Identification of SMA dampers nonlinear restoring force of incompletely excited MDOF structures with double polynomial model

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ABSTRACT: The initiation and development of damage such as cracks in an engineering structure under dynamic loadings is a typical nonlinear process. Strictly speaking, the conventional vibration and eigenvalues extraction based damage identification approaches are suitable for linear systems only. In this paper, based on a double Chebyshev polynomial model, a time-domain NRF identification approach is proposed for identifying the structural nonlinearity as well as the mass distribution under incomplete dynamic loadings. The feasibility and robustness of the proposed approach is validated via a numerical MDOF structure equipped with a shape memory alloy (SMA) damper whose restoring force is modeled by a double-flag-shaped nonlinear model. Results show that the proposed approach is capable of identifying the mass distribution and structural nonlinearity, and could be potentially used for monitoring damage initiation and propagation processes during vibration of engineering structures under dynamic excitations.

Key words: Damage identification; Nonlinear restoring force; Double Chebyshev polynomial model; Incomplete excitations; Double flag-shaped nonlinear model; SMA damper.

1 INTRODUCTION

Due to the rapid increase in the number of deteriorating structures and damaged structures under strong dynamic loadings such as earthquakes, it is crucial to evaluate their current reliability, performance, and condition for the prevention of potentially catastrophic events, as well as for remaining life estimation, retrofitting and strengthening. The development of vibration-based structural damage detection approach has been one of the most active research areas in civil and infrastructural engineering for life-cycle performance evaluation and maintenance. Much progress has been made in this area and comprehensive literatures can be found [1-3].

In most of the nonlinearity-identification approaches, all excitations (inputs) applied to the degrees of the nonlinear structural system are assumed to be known and available for the nonlinearity identification. However, in many practical situations, it is either too difficult to excite all of the DOFs of an engineering structure especially the complex and large-scale structures, or not easy to obtain the complete measurements of the external excitations due to inaccessibility and the limitation of the number of available sensors. More recently, based on the basic idea of equivalent linearization and the symmetry of the identified stiffness matrix, Xu et al. proposed a data-based model-free hysteresis identification approach for nonlinear systems under incomplete excitations [4, 5]. By employing the power series polynomial model (PSPM) for the representation of the structural nonlinearity, Xu et al. proposed a time-domain data-based approach for identifying the nonlinear restoring forces under spatially incomplete excitations [6]. Moreover, He et al. developed nonlinear identification approach for simultaneously identifying the NRF and the partially unknown excitations by the combination of the PSPM and adaptive iterative least-square estimation [7].

In this paper, a double-Chebyshev polynomial modeling involving the instantaneous values of the state variables of a MDOF structural system is proposed to represent the system nonlinearities. The results show that though the constitutive model of the structural members and the structural mass distribution are both assumed to be unknown, the proposed method could still identify NRF with acceptable accuracy, and it provides a promising approach for damage detection where structural nonlinearity needs to be considered.

2 THE DOUBLE CHEBYSHEV POLYNOMIAL MODEL (DCPM)-BASED APPROACH

In most of the currently available vibration-based damage detection algorithms, the damage is identified in the form of the decrease in structural stiffness by the use of structural parameters identification and model updating algorithms, using eigenvalues or eigenvectors extracted from the structural dynamic response time histories. Strictly speaking, the approaches based on eigenvalues extraction are suitable for linear structures only. Instead of stiffness, restoring forces can describe the linear and nonlinear behavior of structures or structural members under dynamic loadings directly, and moreover, the hysteresis curve (typically found in all structural materials undergoing significant deformations) can be employed to evaluate the energy dissipated during vibration, and to identify the damage initiation and development process quantitatively. The identification of restoring forces is more meaningful for nonlinear structures. Unfortunately, the restoring force of a structure under dynamic loadings cannot be measured directly, and the restoring force model of a certain structure such as an RC structure in civil engineering is hard to know a priori and too difficult to be modeled in a parametric model accurately. Consequently, efficient and general restoring
force identification methodologies using structural dynamic measurements are crucial for damage detection, life-cycle performance evaluation, remaining service life forecasting and even retrofiting and strengthening of engineering structures after dynamic loadings.

Consider a discrete n-DOF lumped-mass chain-like structural system incorporating nonlinear non-conservative dissipative members and subjected to directly applied forces $P(t)$. The motion of this nonlinear system can be governed by the following equation of motion:

$$ M\ddot{x}(t) + R[\dot{x}(t), x(t), g^{\text{non}}] = P(t) $$

(1)

where $x(t)$ is the displacement vector of order $n$, $M$ is the constant matrix that characterizes the inertia forces, $R[\dot{x}(t), x(t), g]$ is the nonlinear non-conservative restoring force vector, $g$ is the vector of system-specific parameters, and $P(t)$ is the directly external forces, respectively.

