ABSTRACT: This paper analyzes the validity of assuming that the behavior of a nonideal energy source in a vibration problem is well described by its stationary torque-speed curves, which means neglecting the motor dynamics. In particular, this work considers an unbalanced induction motor as the energy source of a linear mechanical system, and compares the results obtained with the stationary curves to those yielded by the dynamical equations of the motor. Besides these numerical calculations, some analytical tools are used to discuss if it is always acceptable to neglect the energy source dynamics when solving vibration problems with non-ideal excitation. Results show that, for the selected parameter values, electrical state variables respond with a characteristic delay which is much shorter than the characteristic time of the problem. This allows using the torque-speed curves of the motor, what considerably reduces the order and complexity of the problem.

KEY WORDS: Nonideal excitation; Induction motor; Torque-speed curves.

1 INTRODUCTION

In vibration problems, an excitation is said to be ideal if it is unaffected by the vibrating system response. In this case, the amplitude and frequency of the excitation are known data. When this assumption is not valid, the vibrating system motion and that of the energy source are coupled, and the excitation is said to be nonideal. This usually happens when the energy source has a limited power supply.

In other words, if the excitation is considered to be ideal, it is implicitly being supposed that the motor is able to provide any torque at the desired rotational speed which, obviously, is not realistic. However, it can be a good approximation if the energy source is sufficiently powerful. Systems excited by nonideal energy sources are usually referred to as nonideal systems.

Nonideal systems have received considerable attention from researchers since the pioneering work of Sommerfeld in 1904 [1]. He mounted an unbalanced electric motor on an elastically supported table and measured the consumed power as well as the frequency and amplitude of the vibration. Some unexpected issues were found, such as jump phenomena and a great increase in the input power without significant change in the motor rotational speed near the natural frequency of the system (Figure 1). These and other nonlinear phenomena associated to nonideal vibrating systems are known as the Sommerfeld effect.

There have been numerous contributions in this field since the surprising results obtained by Sommerfeld. Without trying to be exhaustive, we will cite some of the most relevant ones.

The most extensive work dealing with nonideal systems was conducted by Kononenko [2], who devoted an entire text to this subject. He faced the problem on how a nonideal energy source affects different kinds of oscillating systems, with particular attention on stationary conditions.

Rand et al. [3] reported the importance of nonideal excitations in a particular failure of dual-spin spacecrafts during despins, referred to as precession phase lock. He built an analytical model of the system, considering no damping and a constant torque provided by the motor.

Krasnopolskaya [5] studied the effect of a limited power supply on an electromotor-pendulum system, finding chaotic behavior under some conditions. Other authors [6, 7] investigated the conditions under which a system is captured at resonance. This is a direct consequence of the motor not having enough power to overcome the energy sink created by structural damping when the oscillation amplitude increases. Dimentberg [6] found that the motor torque needed to pass through resonance is much less for a quick passage than under quasistatic conditions.

El-Badawi [8], among many others, investigated the effect of including some nonlinear terms when modeling the structure behavior. The reason to add these higher order terms is that, near resonance, it is expected a large-amplitude response of the structure not captured with the linear model. The reader interested in an extensive review on nonideal vibration problems is referred to [9].

The fact that we would like to recall is that the motor behavior has usually been described by its torque-speed...
curves. This is a delicate point, as these motor curves are stationary, that is to say, the curves are a collection of equilibrium points, reached once the motor internal variables have stabilized at constant values. It means that, rigorously, the curves should not be used unless stationary conditions have been attained.

The alternative consists in using the dynamical equations describing the motor behavior. These equations, that would model the internal dynamics of the energy source, should be obtained specifically for the particular type of motor being considered. In this paper, a three phase induction motor is modeled [10]. When using the motor dynamical equations, some new state variables (the electrical ones) appear in the formulation. Logically, the number of new variables will equal the number of new equations.

It is clear that this second approach increases the problem dimension, what complicates finding approximate analytical solutions and raises the computational cost of numerical ones. On the other hand, it is a more complete description of the physical problem, as it considers the motor dynamics as a part of the model.

In this paper we analyze whether it is reasonable to use the stationary torque-speed curves, instead of the motor dynamical equations, without losing essential information on the system dynamics. In section 2, we present the governing equations of the system, including the electrical variables. Section 3 deals with the meaning and applicability of stationary torque-speed curves. The characteristic response time for electrical and mechanical variables is investigated in section 4. Section 5 shows some numerical results for passing through resonance in both directions, comparing the use of stationary curves to that of the dynamical motor equations. Finally, section 6 presents the main conclusions of this study.

