Translational and rotational excitation for the seismic analysis of base-isolated structures

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ABSTRACT: The development of a complete “6-dimensional” seismic input model is addressed for the analysis of large structures resting on rigid mat foundations. The topic is first illustrated within the framework of the linearized theory of dynamic soil-structure interaction and the nature of the kinematic transformation between the free-field seismic motion and the motion of the rigid mat is cleared and discussed. On this basis, the complete spectral description (direct and cross power density functions) of the 6D motion of the rigid mat is derived from the space-time stochastic model of the free field displacement. To this aim different forms of the coherency function relating the horizontal and vertical motion at a point are proposed and tested.

The rigid-mat excitation model is then applied to the dynamic analysis of the reactor building designed within the IRIS international project, this being a classical example of rigid system, i.e. of a structural system which, once base-isolated, tends to behave as a rigid body.

The effect of the rotational input components is studied, by a classical random vibration approach, in view of characterizing their effect on the isolators, especially in term of imposed relative displacement and of applied axial forces. The result show how the torsional rotation input has a negligible effect on the horizontal relative displacement of the isolators, while rocking inputs are responsible for a more significant increase in the axial load variation. The effect of the model assumed for the correlation between the horizontal and vertical free-field motion at a point is also enlightened.

KEY WORDS: Seismic excitation; Isolated structures; Soil-structure systems; Kinematic interaction; Rotational input, Random vibration analysis

1 INTRODUCTION

The introduction of seismic isolation systems is likely to become almost mandatory, in the next future, in the design of buildings and facilities for which superior performances are needed against design actions and whose functionality after the event is of utmost importance. This is the typical case of Nuclear Power Plants; in fact, almost all recent (Generation III and IV) designs of nuclear reactor buildings rely on base isolation for reducing the risk associated to earthquake-induced events to the same order of magnitude of the one deriving from internal faults. In addition, the reduction of the acceleration values transmitted to the equipment and their more uniform pattern throughout the building, which tends to behave as a rigid body, is of great help in the standardization of the equipment itself.

It must be observed, however, that in such conditions the isolation devices themselves are prone to become the critical components in terms of seismic fragility, since the response attenuation is obtained at the price of large relative displacements between the building and the foundation; thus, all factors affecting the isolators performance become of utmost importance in risk evaluation. Among these factors, some input related issues have captured the attention of designers and researchers, namely:

- the effect of the torsional rigid-body mode on the isolator shear strain, which is usually taken into account by code provisions in terms of accidental mass eccentricity,
- the effect of rocking on the variation of the axial forces applied to the isolators.

Both aspects can be affected by the rotational input transmitted by the foundation, which can be usually assumed as rigid in reactor buildings, to the isolated structure; to this topic is addressed the research effort whose preliminary results are summarized in this paper in which, after a presentation of the excitation model, an example is shown, regarding a reactor building resting on HDRB (High Damping Rubber Bearing) devices, these representing one of the most diffused and reliable solutions for seismic isolation. For a preliminary evaluation, the response analysis is here based on a linearized model; the use of non-linear models for the isolation devices is left to future developments of the research.

2 EQUATIONS OF MOTION

According to a standard procedure, the motion of the system can be decomposed into a pseudo-static and a dynamic component, i.e.

\[ q(t) = q^{(p)}(t) + q^{(d)}(t) \]  

where the pseudo-static component is defined as the motion occurring when the mass and the damping of the structure are set to zero, i.e. the one due to kinematic interaction, while the dynamic component accounts for inertial interaction. Note that the \( q(t) \) vector has here the dimensions \( n \) of the number of degrees of freedom of the structure+foundation system, since
the ground behavior is represented through impedance functions referred to the “contact” displacements \( q_c \), i.e. the ones of the nodes at the ground-foundation interface. Assuming that the system is supported by a single rigid surface foundation, the latter displacement components can be expressed, in pseudo-static conditions, as

\[
q_c^{(p)}(t) = q_c^{(f)}(t) + q_c^{(a)}(t) = \gamma q_c^{(p)}(t) \tag{2}
\]

where \( \gamma \) is a constant matrix accounting for the kinematic constraint, while the superscripts \( f \) and \( a \) denote respectively the motion in free-field conditions and the added motion due to the introduction of the rigid mat. The vector \( q_0(t) \) lists the 6 motion components of the latter; the displacement components in this vector are the ones of the centroid of the contact surface (Fig. 1).