In this study, the NRF of the system is assumed to be expressed in a general double Chebyshev polynomial form as shown in the following equation:

$$ R_{i,j-l}[\dot{x}(t), x(t), g] \approx R_{i,j-l}[v_{j-l,i}, s_{j-l,i}, g] $$

$$ \approx \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i-j,h,l} T_h(v'_{j-1}, T_j(s'_{j-1})) $$

(2)

where $R_{i,j-l}[\dot{x}(t), x(t), g]$ is the NRF between the $i$-th DOF and the $(i-1)$-th DOF, $g^{\text{non}}_{i-j,h,l}$ is the coefficient of the polynomial, $k$ and $q$ are integers which characterize the nonlinearity of the system, and $T(\alpha)$ is the $i$-th kind of Chebyshev polynomial. $v_{j-l,i}$ and $s_{j-l,i}$ are relative-velocity and relative-displacement vectors between two adjacent DOFs of $i$ and $i-1$, i.e. for a chain-like lumped-mass system, $v_{j-l,i}$ and $s_{j-l,i}$ are the inter-story velocity and inter-story displacement vectors and can be defined as $v_{j-l,i} = \tilde{x}_i - \tilde{x}_{i-1}$, $s_{j-l,i} = \bar{x}_i - x_{i-1}$.

The $T_h(\alpha)$ is orthogonal to each other on the interval [-1, 1]. Because the dynamic response is not exactly within the interval [-1, 1], the measured data of dynamic response should be normalized and be mapped onto the appropriate region of [-1,1] by the linear transformations as follows:

$$ v'_{j-l,i} = \frac{v_{j-l,i} - (v_{j-l,i-\text{max}} + v_{j-l,i-\text{min}})}{(v_{j-l,i-\text{max}} - v_{j-l,i-\text{min}})}/2 $$

$$ s'_{j-l,i} = \frac{s_{j-l,i} - (s_{j-l,i-\text{max}} + s_{j-l,i-\text{min}})}{(s_{j-l,i-\text{max}} - s_{j-l,i-\text{min}})}/2 $$

(3)

Consequently, the equation of motion for each DOF can be rearranged as follows:

$$ m_i \ddot{x}_i(t) + \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i-j,h,l} T_h(v'_{j-1,i}) T_j(s'_{j-1,i}) = P_i(t) $$

$$ \ldots $$

$$ m_{i-1} \ddot{x}_{i-1}(t) + \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i-1-j,h,l} T_h(v'_{j-1,i-1}) T_j(s'_{j-1,i-1}) = P_{i-1}(t) $$

(5)

Since there is only one term of 1 in the $i$-th DOF, the corresponding coefficients of the $i$-th DOF can be identified by implementing least-square algorithms. According to the relationship between the action force and the reaction force, the following equation exists,

$$ \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i,j-h,l} T_h(v'_{j,i}) T_j(s'_{j,i}) = - \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i,j-h,l} T_h(v'_{j,i-1}) T_j(s'_{j,i-1}) $$

$$ \ldots $$

$$ \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{i,j-h,l} T_h(v'_{j,i-n}) T_j(s'_{j,i-n}) = p_{i,n}(t) $$

(6)

Consider the nonlinear system mentioned above under arbitrary incomplete excitations, the rank of $P(t)$ defined in Equation (1) will be less than the order of $n$. Consequently, the unknown coefficients including the mass distribution cannot be uniquely determined by implementing least-square algorithms directly. With loss of the generality, assume the $n$-th DOF of the structure is not excited and the $(n-1)$-th DOF of the structure is excited, the equation of motion of the $n$-th and the $(n-1)$-th DOF can be written in the following two equations respectively,

$$ m_n \ddot{x}_n(t) + \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{n-j,n-h,l} T_h(v_{n,j,n-1}) T_j(s_{j,n-1}) = p_{n,n}(t) $$

(7)

$$ m_{n-1} \ddot{x}_{n-1}(t) + \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{n-1,j,n-h,l} T_h(v_{n,j-2,n-2}) T_j(s_{j,n-2}) $$

$$ \ldots $$

$$ + \sum_{h=0}^{k} \sum_{j=0}^{q} g^{\text{non}}_{n-1,j,n-h,l} T_h(v'_{j,n-1}) T_j(s'_{j,n-1}) = p_{n,n-1}(t) $$

(8)

The algebraic coefficients including the mass of the $(n-1)$-th DOF and the nonlinear restoring force between the $(n-1)$-th DOF and the connecting DOFs can be identified with an optimization algorithm such as least-square techniques using the applied excitations and the corresponding normalized response time series.

For the $n$-th DOF which is not excited, its mass can be identified according the following Equation (9) with linear fitting technique.
\[ m_n \ddot{x}_n(t) = -\sum_{h=0}^{k} \sum_{j=0}^{q} g_{n,h,j}^{\text{non}} T_h(v_{n-1}^{\prime}(s_{n-1}^{\prime})) \]
\[ = \sum_{h=0}^{k} \sum_{j=0}^{q} g_{n-1,h,j}^{\text{non}} T_h(v_{n-1}^{\prime}(s_{n-1}^{\prime})) \]  

(9)

Then, the algebraic coefficients including the mass of the DOFs excited and the corresponding nonlinear restoring force can be identified using least-square techniques in sequence with the normalized dynamic measurement and excitation information. The mass of the DOF with no excitation applied can be identified based on the nonlinear restoring force between it and the DOF with excitation. The process can be carried out in sequence until all of the mass distribution of the structure and the nonlinear restoring force can be identified.

