Damage identification in a parabolic arch through the combined use of modal properties and empirical mode decomposition

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ABSTRACT: The free dynamics of a parabolic arch is studied in order to identify a localized damage. The identification technique is based on the combined use of time-frequency vibration signals processing and modal-based properties referred to finite element models of the arch. By relying on the information gathered for the undamaged and damaged states, the Empirical Mode Decomposition (EMD) is applied to the non-stationary acceleration data and the connection between the intrinsic mode function features and the underlying dynamics is investigated. The approach is devised to obtain a response-based identification technique in a data driven framework.

KEY WORDS: localized damage; empirical mode decomposition; prototype parabolic arch; numerical detection.

1 INTRODUCTION

The continuous progress of signal processing tools and sensing technologies has contributed, in the last two decades, to conceive numerous vibration based approaches for structural damage identification. In essence, their purpose is to identify the presence, type and severity of damage, to locate it, and to predict the remaining service life of the structure from its dynamic response. Depending on the type of structural response considered and how it is used to infer the damage features, three main groups of methods can be roughly identified, based on: direct physical properties (mass, stiffness, damping), modal properties (natural frequencies, mode shapes, modal damping) and signal processing techniques (Fourier, Wavelet, Hilbert and Hilbert-Huang transforms). Several review works providing useful overviews on the main techniques so far proposed in the literature and their evolution are available [1, 2, 3].

The present study proposes a new damage identification technique based on the combined use of the decomposition part of the Hilbert-Huang Transform (HHT), known as Empirical Mode Decomposition (EMD), and the experimental modal analysis approach. Therefore the method aims at merging the time-frequency content of the structural response signals provided by the signal processing phase with the dynamic features of the damaged structure provided by the modal perspective. Since its introduction [4] the HHT has been adopted by several authors for damage detection purposes due to its capability of coping with nonstationary and nonlinear signals. Frequency-time and energy-time plots of the IMFs (Intrinsic Mode Function) obtained from the EMD of propagating wave signals were proposed in [5] to detect damage in beams and plates by distinguishing timings of the direct, damage-reflected and boundary-reflected waves. The instantaneous phase extracted from the HHT was used in [6] for experimental damage detection in a three story building. The EMD was used by several authors to define an energy damage index based on which the healthy and damaged states of the structures can be assessed [7, 8, 9]. More recently, combined damage identification was addressed in [10] by proposing an index representing the difference in the structure’s energy at its healthy and damaged states; the index is derived from a fusion of EMD, Wavelet and neural networks techniques. Several techniques based on the EMD were also proposed to detect damage location and severity by analyzing measured structural response time histories during which abrupt stiffness changes occur [11, 12, 13]. As is known, modal based approaches use the change of either the mode shape or its curvature from intact and damaged structures as a basic feature for damage identification. The baseline data from intact structure can be obtained either from an experimental test or from an accurate numerical model of the undamaged structure [3]. In this work the EMD is applied to the structural response
acceleration data to extract the IMFs at each sensor located along the structure. Then, from the IMFs projections, the experimental pseudo-modal shapes are constructed for the healthy and damaged cases. From the comparison between the derived spatial functions location and severity of damage can be detected. The procedure is applied to the free dynamics of the parabolic arch investigated in [14] to identify damage represented by a notch reducing the arch cross-section height.

2 DAMAGE EVALUATION METHOD

The basic idea of the damage identification technique proposed in this paper is the use of a time-frequency vibration signals processing, known as Empirical Mode Decomposition (EMD) [4], to derive a privileged basis, i.e. the Intrinsic Mode Functions (IMF), for the projection of both the undamaged and damaged states. Due to its empirical nature, the EMD does not strictly belong to the class of modal-based techniques, indeed there’s no guarantee that a vibration signal processed through the EMD provides with results directly linked to the modal properties (frequencies, damping factors and mode shapes) of the structure.

2.1 Empirical Mode Decomposition

The Empirical Mode Decomposition (EMD) is a technique based on the decomposition of a signal in fast and slow oscillations. Roughly speaking, the purpose of the method is to separate the fast oscillation from the slow ones through an iterative scheme in which, at each step, the derived slow oscillations are treated as a new signal. Therefore the original signal is decomposed into a collection of few oscillatory modes, the IMFs, that can be viewed as the counterpart of the harmonic functions representing the usual mode shapes. The resulting decomposition is thereby called empirical, since it is obtained from a procedure without a direct relationship with the underlying parametric modal model.

The main characteristics of EMD are the possibility to operate on nonlinear and non-stationary vibrations, as well as the auto-adaptive nature of the decomposition and the simplicity of use. These properties led to a widespread use of EMD in a variety of engineering research areas, including structural health monitoring and identification.