![Figure 1. Rigid mat foundation](image)

Given the above definitions, standard manipulation allows for expressing the relationship between the foundation motion in pseudo-static conditions \( q_0^{(p)}(t) \) and the free-field motion of the contact points \( q_c^{(f)}(t) \), this being the one for which established models, based on experimental observations, are largely available in the literature. This relationship turns out to be frequency dependent and is thus expressed below in terms of Fourier Transforms (denoted by a tilde) of time functions as

\[
\tilde{q}_0^{(p)}(f) = H_0^{(g)}(f) \gamma^T E_c^{(g)}(f) \tilde{q}_c^{(f)}(f) = \beta^{(g)}(f) \tilde{q}_c^{(f)}(f) \tag{3}
\]

where \( H_0^{(g)}(f) \) and \( E_c^{(g)}(f) \) denote respectively the frequency response function (FRF) matrix of the rigid foundation resting on an elastic soil and the ground impedance matrix (inverse of the FRF one) referred to the contact displacements \( q_c \); note that no confusion should arise from using the letter \( f \) for frequency as well as for free-field motion.

In the case study here shown a simplified, frequency independent definition of the transformation matrix \( \beta \) has been adopted, as usual done in practical applications, based on the following simplified criteria:

- the translational components in \( q_0^{(p)}(t) \) at the centroid are defined as the average of the corresponding free-field components at the contact points;
- the rotational components are defined as the average of the rigid body rotations obtained for all contact points and projected onto the reference axes; each rotation is obtained by subtracting from the relevant local displacement component (e.g. vertical for rocking components) the value at the centroid (average value) and dividing the result by the distance between the two points.

Once the model for the pseudostatic motion of the foundation is established, the equations delivering the dynamic portion, i.e. the motion relative to the rigid mat, can be written in the standard form

\[
mq^{(d)} + cq^{(d)} + kq^{(d)} = -mRq_0^{(p)}(t) \tag{4}
\]

where \( m \), \( k \) and \( c \) are respectively the mass, stiffness and viscous damping matrix, while \( R \) is a \( (n,6) \) matrix expressing the rigid body motion of the structure as a function of the foundation motion.

3 COMPUTATION OF RESPONSE TO RANDOM GROUND MOTION

To apply equations (3) and (4) a complete space-time description of the 3D free-field seismic motion at contact points is required. Input stationarity has been here assumed to the aim of investigating the effect of the rotational input and of the correlation between ground motion components; the extension to quasi-stationary excitation is under development.

3.1 Ground motion model

To simplify the formal definition of the model the vector of contact point free-field motion has been partitioned between the three translational components (along \( x,y,z \)) as

\[
q_c^{(f)}(t) = \{u(t) \ v(t) \ w(t)\} \tag{5}
\]

where \( w(t) \) is the vector of vertical components. The model is defined by means of the cross spectral density between two components of the seismic motion at two contact points; a problem arises, in this respect, since available literature models for seismic motion are restricted to the correlation between parallel components, while the vertical component at a point is usually taken as uncorrelated with respect to the horizontal ones.

Pursuing a more general approach, a complete model has been here formulated, whose form has been suggested by the one currently adopted for turbulence components in wind modeling; accordingly, and focusing, for example, on the horizontal \( v \) component at point \( j \) and on the vertical component \( w \) at node \( k \), the model is first expressed in the usual form

\[
S_{ij,nk}(f) = \sqrt{S_v(f)S_w(f)} C_{ij,nk} \exp\left[i \theta_{ij,nk}(f)\right] \tag{6}
\]

where \( S_v \) and \( S_w \) are the direct spectral densities, assumed to be equal at all contact points, \( C_{ij,nk} \) is the absolute value of the coherency function (lagged coherency) and the complex exponential accounts for the delay effect, which is usually neglected in wind engineering. The lagged coherency is then modeled as the product of a “point” coherency, modeling the...
correlation among the components at a point, times a “space” coherence, modeling the correlation between parallel components at different points; the latter is in turn introduced as the geometrical average between the coherency functions characterizing the two components, i.e.

\[
C_{y,w_k}(f) = C_{yw}^{(P)}(f)C_{y,w_k}^{(S)}(f) = C_{yw}^{(P)}(f)\sqrt{C_{y,y}(f)C_{w,w_k}(f)}
\]  

Note that the delay effect as well can be ascribed to both local (systematic phase difference between components) and spatial properties (wave-passage) of the seismic motion field.