2 GOVERNING EQUATIONS

A linear mechanical system excited by an unbalanced electric AC motor will be considered (Figure 2). This simple system, which has 2 degrees of freedom (the position $x$ and the rotor angle $\phi$), could represent any machine that includes rotating equipment such that, at certain angular velocities, a considerable amplitude vibration is produced in its supporting equipment such that, at certain angular velocities, a considerable amplitude vibration is produced in its supporting structure [8].

The governing equations for this system are well known [2, 4, 11, 12]. They are immediately obtained by using Lagrange’s equations or Hamilton’s principle.

$$
(m_0 + m_1) \ddot{x} - m_1 r (\dot{\phi} \cos \phi - \dot{\phi}^2 \sin \phi) + k x + c \dot{x} = 0 \\
(l + m_1 r^2) \ddot{\phi} - m_1 r \dot{x} \cos \phi = L - H(\phi)
$$

(1)

where $m_1$ represents the unbalanced rotating mass, $m_0$ is the mass of the motor, $l$ is the moment of inertia of the rotor and other rotating parts except $m_1$, $r$ is the distance between $m_1$ and the rotor axis, $k$ is the spring constant, $c$ is the damping coefficient, $L$ is the torque produced by the motor, $H$ is the resisting moment (due to windage and mechanical losses if there is no load torque), which will be considered to be proportional to the rotor angular speed, and the dots denotes derivation with respect to time.

![Figure 2. System scheme [4]](image)

It is illustrative to take the rotating velocity as a constant, which is the usual assumption when modeling the excitation of an ideal unbalanced motor.

$$
(m_0 + m_1) \ddot{x} + k x + c \dot{x} = -m_1 r \dot{\phi}^2 \sin \phi \\
L = H(\dot{\phi}) - m_1 r \dot{x} \cos \phi
$$

(2)

The first of the above equations is that of a 1 d.o.f. linear system excited by the harmonic centrifugal force of the unbalanced mass. The second equation states the torque that must be applied by the motor, which is not constant. Therefore, this situation would only be possible if the motor was capable of producing any torque for the same angular velocity. This could be an acceptable hypothesis if its torque-speed curves (assuming those curves can legitimately be used) are almost vertical.

In the second of equations (1), it has been intentionally unspecified the dependence of torque $L$ on other system variables. The classical approach [2, 4, 6, 8, 11, 12] consists in taking the motor torque as a function of its rotational velocity,

$$
L = L(\dot{\phi})
$$

This is equivalent to characterizing the motor behavior by means of its stationary torque-speed curves which, for an induction motor have the shape shown in Figure 3. As far as the authors know, the use this approach has not been properly justified yet.

![Figure 3. Typical torque-speed curve for an induction motor](image)
and assuming linear magnetic circuits, the electrical equations [10] are

\[
\frac{d\psi_{rd}}{dt} = -\alpha \psi_{rd} + (\omega_0 - \phi)\psi_{rq} + \alpha M i_{sd}
\]
\[
\frac{d\psi_{rq}}{dt} = -\alpha \psi_{rq} - (\omega_0 - \phi)\psi_{rd} + \alpha M i_{sq}
\]
\[
\frac{di_{sd}}{dt} = -\gamma i_{sd} + \omega_0 i_{sq} + \delta \alpha \psi_{rq} + \delta \phi \psi_{rd} + \frac{V}{\eta}
\] (3)
\[
\frac{di_{sq}}{dt} = -\gamma i_{sq} - \omega_0 i_{sd} + \delta \alpha \psi_{rd} - \delta \phi \psi_{rq}
\]
\[
L = \frac{M}{L_r}(\psi_{rd}i_{sq} - \psi_{rq}i_{sd})
\]

where \(V\) is the r.m.s line stator voltage (assuming the stator is star connected). \((\psi_{rd}, \psi_{rq})\) are the rotor magnetic fluxes and \((i_{sd}, i_{sq})\) are the stator currents in a \((d,q)\) reference frame which rotates at \(\omega_0\) velocity, with axis \(d\) always aligned with the stator voltage vector. \(\omega_0\) is the frequency of the stator sinusoidal voltages. The constant model parameters are: rotor and stator windings resistances \((R_r, R_s)\) and inductances \((L_r, L_s)\), and mutual inductance \(M\). The rest of parameters are functions of them:

\[
\eta = L_x \left(1 - \frac{M^2}{L_x L_r}\right), \alpha = \frac{R_r}{L_{sr}}, \delta = \frac{M}{\eta L_r}, \gamma = \frac{R_s}{\eta} + \delta \alpha M
\] (4)

Let us note that, according to the motor dynamical model, torque \(L\) is a function of rotor magnetic fluxes and stator currents. Combining equations (1) and (3), a system of 6 nonlinear differential equations with 6 unknowns \((x, \phi, i_{sd}, i_{sq}, \psi_{rd}, \psi_{rq})\) is obtained. The motor can be controlled by means of the input variables \((V, \omega_0)\).