The signal decomposition scheme can be summarized in the following steps (for sake of brevity we describe the procedure with reference to the continuous case, the extension to the discrete case being straightforward):

- identify all the extrema of the given signal $g(t)$;
- interpolate the minima (maxima) obtaining a lower (upper) envelope $e_u^{(0)}(t)$ ($e_l^{(0)}(t)$);
- evaluate the mean function $m_1^{(0)}(t)$ as the half-sum of the upper and lower envelopes $m_1^{(0)}(t) = (e_u^{(0)}(t) + e_l^{(0)}(t))/2$;
- subtract the mean function from the original signal $g_1^{(0)}(t) = g(t) - m_1^{(0)}(t)$;
- repeat the previous four steps, treating the function $g_1^{(0)}(t)$ as the original signal, updating the iteration order from 0 to 1;
- said $K$ the number of iteration necessary for reaching a chosen (local) stopping criterion, the function $g_1^{(K)}(t)$, or more simply $g_1(t)$ is called the first Intrinsic Mode Function (IMF);
- evaluate the first residue $r_1(t) = y(t) - g_1(t)$;
- repeat all the previous steps considering the residue $r_1(t)$ as the starting signal and so on;
- when a (global) stopping criterion is reached, the original signal $y(t)$ can be reconstructed as $y(t) = \sum_{n=1}^{N} g_n(t) + r_n(t)$, with $N$ the number of IMF $g_n(t)$ and $r_n(t)$ the final residue.

In other words, the EMD algorithm can be viewed as a sifting process, in which a nonlinear filtering operator is applied to the signal until a chosen stopping criterion is reached. The various versions of EMD proposed in the literature differ for the choice of interpolation rule and stopping conditions (local and global). In this work we use a cubic spline as interpolation rule, and, as local stopping criterion, a tolerance value of 0.2 on the standard deviation between two consecutive iterations; conversely, no global stopping criterion is assumed, except for the existence condition of at least two minima and two maxima, necessary to realize the upper and lower envelopes. Moreover, to reduce edge effects, dummy maxima and minima are added on both edges of the analysed time history, mirroring asymmetrically the closest extrema.

Completeness and orthogonality of the decomposition play a crucial role in the proposed damage identification technique. Therefore the Gram-Schmidt orthogonalisation method proposed in [15], is implemented in order to obtain sets of Orthogonal Intrinsic Mode Functions (OIMFs).
2.2 Damage index

The response-based damage identification method proposed in this work is based on the comparison between the EMD components of the undamaged and damaged free dynamic responses. The method is devised to detect and localize damage, with the only requirements that the same sensors configuration and excitation points are considered in the undamaged and damaged states.

The first step is to evaluate the averaged power $P_{ij}$ of the IMF$_{ij}$ (or OIMF$_{ij}$), where $i = 1, \ldots, N$ and $j = 1, \ldots, M$ are the sensor index and the IMF or OIMF order, respectively. The elements $\varphi_{ij} = \sqrt{P_{ij}}$ represent the components of $M$ pseudo-modes and the damage index can be derived from the comparison between pseudo-modes obtained from undamaged and damaged states. Accordingly, we introduce the following Pseudo-Mode Index (PMI):

$$\text{PMI}(i) = \frac{| \varphi_{ij}^d - \varphi_{ij}^w |}{\varphi_{ij}^w}$$

where $J$ is the index of the pseudo-mode in which the energy measure $\Delta P(j) = | \sum_{i=1}^{N} P_{ij}^d - \sum_{i=1}^{N} P_{ij}^w |$ exhibits a maximum, i.e. the pseudo-mode with the maximum power variation due to damage.

At first, the undamaged steel arch adopted as prototype model is considered in the next section. Its mechanical properties are identified by combining the data reported by the manufacturer and the results of a series of impulsive tests. Then pseudo-experimental data are considered for the damaged structure in order to assess the performance of the proposed technique in noise-free conditions.

3 UNDAMAGED STATE: PROTOTYPE AND MODEL UPDATING

We summarize the properties of the adopted prototype, a double-hinged parabolic steel arch, in the undamaged state, see Fig. 1.

The centerline of the arch has a span of 1010 mm and a rise of 205 mm (span-to-rise ratio of 4.81); the cross-section is rectangular with a width $b_a$ of 40 mm and a height $h_a$ of 8 mm. The material properties are the Young’s modulus $E = 2.050 \times 10^5$ N/mm², the Poisson’s ratio $\nu = 0.3$ and the mass density $\rho = 7.849 \times 10^{-6}$ kg/mm³. All the above values are those declared by the manufacturer, except for the dimension of the half-length of the hinges (5 mm), directly measured on the prototype.