In the case study here shown, available literature models have been assumed for space coherency functions, e.g. \(C_{y,w_k}(f)\), between parallel components; some rather crude assumptions, on the other hand, have been introduced for the expression of the point coherency, especially when the correlation between horizontal and vertical components have been considered. These assumptions seem to be justifiable within the context of an initial investigation of the effect of the ground motion properties on the response of isolated systems; in parallel activity, a number of real three-component acceleration records are under examination to the aim of establishing more reliable models.

More precisely, two assumptions have been tested for the properties of the point correlation between horizontal and vertical free-field motion, summarized in the following.

The first model, named Constant Coherency model (CCM) in the following, is based on the assumption that the cross “point” spectral density between the vertical component and each of the horizontal ones is real (no local delay effect) and characterized by a constant point-coherency function, whose value \(C_{yw}^{(P)}(f) = C_{w,w}^{(P)}(f) = \overline{C}^{(P)}\) is computed by assigning a value to the correlation coefficient \(\rho = \rho_{yw} = \rho_{ww}\); horizontal components are here assumed as uncorrelated.

In the second model, termed the linear combination model (LCM), it is assumed that the components of the free-field motion at a generic point can be defined as the sum of totally uncorrelated time functions and a coherent wave. This is the case, for example, of the contribution of simple Rayleigh waves, propagating at an angle \(\phi\) with respect to the \(x\) axis; the free-field motion at a point can be expressed, in such situation, as

\[
\begin{align*}
\omega(t) &= \omega_1(t) + \omega_2(t) \\
u(t) &= u_1(t) + \beta \cos \phi \omega_2(t + \tau) \\
v(t) &= v_1(t) + \beta \sin \phi \omega_2(t + \tau)
\end{align*}
\]  

where \(\omega_1(t), \omega_2(t), u_1(t)\) and \(v_1(t)\) are uncorrelated time-histories, \(\beta = 0.6813\) at the surface and \(\tau\) is equal to one quarter of the wave period.

If we assume that \(\omega_1(t)\) and \(\omega_2(t)\) share the same spectral shape and that the contribution of the correlated wave can be defined, in terms of standard deviation, as \(\sigma_{\omega_2} = \alpha_c\sigma_{\omega}\), the local correlation model turns out to be the following:

\[
S_{\omega_2}(f) = \frac{\alpha_c^2}{2} \sin 2\phi S_{\omega}(f)
\]

\[
S_{\omega_1}(f) = \alpha_c^2 \beta \cos \phi S_{\omega}(f) \exp(-i\frac{\pi}{2})
\]

\[
S_{v_1}(f) = \alpha_c^2 \beta \sin \phi S_{\omega}(f) \exp(-i\frac{\pi}{2})
\]

from which point coherency functions can be easily derived.

The stochastic model of the free-field motion is thus established once the direct spectral densities of the local components are defined, along with the space coherency functions; both aspects are largely covered in the literature. The matrix of the spectral densities of the pseudostatic motion can in turn be obtained, from (3), as

\[
S_{\omega}^{(t)}(f) = \mathbf{B}^{(t)} S_{\omega}^{(t)}(f) \mathbf{B}^{(t)T}
\]

where \(S_{\omega}^{(t)}(f)\) is the spectral density matrix of the free-field motion, resulting from the above described model.

3.2 Random response computation

Once the stochastic model for the input (see equations 10) is defined, the spectral density matrix of the response, can be expressed, according to equations (4) and to classical random vibration theory, in the following form

\[
S_{\omega}^{(t)}(f) = \mathbf{H}(f) \mathbf{m} \mathbf{R}_{\omega}^{(t)}(f) \mathbf{R}^T \mathbf{m} \mathbf{H}^*(f)
\]

(11)

where \(\mathbf{H}(f) = \left[ -(2\pi f)^2 \mathbf{m} + i2\pi f \mathbf{c}_r + i\mathbf{c}_h + \mathbf{k} \right]^{-1}\)

is the matrix of frequency response functions of the model, in which a combined damping model has been introduced, encompassing both a viscous and a hysteretic damping matrix, named \(\mathbf{c}_r\) and \(\mathbf{c}_h\) respectively. From (11) all quantities which are linearly related to relative motion, such as internal forces, stresses, etc., can be derived. In fact, if \(Z(t) = \mathbf{b}^T \mathbf{q}^{(d)}(t)\) is, for example, an internal force component, its spectral density can be computed as

\[
S_Z(f) = \mathbf{b}^T S_{\omega}^{(t)}(f) \mathbf{b}
\]