Let us focus on the mechanical part of the problem i.e. equations (1). It is classical to use the averaging method in order to simplify the system of equations and better understand its behavior [2, 4, 8, 11]. Firstly, a hypothesis will be made on the relative sizes of the terms in equations (1). It is classical to use the averaging method in order to simplify the system of equations and better understand its behavior [2, 4, 8, 11].

The amplitude of the motion. It is useful to make a change of variables

\[
\phi'(\tau) = 1 + \Delta(\tau) = 1 + \varepsilon \sigma(\tau)
\]
\[
u(\tau) = a(\tau) \cos(\phi(\tau) + \beta(\tau))
\] (7)

The first of the above equations expresses that motions being considered are those where the rotation velocity of the motor is near the natural frequency of the system.

The second equation substitutes the variable \(u\) for two new variables: amplitude \(a\) and phase \(\beta\). This allows imposing an additional constraint in order to have as many equations as unknowns. The formulation becomes simpler if this condition is selected as

\[
u'(\tau) = -a(\tau) \sin(\phi(\tau) + \beta(\tau))
\] (8)

which is the form of \(x'\) supposing \(a\) and \(\beta\) were constants and \(\phi'\) was equal to 1. The direct consequence of this restriction is

\[
a' \cos(\phi + \beta) - a(\Delta + \beta') \sin(\phi + \beta) = 0
\] (9)

Substituting (7) in (5) and using (9), a linear system of 3 equations is obtained for \(a', \beta\) and \(\Delta'\). By solving the system and neglecting terms \(O(\varepsilon^2)\), we obtain

\[
a' = \varepsilon(\sin \phi - 2\mu a \sin(\phi + \beta)) \sin(\phi + \beta)
\]
\[
\beta' = -\varepsilon \left[ \sigma + \frac{1}{a}(2\mu a \sin(\phi + \beta) - \sin \phi) \cos(\phi + \beta) \right]
\]
\[
\Delta' = \varepsilon[T - ab \cos(\phi + \beta) \cos \phi]
\]
\[
\phi' = 1 + \Delta
\] (10)

Based on the fact that \(a', \beta\) and \(\Delta'\) are small quantities compared to \(\phi'\), the method of averaging [2] can be applied. With this approximate technique, each equation is averaged along a revolution of the rotor, taking \(a\), \(\beta\) and \(\Delta\) as constants during one cycle, so that variable \(\phi\) no longer appears in the formulation. For a detailed explanation of the method and its applicability, the reader is referred to [11, 13]. After averaging, the system becomes

\[
\begin{cases}
a' = \varepsilon \left[ \frac{1}{2} \cos \beta - \mu a \right] \\
\beta' = -\varepsilon \left[ \sigma + \frac{1}{a} \sin \beta \right] \\
\Delta' = \varepsilon \left[ T - \frac{1}{2} abc \cos \beta \right]
\end{cases}
\] (11)

It has been obtained an autonomous system of three ordinary nonlinear differential equations. It is interesting to find the equilibrium points in system (11):

\[
\begin{cases}
a' = 0 \\
\beta' = 0 \\
\Delta' = 0
\end{cases} \Rightarrow \begin{cases}
a = \frac{1}{2} (\mu^2 + \sigma^2)^{-1/2} \\
T = \frac{\mu a^2 b}{2} \\
\beta = \sin^{-1}(-2a\sigma) = \cos^{-1}(2\mu a)
\end{cases}
\] (12)

Combining the first two equations in (12), we arrive to

\[
T = \frac{\mu b}{4(\mu^2 + \sigma^2)}
\] (13)
Using dimensional variables

\[ L(\dot{\phi}) = H(\dot{\phi}) + \frac{c_0^2 (m_1 r)^2}{2 (c^2 + 4(\phi - \omega)^2(m_0 + m_1)^2) \omega^2} \]