In order to verify the parameters declared by the manufacturer, an extensive campaign of experimental tests has been performed on the undamaged arch. The experiments, performed at the Laboratory of the Department of Structural and Geotechnical Engineering, (SAPIENZA University of Rome), consist of dynamic tests, developed using an instrumented hammer as excitation instrument and recording the response of the structure with seven uniaxial piezoelectric accelerometers, positioned as shown in Fig. 1.

The impact hammer has a length of 216 mm, a mass of 160 g, a sensitivity of 2.25 mV/N (± 15 %) and a resonant frequency $\geq 22$ kHz. The piezoelectric accelerometers have a sensitivity of 10 mV/g and a dynamic range of 500 ± g; moreover, three of them have a frequency range from 0.005 to 5000 Hz, with a mass of $27 \times 10^{-3}$ kg, and the other four have a frequency range from 1 to 7000 Hz, with a mass of $13 \times 10^{-3}$ kg for the first three and $12 \times 10^{-3}$ kg for the last one.

Four different locations have been assumed as excitation points: in correspondence of the first, the second and the fourth accelerometer, and in between the second and the third. For each excitation point we have repeated the test ten times, so that, given that seven accelerometers are used, 280 recordings in total have been considered. Performing a Fast Fourier Transform and using a standard peak picking approach, each test of this database was used for identification of the first six frequencies of in-plane vibrations of the arch.

Mean values $\mu$, standard deviations $\sigma$ and coefficients of variation $CV = \mu/\sigma$ of the identified frequencies are shown in Table 1. Given the high ratio between the accelerometers bandwidth and the analysed frequency range (0 -700 Hz), as well as the number of recordings, a rather reliable estimation of the frequencies is obtained. As expected, the maximum value of the coefficient of variation, as reported in Table 1, is just 1.41 % (bold).

Starting from the properties illustrated in Fig. 1 and already discussed above, a finite element model of the prototype is constructed. Along the clear span (1000 mm) the centerline of the arch is modelled with 8 curved Timoshenko-like beam element (each of them postprocessed in 100 straight frames). The cylindrical hinges are modelled with a rigid link that describes the constraint dimension (5 mm). Moreover, in order to accurately reproduce the numerical model, also the masses of the accelerometers are taken into account.

A comparison between the experimental and numerical frequencies is shown in Table 2. The experimental values refer to an overall mean of the 280 recordings discussed above; indeed, the low coefficients of variation presented in Table 1 suggest that all the data can be regarded as...
Figure 1. The prototype arch (top left), the instrumented hammer and one of the seven accelerometers (top right), the geometrical properties (bottom left) and the cross-section data (bottom right). Red circles indicate the instrumented sections.

Table 1. Identified frequencies: mean values $\mu$ (Hz), standard deviations $\sigma$ (Hz) and coefficients of variation $CV$. 

<table>
<thead>
<tr>
<th>Mode</th>
<th>Impulse in 1</th>
<th>Impulse in 2</th>
<th>Impulse in 2-3</th>
<th>Impulse in 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\sigma$</td>
<td>$CV$</td>
<td>$\mu$</td>
</tr>
<tr>
<td>1</td>
<td>50.21</td>
<td>0.32</td>
<td>0.64%</td>
<td>50.07</td>
</tr>
<tr>
<td>2</td>
<td>124.06</td>
<td>1.27</td>
<td>1.02%</td>
<td>123.84</td>
</tr>
<tr>
<td>3</td>
<td>224.45</td>
<td>0.77</td>
<td>0.34%</td>
<td>225.24</td>
</tr>
<tr>
<td>4</td>
<td>359.32</td>
<td>0.41</td>
<td>0.11%</td>
<td>359.31</td>
</tr>
<tr>
<td>5</td>
<td>509.74</td>
<td>2.06</td>
<td>0.40%</td>
<td>510.30</td>
</tr>
<tr>
<td>6</td>
<td>716.60</td>
<td>2.04</td>
<td>0.28%</td>
<td>717.35</td>
</tr>
</tbody>
</table>

$E = 2.050 \times 10^5$ N/mm$^2$  
$\nu = 0.3$  
$\rho = 7.849 \times 10^6$ kg/mm$^3$
reliable. As far as the numerical model, the values of the percentage difference $\Delta = \left(\frac{f_n - f_x}{f_x}\right)$ (where $f_n$ and $f_x$ mean experimental and numerical frequency, respectively), suggest to develop an updating of the numerical model.