(12)

Finally, if the total motion is needed, e.g. in terms of accelerations, taking (1) and the definition of \(\mathbf{R}\) into account, the following expression can be derived

\[
S_q(f) = \mathbf{G}(f) \mathbf{R}_{\omega}^{(t)}(f) \mathbf{R}^T \mathbf{G}^*(f)
\]

(13)

where \(\mathbf{G}(f) = \left[ \mathbf{I} + (2\pi f)^2 \mathbf{H}(f) \mathbf{m} \right]\)

4 EXAMPLE OF APPLICATION

4.1 The IRIS reactor building

A medium power (335 MWe) pressurized light water reactor, the International Reactor Innovative and Secure (IRIS), was preliminarily designed by an international consortium which includes more than 20 partners from 10 countries (Carelli et al [1]). A site characterized by a low-to-average seismicity level has been herein assumed as possible installation. In a tentative
design of the Nuclear Steam Supply System (NSSS) building (see Fig. 2), the introduction of an isolation system was considered; the system is made by 120 High Damping Rubber Bearings (HDRB) installed between the foundation slab and the base (Fig. 3). HDRB devices are made of alternated rubber layers and steel plates, glued through vulcanization. Damping factor intrinsic to this technology ranges generally from 10% to 20%, shear modulus in the range 0.8-1.4 MPa.

Steel plates give a high vertical stiffness to the isolator with low deformability under dead loads (structural weights), though allowing large horizontal deformations. Therefore, the isolated building has lower natural frequencies, with respect to the fixed base version, for motions lying in the horizontal plane (typically 0.5 - 0.7 Hz), moving the natural vibration characteristics of the structure where the spectrum of ground motion has generally quite low energy. Consequently, the isolated building moves like a rigid body (Forni et al [2]) over the isolation system, which is strained in shear, continuously carrying the dead load. Furthermore, the absolute acceleration of the building is much smaller than the PGA, avoiding amplification at highest floors. This result is reached at price of large relative displacements between the building and the foundation slab.

In this light, the design of the isolation system must reach a reasonable compromise between limitation of absolute accelerations and relative displacements. For the case of the IRIS NSSS this led to a 0.7 Hz isolation frequency, i.e. to a value which can be seen as an upper limit for the parameter. It is worth noting that, for the IRIS non-isolated case the lowest natural frequency of the building, associated to a global rocking mode, is in the range 2-6 Hz according to the soil stiffness (shear modulus approx ranging from 0.1 to 1 GPa).

In view of the above considerations, a 0.7 Hz was set as isolation frequency, thus leading to a somehow limited degree of decoupling between the structural and ground motion, but limiting the relative displacement between the isolated building and the ground to a value of 10 cm at the SSE level (Peak Ground Acceleration equal to 0.3 g).

This is advantageous both for the performance of the isolators in beyond design conditions (limitation of the maximum span displacements under severe earthquakes) and for the design of steam lines connecting the NSSS building with the turbine units.

The characteristics of the isolators are given in Table 1. The resulting natural frequencies of the NPP isolated model are given in the Table 2 both for the fixed-base building and for the model taking soil-structure interaction into account (for comparison with the IRIS non isolated version see also De Grandis et al. [3]). In Perotti et al. [4] a methodology for the computation of the seismic fragility for the IRIS isolation system is presented as well as an overview of numerical models for the numerical simulation.

4.2 Simplified dynamic model of the building

The model here adopted for the IRIS reactor building is based on the assumption that the building moves as a rigid body supported by the isolation system; this hypothesis is justified for translational and torsional global modes. When the vertical and rocking modes are considered, the assumption leads to significant errors in the corresponding natural frequencies of the fixed-base model, in which soil structure interaction is neglected. This is because the deformability of the isolated structure is not negligible when compared to the vertical
stiffness of the isolators. For this reason the latter values have been fictitiously corrected for matching the natural frequencies of the first six normal modes of the fixed-base model, which were known from a refined FE 3D analysis.

As a further simplifying hypothesis, a rigid-body behavior has been assumed for the foundation as well; in this respect it must be observed that the horizontal foundation slab is stiffened by two contributions. First by the “box effect” due to the vertical walls; secondly by the isolators which, by virtue of their large axial stiffness, introduce an efficient vertical connection between the slab and the superstructure. Given the stiffness of the latter, the assumption that the building+foundation system behaves as a rigid body for the vertical and rocking modes involving ground deformation seems a very sound one (Figure 3). In addition, the foundation slab provides enough in-plane stiffness to justify the assumption of rigid-body motion for the translational and torsional global modes.