3 NUMERICAL SIMULATION VALIDATION

3.1 Description of the 4-DOF nonlinear numerical model with a SMA damper

To illustrate the accuracy of the proposed DCPM-model approach, a 4-DOF nonlinear lumped-mass structure equipped with an SMA damper is considered as a numerical simulation example as shown in Fig. 1. Each story of the model is associated with one horizontal DOF. The properties of the linear part of the structure without the SMA damper are \( m_i = 300 \text{kg}, k_i = 4 \times 10^7 \text{N/m}, \) and \( c_i = 200 \text{N/m/s} \) (\( i = 1, 2, 3, 4 \)). In order to mimic the nonlinear behavior of the numerical model, an SMA damper, which is widely used as a typical energy dissipation device in engineering structures for vibration control, is introduced on the 4th floor of the numerical model as shown in Fig. 1.

The expression of the SMA damper is given by the following equations:

\[ F_{\text{int}}(S) = \begin{cases} k_1 S & \text{if } S < 0.4 \times 10^4 \text{N/m} \\ \frac{k_2 S}{S^2 + S_0^2} & \text{if } S \geq 0.4 \times 10^4 \text{N/m} \end{cases} \]

(10)

where \( k_1 \) and \( k_2 \) are the stiffness coefficient, \( \text{sgn}(*) \) is the sign function and \( S \) is relative-displacement vectors. In this example, the following numerical values for the SMA damper model are used: \( k_1 = 1.5 \times 10^7 \text{N/m}, k_2 = 3.0 \times 10^7 \text{N/m}, S_0 = 0.006 \text{m}, \)

3.2 Identification of NRF provided by SMA damper under incomplete excitations

SMA has unique shape memory effect, super elastic property, high damping, good durability and corrosion resistance, and provides the characteristics of large deformations and that the deformations can be restored, so it is widely used in the field of building structural vibration control [8, 9]. SMA has the distinctive nonlinear characteristics and in this study a double flag constitutive model for the numerical simulation of the restoring force characteristics is adopted [10-12].
To further investigate the effectiveness of the proposed DCPM-based approach, another nonlinear identification approach i.e. PSPM-based approach [6] is employed herein and compared with the proposed approach. The errors of the identified SMA damper force by the PSPM-based approach are plotted in Fig. 4 as a solid line whereas those by the proposed PSPM-based approach are shown as a dashed line. For ease and clarity of comparison, only the time segment from 0.57s to 0.63s is given in Fig. 4. It can be seen that the identified errors determined by the proposed methods are relatively smaller than those of the PSPM-based approach. Similar results can be found in the remaining time segments.

Fig.3 The SMA damper force (noise-free): (a) on the 1st floor; (b) on the 2nd floor; (c) on the 3rd floor; (d) on the 4th floor
Additional case: 3% noise level in dynamic response measurements

In practical situations, the noise is inevitable to be included in the response measurements. From this point of view, in order to investigate the robustness of the proposed approach, it is necessary to consider the effect of the noise. In this case, the identical random excitation is employed, and all the structural response measurements are simulated by the theoretically computed responses superimposed with the white noise with a 3% noise-to-signal ratio in terms of root mean square (RMS). Similar procedures shown in the previous section are implemented to identify the NRF of the system and SMA damper force. The SMA damper forces in this case are plotted in Fig. 5. It is obvious from Fig. 5 that the SMA damper should be located on the 4th floor because the SMA damper forces on the 1st, 2nd, and 3rd floor are close to zero.
4 CONCLUDING REMARKS

In this paper, based on a double Chebyshev polynomial model, a time-domain NRF identification approach is proposed for identifying the structural nonlinearity as well as the mass distribution under incomplete dynamic loadings. The feasibility and robustness of the proposed approach is validated via a numerical MDOF structure equipped with a shape memory alloy (SMA) damper whose restoring force is modeled by a double-flag-shaped nonlinear model. Results show that the proposed approach is capable of identifying the mass distribution and structural nonlinearity, and could be potentially used for monitoring damage initiation and propagation processes during vibration of engineering structures under dynamic excitations.

A distinguishing feature of the proposed nonlinear restoring force identification approach is that, other than the assumption of chainlike topology, it does not need information about the structure (such as structural characteristics or model class) and only the applied excitations on part of the DOFs of the structure and the corresponding response measurements are required. It provides a general methodology for the identification of NRFs of engineering structures, which can be used for the monitoring of damage initiation and development, and for evaluation of damage severity of engineering structures under dynamic loadings, where significant nonlinearities may be induced. Future studies should include further investigation on the performance of the proposed approach on structural systems with different classes of incorporated nonlinearities in a distributed system and the experimental validation with dynamic test measurements.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support provided through the National Natural Science Foundation of China (NSFC) under grant No 50978092.

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