This can be rewritten as follows

\[ L(\dot{\phi}) = H(\dot{\phi}) + S(\dot{\phi}) \]

where a vibrational torque \( S \) [12] has been defined as

\[ S(\dot{\phi}) = \frac{c_0^2 (m_1 r)^2}{2 (c^2 + 4(\phi - \omega)^2(m_0 + m_1)^2) \omega^2} \]

Equation (15) means that, in stationary conditions, the torque provided by the motor, \( L \), must be equal to the sum of resisting moment, \( H \), and the torque demanded by the vibrating system, \( S \). As we are looking for stationary solutions, the characteristic torque-speed curves of the motor can legitimately be used. The solutions of equation (9) can easily be found in a graphical way by plotting the torque-speed curves (Figure 4). For the particular case shown in Figure 4, the system exhibits 3 stationary points. Although a detailed stability analysis will not be carried out here, it can be shown that the middle equilibrium point in Figure 4 is unstable, while the other two are stable [2, 4, 6].

![Figure 4. Graphical calculation of the equilibrium points](image)

**a)** general view, **b)** zoom near resonance

### 3 STATIONARY TORQUE-SPEED CURVES

This section is intended to give some insight into the nature of the different torques acting on the rotor, and their dependence on the system state variables.

Maintaining all the assumptions of the previous section, system (3) can be added to system (11), using dimensionless time \( \tau \) and writing the rotor speed in terms of \( \Delta \). Thus a dynamical system of dimension 7 is obtained in the form

\[ x' \in f(x, \Delta, V, \omega_0) \]

\[ \Delta' = g(x, \Delta, x_m) \]

\[ x_m' = h(x_m, \Delta) \]

where the state vector \( x \) has been decomposed in three parts

\[ x = \begin{pmatrix} x_e \\ x_m \end{pmatrix}, x_e = \begin{pmatrix} \phi_{rd} \\ \phi_{rq} \end{pmatrix}, x_m = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \]

and functions \( f, g \) and \( h \) have the following expressions.

\[ f(x_e, \Delta, V, \omega_0) = \begin{cases} \frac{1}{\omega} \left( -\gamma l_{sd} + \alpha \psi_{rd} + \Delta \omega (1 + \Delta) \psi_{rq} \right) \\ -\beta \psi_{rd} + \Delta \omega (1 + \Delta) \psi_{rq} + \alpha M_{rd} \\ -\psi_{rd} - \alpha M_{rd} \end{cases} \]

\[ g(x_e, \Delta, x_m) = \epsilon T_1(x_e) - T_2(\Delta) - T_3(x_m) \]

\[ h(x_m, \Delta) = \begin{pmatrix} \frac{1}{2} \cos \beta - \mu \Delta \\ -\Delta - \frac{\epsilon}{2a} \sin \beta \end{pmatrix} \]

Torques \( T_1, T_2 \) and \( T_3 \) are given by

\[ \epsilon T_1 = \frac{M}{K_p} \left( \psi_{rd} l_{sd} - \psi_{rq} l_{sd} \right) \\
\epsilon T_2 = \left( \frac{l_{sd}}{l_{sd}^2} \right)(1 + m_1 r)^2 \omega^2 \\
\epsilon T_3 = \frac{\epsilon}{2} a b \cos \beta \]

It is clear that the rotor angular acceleration is proportional to the difference between an active torque, produced by the electric motor, and two resisting torques (one is due to vibration and the other one corresponds to mechanical losses in the rotating parts).

Let us focus on the first vector equation in (17), i.e., the one concerning the evolution of the electrical variables. If we solve

\[ f(x_e, \Delta, V, \omega_0) = 0 \]

an expression can be obtained for the electrical variables as functions of the inputs \( V \) and \( \omega_0 \), which can be directly controlled by means of a variable frequency drive, and the rotor angular velocity

\[ x_e = x_e' (\Delta, V, \omega_0) \]

The interpretation is clear: given the amplitude and frequency of the stator voltages and a particular value of the rotor speed, there is a set of values of the electrical variables that makes them constant. This could be called a **partial**
equilibrium, as it refers only to a set of the 7 equations that govern the complete problem. We shall see in next section that this partial equilibrium points are stable, which means that, when the variation of the input variables and/or the rotor velocity is slow enough, the electrical variables will follow the values given by $x^*_m(\Delta, V, \omega_0)$. 