Figure 4. Plan layout of the isolators.

Summing up, the simplified dynamic model of the building encompasses two rigid bodies connected by a set of springs representing the isolators; the damping of these springs is modeled as hysteretic, according to a damping factor (equivalent viscous) equal to 10%. The plan layout of the isolation system is depicted in Figure 4.

The lower rigid body is in turn connected to the reference system by means of six springs and six dashpots, modeling the effect of an homogeneous soil having a tangential elastic modulus $G = 200$ MPa and a Poisson ratio equal to 0.25.

4.3 Seismic excitation model

As previously stated the definition of the stochastic model of the free-field excitation needs to be completed by assigning:

- the direct spectral densities of the three components of motion, which assumed to be the same at all contact points,
- the form of the lagged space coherency functions.

The direct Power Spectral Densities of the components of motions are based on the well known one by Kanai-Tajimi, as modified by Clough and Penzien, here denotes as MKT PSD:

$$S_{\text{MKT}} = S_0 \left[ \frac{\omega_1^4 + 4\xi_1^2 \omega_1^2 \omega_1^2}{(\omega_1^2 - \omega_2^2)^2 + 4\xi_1^2 \omega_2^2 \omega_1^2} \right] \left[ \frac{\omega_2^4}{(\omega_1^2 - \omega_2^2)^2 + 4\xi_2^2 \omega_2^2 \omega_1^2} \right]$$

(14)

The MKT PSD can be viewed as the effect of a filter, representing the soil, on a white noise process of intensity $S_0$, which in turn represents the motion of the bedrock. The parameters $\omega_1$ and $\xi_1$ represent the soil natural frequency and damping ratio, respectively, while $\omega_2$ and $\xi_2$ are the parameters of an additional high-pass filter introduced by Clough and Penzien to guarantee finite-power displacements.

The values for the parameters in (14) were selected so that the corresponding horizontal and vertical acceleration response spectra $R_S$ are on average compatible with the elastic ones in EN 1998-1 [9] for a selected spectral type, either Type 1 for a far field (FF) event, or Type 2 for a near field (NF) one, and a selected soil type (“C”). The procedure for selecting the parameters is thoroughly described in Martinelli et al. [5]. These values are listed in table 3, where the lower rows are relative to the vertical spectra $S_{ve}$; the latter satisfy, in terms of PGA (last column in the table), the ratios given in Table 3.4 of EN 1998-1.

Table 3. SPD parameters matching Eurocode [9] response spectra ($\omega_8$ in rad/s, $S_0$ in m$^2$ s$^{-4}$ Hz$^{-1}$, $a_8$ in g units)

<table>
<thead>
<tr>
<th>$S_{ve}$</th>
<th>$\omega_8$</th>
<th>$\xi_8$</th>
<th>$\omega_r$</th>
<th>$\xi_r$</th>
<th>$S_0 \times 10^4$</th>
<th>$a_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>12.02</td>
<td>0.6926</td>
<td>0.318</td>
<td>3.97</td>
<td>1.953</td>
<td>0.30</td>
</tr>
<tr>
<td>T2</td>
<td>30.46</td>
<td>0.6326</td>
<td>0.634</td>
<td>4.25</td>
<td>2.023</td>
<td>0.30</td>
</tr>
<tr>
<td>T1</td>
<td>53.95</td>
<td>0.6338</td>
<td>3.50</td>
<td>1.22</td>
<td>0.3382</td>
<td>0.27</td>
</tr>
<tr>
<td>T2</td>
<td>53.95</td>
<td>0.6338</td>
<td>3.50</td>
<td>1.22</td>
<td>0.3382</td>
<td>0.135</td>
</tr>
</tbody>
</table>

5 LAGGED COHERENCY MODELS

As explained in Section 3, the lagged coherency is obtained by the composition of a spatial coherency and a point coherency. Beside the two models for the point coherency, described in Section 3, three forms of the space coherency functions are considered in this work. One is suggested for short separation distances (Abrahsonson et al [8]), one can be also used for the vertical component of the seismic motion (Abrahamson [6]) and one is widely adopted and retained as valid for large separation distances (Luco and Wong [7]).

