shear damping is not so effective in the walls. Even in case of damping part in Fig.8, there is no sensible effect due to shear damping. Fig.7 clearly shows that both internal bending $c_b$ and external viscous damping $c$ are able to destroy the oscillating part of $\omega_1$. Note that when the oscillating part vanishes, the damping is reached its critical level.

Concerning the damping part diagrams (Fig.8), the external damping curve linearly varies up to the critical value. There is an overlap with bending diagram from origin to 0.2 damping ratio. For damping ratios higher than 0.2, the bending diagram goes up in comparison with the external damping one and finally gives a critical damping part at 17.89 Hz, while the value produced by external damping mechanism is 13.09 Hz.

![Figure 7](image)

**Figure 7.** Trend of oscillating part of $\omega_1$ versus modal damping ratio in walls

![Figure 8](image)

**Figure 8.** Trend of damping part of $\omega_1$ versus modal damping ratio in walls

### 3.3 Verification of Damped GSB Model

In order to verify the accuracy of damped GSB, maximum dynamically deflection of the reference problem subjected to a harmonic sine load is graphed by both real (Fig.1) and generalized sandwich beams (Fig.3) while the internal bending damping multiplier $\eta_b$ in connecting beams is uniformly 0.005 (see Fig.9). Note that only the bending damping is taken into account because of its dominant performance. It can be observed that maximum difference between two methods is about 1.16% in tip-displacement value. Additionally, the comparison of first damped eigenfrequency is plotted for both damping and oscillating parts in Fig.10. There can be also observed a good agreement between discrete sandwich beam and continuum GSB results.

![Figure 9](image)

**Figure 9.** Maximum dynamically lateral deflection resulted by damped real and generalized models

<table>
<thead>
<tr>
<th>$\eta_b$</th>
<th>RSB-Real</th>
<th>GSB-Real</th>
<th>RSB-Imaginary</th>
<th>GSB-Imaginary</th>
</tr>
</thead>
<tbody>
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<td>1.005</td>
<td>1.005</td>
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</table>

![Figure 10](image)

**Figure 10.** Trend of damping and oscillating parts of $\omega_1$ resulted by damped RSB and GSB
3.4 Efficiency of GSB with DIVD models in dynamic deflection and vibration control

Concerning the DIVD model, previous investigations have been performed with identical internal damping in all connecting beam, but imagining the structural deformation of coupled shear walls subjected to lateral loads, the most impressive effect on vibration control is corresponded to connecting beams located at higher elevations [17]. Accordingly by means of the GSB, there is possible to investigate the equivalent effect of non-identical internal damping in connecting beams in order to simulate the existence of additional dissipation devices in coupled walls (i.e. viscous or friction dampers). For this purpose, a non-uniformly DIVD model \( \tau_{eq} \), is defined in the continuum core of GSB model with a zero damping coefficient at its lower end and a maximum coefficient at higher end (see Fig.11).

Further analysis is devoted to controlling tip-displacement dynamic magnification factor by such non-uniformly damping model and optimization of damping length \( L_d \), while having uniformly full damped walls with 5% of bending critical damping \( C_b \). Because of variation in non-uniformly DIVD along \( L_d \) of core, its maximum amount \( \left( \tau_{eq} \right)_{max} \) is considered as the core damping candidate for assigning the damping value. Therefore two main damping contributions are conceived acting on the beam system and defined as follows:

\[
\begin{align*}
C_b &= \left( \tau_{eq} \right)_{max} A_s L_s = \left( \frac{C_{eq}}{C_b} \right)_{max} L \Rightarrow \eta = \frac{C_{eq}}{C_b} \\
C_{eq} &= \left( \tau_{eq} \right)_{min} A_s L_s = \left( \frac{C_{eq}}{C_b} \right)_{min} L \Rightarrow \eta = \frac{C_{eq}}{C_b}
\end{align*}
\]

Where \( A_s \) and \( A_s \) are cross-section areas of left and right walls, and \( t \) and \( t \) are core thickness and width. According to Eq.24, a parameter \( \eta \) is defined as the ratio between resultant damping in the continuum core \( C_{eq} \) and the one in walls \( C_b \) in order to consider different values of core damping. The cantilevered GSB model is loaded by an external distributed sinusoidal dynamic load, \( f(t) \), with amplitude \( F=16500 \) \( N \) and frequency ratio 0.9 between excitation and fundamental frequency, as a near-resonance situation.