If, during the evolution of the system, the electrical variables were always close enough to their partial equilibrium values, it could be assumed that (22) holds at every instant and, substituting in the first of equations (20), the torque produced by the motor would be a function of the rotor speed and the input variables.

$$T^*_1(\Delta, V, \omega_0) = T_1[x^*_m(\Delta, V, \omega_0)]$$

(23)

Following this procedure, the expression of $T^*_1$ can easily be obtained but, because of its long and complicated form, it is not shown here. It may be consulted in [10]. For fixed values of the input variables, $T^*_1$ is a function of the rotor angular velocity and its shape is shown in Figure 3, which is the well-known torque-speed characteristic curve of an induction motor. Although in Figure 3 and Figure 4a the complete curve is shown, the only part that must be considered in this problem is the one near the natural frequency of the system (Figure 4b), as the rotor velocity is supposed to be close to $\omega$.

Thus, the stationary torque-speed curves of the motor are the consequence of assuming that the electrical variables take, at every instant, their partial equilibrium values given by the rotor speed and the input variables.

It is obvious that the above analysis could also be applied to the vibrational torque

$$h(x_m, \Delta) = 0 \rightarrow x_m = x^*_m(\Delta)$$

$$T^*_2(\Delta) = T_2[x^*_m(\Delta)] = \frac{\mu b}{4\left(\mu^2 + (\Delta/\varepsilon)^2\right)}$$

The last expression had already been obtained when calculating the equilibrium points of system (11). The interpretation of $T^*_2$ is analogous to that of $T^*_1$. Moreover, it will be shown that the partial equilibrium given by $x^*_m(\Delta)$ is also stable.

If both torques $T^*_1$ and $T^*_2$ were used in the second of equations (10), then a single differential equation would be obtained for a single state variable, $\Delta$. However, it will be justified in next sections that, while replacing $T_1(x_m)$ with $T^*_1(\Delta, V, \omega_0)$ may not essentially alter the system dynamics, replacing $T_2(x_m)$ with $T^*_2(\Delta)$ does.

4 RESPONSE VELOCITY OF ELECTRICAL AND MECHANICAL VARIABLES

Section 3 has shown that using stationary torque-speed curves means assuming that certain state variables always take their partial equilibrium values given by the rotor speed. For this to be acceptable, the partial equilibrium must be stable and, as it is known from classical local theory of dynamical systems, the stability is given by the eigenvalues of the jacobian matrix evaluated at the equilibrium points.

$$\frac{\partial f_j}{\partial x_{e_j}}^T \frac{1}{\omega} \begin{bmatrix} -\gamma & \omega_0 & -\delta \alpha & \delta \omega(1+\Delta) \\ -\alpha \omega_0 \gamma & -\omega(1+\Delta) & -\delta \alpha & 0 \\ 0 & aM(1+\Delta) \omega_0 - \omega(1+\Delta) & 0 \end{bmatrix}$$

(24)

$$\frac{\partial h_i}{\partial x_{m_j}}^T = \begin{bmatrix} -\epsilon \mu & a' \Delta \\ -\delta / a' & -\epsilon \mu \end{bmatrix}$$

(25)

Let us give numerical values to the parameters. The electrical parameters have been taken from [10].

$$I = 0.0075 kgm^2$$
$$R_s = 5.3 \Omega$$
$$R_r = 3.3 \Omega$$
$$L_s = 0.365 H$$
$$L_r = 0.375 H$$
$$M = 0.34 H$$
$$\omega_{0N} = 2\pi \cdot 16.7 rad/s$$
$$V_N = \frac{3}{\sqrt{2}} \cdot 110 V$$

$\omega_{0N}$ and $V_N$ are the nominal values for the sinusoidal input voltages. In order to control the motor, input variables can be modified by means of a Variable Frequency Drive. The simplest control method is the so called V/f control, shown in Figure 5 ($f = \omega_0/2\pi$). We will consider the motor to be working in the constant flux region, which means

$$V = p \cdot V_N$$
$$\omega_0 = p \cdot \omega_{0N}$$

(26)

Mechanical parameter values will be taken as

$$m_0 = 6 kg$$
$$m_1 = 0.2 kg$$
$$r = 0.01 m$$
$$X = 0.2 m$$
$$\omega = 69 rad/s$$
$$\mu = 1$$

$$H = c_1 \phi, \ c_1 = 0.008 \frac{Nm}{rad/s}$$

And the rest of parameters can be obtained from these ones.