5.1 Abrahamsson et al. (1991) Lagged Coherency

The first model of the lagged coherency function we consider is the one developed by Abrahamson et al. [8] by performing non-linear regression analyzes of coherence function estimates:

$$\tan^{-1} \left[ \frac{y(f, \xi)}{(a-b\xi)} \right] = \exp \left[ -c \frac{d\xi}{\xi^2} \right] + \frac{f^2}{3}$$

(15)

where the values of the parameters are given in the Appendix. The model is effective for the evaluation of the coherency in the case of short separation distances (<100 m) between the pairs of recording stations.

Is one of the few functions available in the literature for the vertical component of the seismic ground motion. It is defined by the following two equations:

\[
\tanh \left( \frac{1}{V_s} \right) = \frac{c_4^H (\xi)}{1 + c_4^H (\xi) f^2} + \left( 4.8 - c_1^H (\xi) \right) \xi^2 f^2 + 0.35 \quad (16)
\]

\[
\tanh \left( \frac{1}{V_s} \right) = \frac{c_4^V (\xi)}{1 + c_4^V (\xi) f^2} + \left( 4.65 - c_3^V (\xi) \right) \xi^2 f^2 + 0.35 \quad (17)
\]

where the functions \( c_1^H \cdots c_4^H \) and \( c_1^V \cdots c_4^V \) can be found in the Appendix.

It should be noted that most of the recordings considered in deriving the previous two forms of the lagged coherency functions (Eq 15-17) were obtained on alluvial soils, and therefore the above relations can be seen as valid parametric models for this soil type.

5.3 Luco & Wong (1986) Lagged Coherency

The Luco and Wong [7] model is regarded as being particularly effective in the case of large distances between the recording locations. It is defined by the following equations:

\[
\psi(f, \xi) = e^{-\alpha^2 \pi^2 \sigma^2 \xi^2} \quad ; \quad \nu = \mu \sqrt{R / r_0} ; \quad \alpha = \nu / V_s \quad (18)
\]

In these: \( V_s \) represents an estimate of the propagation velocity of the S-waves, \( R \) is the distance from the source traveled by the waves, \( r_0 \) is an index of the lengthscale of the inhomogeneities in the soil medium, \( \mu \) is a measure of the variation of its elastic properties. The parameter \( \alpha \), having dimension [s/m], is also an index of the properties of the ground and is related to the inverse of \( V_s \). In practice, \( \alpha \) regulates the exponential decay of the lagged coherency with frequency and distance; Luco and Wong suggest for it a value equal to \( 2.5 \times 10^{-4} \) s/m.

A plot of the lagged coherencies herein considered is presented in Figure 5, from which one can appreciate the different behavior of the considered coherency functions. The ones from Abrahamson and Abrahamson et al. are always very close, while the one by Luco and Wong shows higher correlation at short distances (10 m) and lower correlation at larger distances and higher frequencies (50 m and \( f > 11 \) Hz) also when a reduced value (\( \alpha = 2.0 \times 10^{-4} \)) has been adopted for the correlation parameter.

6 PARAMETRIC ANALYSES AND RESULTS

Results are presented in terms of the relative displacement components over the isolation devices, being the vertical one directly related to the axial force variation. The isolation devices here considered are those on the x axis in Figure 4.

In Figure 6 the spectral power density of the vertical relative displacement across isolator 113 is given, as an example, for the Abrahamson coherency model; the Constant Coherency Model (CCM) is here tested with two values of the correlation coefficient (±0.5) while for the Linear Combination Model a large value of the \( \alpha_c \) parameter, i.e. 0.8, is adopted.

![Figure 5. Lagged coherencies at representative separation distances (top 50 m, bottom 10 m)](image_url)
Figure 6. Isolator 113; (top) SPD of horizontal relative displacement for model (16)+(17) and type 1 spectrum, (bottom) enlargement between 4 and 10 Hz

Figure 7. Isolator 113; SPD of horizontal relative displacement for model (16)+(17) and type 1 spectrum.

Figure 8. Isolator 113; SPD of vertical relative displacement for model (16)+(17) and type 1 spectrum – comparison between 6D, 3D and 2D excitation.

Figure 9. Isolator 3; SPD of vertical relative displacement for model (16)+(17) and type 1 spectrum – comparison between 6D, 3D and 2D excitation.

Figure 10. Isolator 113; SPD of vertical relative displacement for model (16)+(17) – comparison between type 1 and 2 spectra.