The tip-displacement dynamic magnification factor of the reference structure is plotted versus the ratio between damped core length \( L_d \) and total building height \( L \) for different \( \eta \) values (see Fig.12). It can be observed that for the damped length ratio \( L_d/L=0.05 \), there is a suddenly decrease in the dynamic magnification factor for all damping values \( \eta \). This length ratio corresponds to a damped core length as 2.8 m (equivalent with having damping device at the top). It means that even having a short damped core length \( L_d \) at the structure top can control considerably the dynamic response. Increasing the core damping length from 0.05 suppresses the magnification factor up to a minimum value which is associated to optimal length of damping distribution. This optimum value is indicated on each diagram related to each \( \eta \) by a small circle (see Fig.12). For example with \( \eta=10^{-4} \), its optimum length is about 75% of total height producing the response as 4.26. It can be also seen that from \( \eta=2\times10^{-4} \) to \( \eta=5\times10^{-4} \), the optimum damping length is the same as 0.75L. For \( \eta=7.5\times10^{-3} \) and \( \eta=1\times10^{-3} \), the optimum length is positioned at 0.85 and 0.9 of total height, respectively. Also when \( \eta \) is \( \eta=5\times10^{-3} \), \( \eta=1\times10^{-2} \), and \( \eta=5\times10^{-2} \), the whole structure height should be damped in order to get minimum response. It can be concluded from Fig.12 that greater the damping amount \( \eta \), generally higher the optimal core damping length \( L_d \).
CONCLUSION

The replacement beam method can be an approximate and simple tool for capturing some basic dynamic characteristics of tall building structural systems such as coupled shear walls. Dealing with such method in literature, there is focused mostly on equivalent stiffness properties addressing bending and shear behavior. However this study gives an opportunity to generalize this kind of tall building modeling by introducing proper dissipation mechanisms (i.e. bending and shear DIVD) associated to the corresponding stiffness mechanisms in all structural elements. There is also definable an external viscous damping accompanied with internal damping mechanisms. As it is indicated, coupled shear walls can be modeled by real sandwich beam (RSB) and even simpler with continuum generalized sandwich beam (GSB) both equipped with DIVD model inside elements. It is shown that application of such damping model is simple and effective as uniformly by RSB reflecting the shear wall system dissipation and also non-uniformly by GSB simulating also smeared equivalent of the (discrete) presence of external damping devices. By means of such replacement beam models with DIVD, the damped eigenproblems, modal damping ratios, and also transversal dynamic responses can be controlled properly. It was shown that bending DIVD is dominant in both walls and connecting beams and can be used as proper damping mechanism in tall coupled walls. These modeling approaches can be primarily conceived as a first step in dealing with practical structural problems such as approximate dynamic modeling and pre-design of high-rise and tall building sub-systems.

REFERENCES

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- APPENDIX

\[
\begin{bmatrix}
2(A_1 + A_2) & 0 & 0 & (A_1 + A_2) & 0 & 0 \\
0 & 2I_1 & 0 & 0 & I_1 & 0 \\
0 & 0 & 2I_2 & 0 & 0 & I_2 \\
(A_1 + A_2) & 0 & 0 & 2(A_1 + A_2) & 0 & 0 \\
0 & I_1 & 0 & 0 & 2I_1 & 0 \\
0 & 0 & I_2 & 0 & 0 & 2I_2 \\
\end{bmatrix}
\]

\[
M_s = \frac{\rho l^4}{6}
\]