Figure 6 shows the eigenvalues of (13) for three different values of $\omega_0$ and the rotor velocity varying from 0 to $\omega_0$. All of the eigenvalues have negative real parts, so the partial equilibrium points are stable. In addition, it is known that the time needed by the variables to reach their equilibrium values is related to the inverse of the absolute value of the eigenvalues real parts.
The eigenvalues that control the dynamics are the slowest ones i.e. those with a closest to zero real part. In this case, it can be observed that the closest to zero real parts are in the order of $-0.1$. Then, turning back to dimensional variables, the time needed by electrical variables to follow their partial equilibrium values can be estimated

$$t_e \approx \frac{1}{0.1\omega} \approx 0.14s$$

On the other hand, the eigenvalues of (14) can easily be calculated:

$$\lambda_m = -\varepsilon\mu \pm i\Delta \Rightarrow Re(\lambda_m) = -\varepsilon\mu = -0.0016$$

$$t_m \approx \frac{1}{0.0016\omega} \approx 65s$$

Then, the time response of mechanical variables is much greater than that of electrical variables. The conclusion is that, if along the system evolution $x_i^e(\Delta, V, \omega_0)$ varies with a characteristic time that is much greater than $t_e$, then the electrical variables will be able to closely follow their partial equilibrium values and, therefore, the stationary torque-speed curves of the motor can be used.

The characteristic time of $x_i^e(\Delta, V, \omega_0)$ will depend on that of $\Delta, V$ and $\omega_0$. While $V$ and $\omega_0$ can be directly controlled, $\Delta$ will vary according to the dynamic behavior of the complete system. The same can be stated about mechanical variables with respect to $t_m$.

5 NUMERICAL SIMULATIONS

Some simulations have been carried out in which $V$ and $\omega_0$ have been varied linearly with time in order to pass through resonance in both directions (Figure 7 and Figure 8), according to (26). Figure 9 shows the evolution of the rotor angular velocity for different values of duration $d$ and various assumptions on the system behavior.
Let us remark some issues concerning the results:

- Curves red and blue virtually coincide in every case except d), which is the only one with $d \approx t_c$.
- The only case in which the Sommerfeld effect can be clearly observed is a). The rotor velocity remains almost constant when it approaches $\omega$ and, after that, there is a jump through resonance. The reason why this phenomenon is not observed in the rest of cases is that duration $d$ is not much greater than $t_m$. This means that variables $x_m$ are not able to follow their partial equilibrium values. As the vibration amplitude does not have time to grow up, the energy sink which produces the Sommerfeld effect does not appear. Note that the effect can be observed in b) and c) when assuming $x_m = x_m^*(green \ curves)$. In simulation d) even the green curve is unable to clearly show the jump phenomenon but, in this case, the reason is that the inertia $I + m_1r^2$ prevent $\phi$ from varying so rapidly.
- In case a), although $d$ is much greater than $t_m$, the characteristic time associated to the variation of the rotor velocity immediately after the jump (around 2s) is not. This justifies that, assuming $x_m = x_m^*$ considerably alters the system response (the green curve shows no oscillation after the jump), while taking $x_c = x_c^*$ is consistent ($t_c \ll 2s$).

Finally, Figure 10 compares the solution of equations (17) for $d = 100s$ with the solution of the original equations (1) + (3) for the same input, in order to show that the averaging method is suitable for this case.

![Figure 9. Evolution of the rotor velocity, a) $d = 100s$, b) $d = 10s$, c) $d = 1s$, d) $d = 0.1s$](image)

![Figure 10. Comparison between the solution of the averaged equations and that of the original ones](image)

6 CONCLUSIONS

We have investigated how the internal dynamics of an energy source affects the dynamical behavior of the whole system in a nonideal vibration problem. The following conclusions have been extracted:

In the case of an induction motor, and for the particular values assigned to the model parameters, the internal motor dynamics can be neglected (i.e. the stationary torque-speed curves can be used) without essentially altering the system behavior when studying the Sommerfeld effect.

The reason is that, for the electrical dynamics to be significant, the variation of the motor input voltage needs to be so fast that the mechanical part of the system is not able to respond and, therefore, the Sommerfeld effect does not manifest. Electrical variables respond in a characteristic time that is much less than that of mechanical variables. In other words, motor stationary curves should not be used if the characteristic time response for electrical and mechanical variables were of the same order.

For any other kind of nonideal energy source producing vibrations e.g. an internal combustion engine, the same procedure could be followed in order to investigate whether its dynamical equations should be included in the model. The eigenvalues associated to internal motor variables (analogue to those in Figure 6) should be calculated. This would allow...
knowing the characteristic response time of these variables. If it was much less than the characteristic time of variation of the motor input variables and of the rotor angular velocity, then the motor dynamics could legitimately be neglected.

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