In Figure 9 isolator 3, located close to the building center, is considered, showing a small effect of the overturning moment due to the first mode response; removal of the vertical excitation causes here a one-order-magnitude drop in the response at intermediate frequencies.

Figure 10 compares responses to different Eurocode 8 response spectra, illustrating the more severe effect of type 1 excitation, characterized by a richer spectrum in the intermediate-to-low frequency range.

In the following Tables 4 to 6 a rough estimation of the extreme values of axial load variation is given, listing the $3\sigma$ levels for the worst case response between isolators 3 and 113, for the two Eurocode 8 spectra (T1 and T2). The Abrahamson et al space coherency is considered in Table 4 along with different parametrization of the point coherency, while Table 5 and 6 show the response to the Luco and Wong spatial model with two different values of the coherency decay coefficient. Note that the static axial load on the isolators is about 6600 kN. In the tables, point coherency 1 and 2 denote respectively the CCM and LCM models.

Results in terms of horizontal relative displacements across isolators are not given, since the rotational input effect on them is totally negligible.

As already detected from the spectral curves, the axial force variation is smaller for the type 2 (here T2) Eurocode spectrum. The effect of rotational and twisting components of the ground motion is responsible for a 5% increase in the case of no local correlation and for a 8% increase in the worst case here considered (CCM model with correlation coefficient between horizontal and vertical components equal to 0.5).
Finally, it can be noted that, in spite of the much higher correlation at short distances (see Figure 5), the effects of the Luco and Wong coherency function are of the same order of magnitude as those due to the other models here considered.

The results here shown demonstrate how for an isolated carefully analyzed, with particular reference to beyond design the isolators on the response to rotational excitation must be deviated to the analysis of recorded events in order to confirm structural models; for the first aspect an ongoing activity is must be confirmed by the point of view of both excitation and appreciated only in terms of axial load variation on the of rotational excitation due to kinematic interaction can be devices. These findings are to be regarded as qualitative and recognized as seismic and to the behavior of buildings having a larger plan size.

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REFERENCES


APPENDIX

Parameters for Abrahamson et al [8] lagged coherency

\[ a = 2.54; \quad b = 0.012; \quad c = 0.115; \quad d = 0.00084; \quad e = 0.878; \quad g = 0.35 \]

Functions defining Abrahamson [6] lagged coherency:

\[
\begin{align*}
&c_H^H(\xi) = \frac{3.95}{1+0.0077\xi + 0.000023\xi^2} e^{-0.85[\xi-0.00012]} \\
&c_H^H(\xi) = 0.4 \left[ \left( \frac{\xi}{5} \right)^3 \right] \left[ \left( \frac{\xi}{190} \right)^3 \right] \left( 1 + \left( \frac{\xi}{180} \right)^3 \right) \\
&c_Y^H(\xi) = 3 \left( e^{-0.05\xi} -1 \right) -0.0018\xi \\
&c_Y^H(\xi) = -0.598 + 0.106ln(\xi + 325) -0.01513e^{-0.6\xi} \\
&c_Y^X(\xi) = 3.5 - 0.37ln(\xi + 0.04) \\
&c_Y^X(\xi) = 0.65 \left[ 1 - \left( 1 + \frac{\xi}{5} \right)^{-2} \right] \\
&c_Y^X(\xi) = 3 \left( e^{-0.05\xi} -1 \right) -0.0018\xi
\end{align*}
\]

Table 4. Extreme axial load variation on isolators for space coherency model (15)+(17)

<table>
<thead>
<tr>
<th>Point coh.</th>
<th>6D T1</th>
<th>3D T1</th>
<th>6D T2</th>
<th>3D T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NULL</td>
<td>8022</td>
<td>7674</td>
<td>5206</td>
<td>5073</td>
</tr>
<tr>
<td>1- ( \rho )=0.2</td>
<td>7930</td>
<td>7740</td>
<td>5169</td>
<td>5136</td>
</tr>
<tr>
<td>1- ( \rho )=0.5</td>
<td>7948</td>
<td>7837</td>
<td>5175</td>
<td>5229</td>
</tr>
<tr>
<td>1- ( \rho )=0.7</td>
<td>8135</td>
<td>7740</td>
<td>5324</td>
<td>5136</td>
</tr>
<tr>
<td>1- ( \rho )=0.8</td>
<td>8302</td>
<td>7837</td>
<td>5498</td>
<td>5229</td>
</tr>
<tr>
<td>2- ( \alpha )=0.2</td>
<td>8023</td>
<td>7679</td>
<td>5206</td>
<td>5075</td>
</tr>
<tr>
<td>2- ( \alpha )=0.5</td>
<td>8030</td>
<td>7704</td>
<td>5209</td>
<td>5084</td>
</tr>
<tr>
<td>2- ( \alpha )=0.8</td>
<td>8043</td>
<td>7750</td>
<td>5214</td>
<td>5102</td>
</tr>
</tbody>
</table>