\[
K_s = \frac{1}{2r^2} \begin{bmatrix}
2K_1 & K_\alpha & K_\beta & -2K_1 & K_\alpha & K_\beta \\
K_\alpha & K_1 & K_\beta & K_\alpha & K_1 & K_\beta \\
K_\beta & K_\beta & K_1 & K_\alpha & K_1 & K_\beta \\
-2K_1 & -K_\alpha & -K_\beta & 2K_1 & -K_\alpha & -K_\beta \\
K_\alpha & K_\alpha & K_\alpha & K_\alpha & K_\alpha & K_\alpha \\
K_\beta & K_\beta & K_\beta & K_\beta & K_\beta & K_\beta \\
\end{bmatrix}
\]

Where

\[
K_1 = \frac{E h}{\alpha^3} \left(\frac{12\alpha^2}{\ell_s^2} + \frac{12\alpha}{\ell_s} + 3 + \lambda\right) ; K_{s1} = G\kappa A_1; K_{s2} = G\kappa A_2
\]

\[
K_2 = \frac{E h}{\alpha^3} \left(\frac{12\beta^2}{\ell_s^2} + \frac{12\beta}{\ell_s} + 3 + \lambda\right) ; K_{s1} = E I_1; K_{s2} = E I_2
\]

\[
K_3 = \frac{E h}{\alpha^3} \left(\frac{24\beta\beta}{\ell_s^2} + \frac{12\alpha + 12\beta}{\ell_s} + 6 - 2\lambda\right)
\]

\[
\lambda = 1 + 12r^4 + \frac{E h}{G\kappa A_1} ; \alpha = -\frac{B_1}{2} ; \beta = \frac{B_2}{2}
\]

\[
C_s = \frac{1}{3} \begin{bmatrix}
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
\end{bmatrix}
\]

\[
C_e = \frac{1}{2} \begin{bmatrix}
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
c_{1A} & c_{2A} & c_{3A} & c_{1A} & c_{2A} & c_{3A} \\
\end{bmatrix}
\]
\[

\begin{align*}
C_1 &= c_s I_b \left( \frac{4}{3 \lambda^2} \ell_b^3 + m_b^2 \ell_b - \frac{2m_b}{\lambda} \right) + \ell_s c_s \kappa A_s m_b^2 \\
C_2 &= c_s I_b \left( \frac{4}{3 \lambda^2} \ell_b^3 + m_b^2 \ell_b + \frac{2m_b}{\lambda} \right) + \ell_s c_s \kappa A_s m_b^2 \\
C_3 &= 2c_s I_b (2m_2 \ell_b - \frac{8}{3 \lambda^2 \ell_b} + \frac{2m_1 - 2m_2}{\lambda}) + 2c_s \kappa A_s (m_1 \ell_b)
\end{align*}
\]

Where
\[

\begin{align*}
m_1 &= \frac{3 + \lambda}{\lambda \ell_b} - \frac{3B_1}{\lambda \ell_b^2} ; m_2 = \frac{3 - \lambda}{\lambda \ell_b} - \frac{3B_2}{\lambda \ell_b^2} \\
m_3 &= \frac{\lambda - 1}{2 \lambda} (1 - \frac{B_1}{\ell_b}) ; m_4 = \frac{\lambda - 1}{2 \lambda} (1 - \frac{B_2}{\ell_b})
\end{align*}
\]

\[
Q_0 = \frac{f(t)}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

\[
v(y) = \left[ (m_1 + 1)y + \frac{1}{2} m_2 y^2 - \frac{y^3}{3 \ell_b^2} - \frac{B_1}{2} \right] \theta + \left[ m_1 y + \frac{1}{2} m_2 y^2 + \frac{y^3}{3 \ell_b^2} \right] \psi
\]

\[
\varphi(y) = \left[ m_1 y - \frac{y^2}{\ell_b} + 1 \right] \theta + \left[ m_2 y + \frac{y^2}{\ell_b} \right] \psi
\]