Table 5. Extreme axial load variation on isolators for space coherency model (18)+(17) with \( \alpha = 2.5E-4 \)

<table>
<thead>
<tr>
<th>Point coh.</th>
<th>6D T1</th>
<th>3D T1</th>
<th>6D T2</th>
<th>3D T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NULL</td>
<td>8044</td>
<td>7697</td>
<td>5297</td>
<td>5167</td>
</tr>
<tr>
<td>1- ( \rho )=0.2</td>
<td>7953</td>
<td>7765</td>
<td>5262</td>
<td>5231</td>
</tr>
<tr>
<td>1- ( \rho )=0.5</td>
<td>7973</td>
<td>7865</td>
<td>5270</td>
<td>5326</td>
</tr>
<tr>
<td>1- ( \rho )=0.8</td>
<td>8160</td>
<td>7765</td>
<td>5417</td>
<td>5231</td>
</tr>
<tr>
<td>1- ( \rho )=0.2</td>
<td>8300</td>
<td>7865</td>
<td>5592</td>
<td>5326</td>
</tr>
<tr>
<td>2- ( \alpha )=0.2</td>
<td>8046</td>
<td>7702</td>
<td>5298</td>
<td>5169</td>
</tr>
<tr>
<td>2- ( \alpha )=0.5</td>
<td>8053</td>
<td>7728</td>
<td>5301</td>
<td>5178</td>
</tr>
<tr>
<td>2- ( \alpha )=0.8</td>
<td>8066</td>
<td>7776</td>
<td>5306</td>
<td>5196</td>
</tr>
</tbody>
</table>

Table 6. Extreme axial load variation on isolators for space coherency model (18)+(17) with \( \alpha = 8.0E-5 \)

<table>
<thead>
<tr>
<th>Point coh.</th>
<th>6D T1</th>
<th>3D T1</th>
<th>6D T2</th>
<th>3D T2</th>
</tr>
</thead>
<tbody>
<tr>
<td>NULL</td>
<td>8057</td>
<td>7711</td>
<td>5355</td>
<td>5226</td>
</tr>
<tr>
<td>1- ( \rho )=0.2</td>
<td>7965</td>
<td>7780</td>
<td>5319</td>
<td>5291</td>
</tr>
<tr>
<td>1- ( \rho )=0.5</td>
<td>7983</td>
<td>7882</td>
<td>5324</td>
<td>5387</td>
</tr>
<tr>
<td>1- ( \rho )=0.8</td>
<td>8167</td>
<td>7780</td>
<td>5478</td>
<td>5291</td>
</tr>
<tr>
<td>1- ( \rho )=0.2</td>
<td>8353</td>
<td>7882</td>
<td>5657</td>
<td>5387</td>
</tr>
<tr>
<td>2- ( \alpha )=0.2</td>
<td>8059</td>
<td>7716</td>
<td>5355</td>
<td>5228</td>
</tr>
<tr>
<td>2- ( \alpha )=0.5</td>
<td>8065</td>
<td>7743</td>
<td>5358</td>
<td>5238</td>
</tr>
<tr>
<td>2- ( \alpha )=0.8</td>
<td>8077</td>
<td>7792</td>
<td>5362</td>
<td>5256</td>
</tr>
</tbody>
</table>

7 CONCLUSIONS

The results here shown demonstrate how for an isolated building of fairly compact plan (diameter is 56 m) the effect of rotational excitation due to kinematic interaction can be appreciated only in terms of axial load variation on the devices. These findings are to be regarded as qualitative and must be confirmed by the point of view of both excitation and structural models; for the first aspect an ongoing activity is devoted to the analysis of recorded events in order to confirm and refine the point coherency models.

On structural grounds, the effect on non-linear behavior of the isolators on the response to rotational excitation must be carefully analyzed, with particular reference to beyond design conditions and to the behavior of buildings having a larger plan size.