Choosing the Young’s modulus $E$ as parameter and the quantity:

$$e = \sqrt{\sum_{i=1}^{6} \Delta_i^2}$$

as the error measure, we obtain the objective function illustrated in Fig. 2. Table 3 shows the quantitative comparison among experimental and numerical frequencies as obtained using the optimal value of $2.000 \times 10^5$ N/mm$^2$ for the Young’s modulus $E$, instead of the initial value of $2.050 \times 10^5$ N/mm$^2$.

Finally, in good agreement with the mean values obtained in the experimental tests, a damping ratio of 1% is assumed for all the modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency $f$, Hz</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.35</td>
<td>3.10%</td>
</tr>
<tr>
<td>2</td>
<td>123.85</td>
<td>0.81%</td>
</tr>
<tr>
<td>3</td>
<td>224.76</td>
<td>1.69%</td>
</tr>
<tr>
<td>4</td>
<td>359.31</td>
<td>-0.47%</td>
</tr>
<tr>
<td>5</td>
<td>509.94</td>
<td>2.11%</td>
</tr>
<tr>
<td>6</td>
<td>716.35</td>
<td>-1.15%</td>
</tr>
</tbody>
</table>

Table 3. Quantitative comparison between experimental and numerical frequencies (updated model).

Figure 3. Perspective view of the notch.

4 DAMAGED STATE: NUMERICAL RESULTS

Starting from the updated numerical model, assumed as undamaged scenario, a localized damage is introduced as a strong reduction of the section height from $h_u = 8$ mm to $h_d = 2$ mm (percentage decrease $(h_d - h_u)/h_u$ of 75%). This damaged section is assigned to a straight frame of length 1 mm centred in the abscissa $x = 692$ mm (between the instrumented sections 5 and 6 of Fig. 1), see Fig. 3. Table 4 shows a quantitative comparison among the frequencies of undamaged and damaged numerical model; the values of the percentage difference $\tilde{\Delta} = \left(\frac{f_d - f_u}{f_u}\right)$ ($f_u$ and $f_d$ mean undamaged and damaged frequency, respectively), illustrate a small reduction of the frequencies, at most of 3.76 % for the first mode.

In order to perform a numerical simulation able to reproduce the undamaged experimental conditions reported in Section 3, we assume the same measurement and excitation points. Therefore, we apply in both, the undamaged and damaged numerical models, an impulsive force, located in four different positions (namely, in correspondence of the first, the second and the fourth accelerometer, and between the second and the third, see Fig. 1), and then, for each scenario, we record seven accelerations (in the direction normal to the arch curve, in agreement with the mean values obtained in the experimental tests, a damping ratio of 1% is assumed for all the modes.
Table 4. Quantitative comparison between undamaged and damaged (numerical) frequencies.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency $f_i$ Hz</th>
<th>∆</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Undamaged</td>
<td>Damaged</td>
</tr>
<tr>
<td>1</td>
<td>51.27</td>
<td>49.35</td>
</tr>
<tr>
<td>2</td>
<td>123.32</td>
<td>123.32</td>
</tr>
<tr>
<td>3</td>
<td>225.75</td>
<td>218.21</td>
</tr>
<tr>
<td>4</td>
<td>353.22</td>
<td>341.95</td>
</tr>
<tr>
<td>5</td>
<td>514.30</td>
<td>514.16</td>
</tr>
<tr>
<td>6</td>
<td>699.44</td>
<td>680.29</td>
</tr>
</tbody>
</table>

Figure 4. Impulse time-history.

with the use of seven uniaxial accelerometers, see Fig. 1), for a total number of 56 accelerations.

An impulse time-history stored during the experimental tests is used in the numerical simulations. This signal has a time step of $5.8594 \times 10^{-5}$ s (sampling frequency of about 17000 Hz) and 24575 steps, obtaining a signal response duration of about 1.44 s, see Fig. 4. The accelerations are stored using a time step of $2 \times 10^{-4}$ s (sampling frequency of 5000 Hz) and 7200 steps, obtaining a signal response of 1.44 s.

The results obtained by performing the empirical mode decomposition and applying the damage identification method based on the PMI index introduced in the previous sections (both in the IMF and OIMF versions), are shown in Fig. 5. The PMI values show that the technique works properly if the orthogonal intrinsic mode functions together with an appropriate selection of IMFs are employed. Indeed, only the OIMF($J$) are characterised by a single peak always located on one of the two sensors closest to damage. In particular, it is worth mentioning that the first IMF or OIMF does not always guarantee the best identification performance; moreover, as expected, the results improve when the excitation is applied closer to the damage (impulse in 4) location.

More insights on the proposed technique are provided by Fig. 4, where a time-frequency analysis of the IMF$_{52}$ is shown. The time-history closer view in the middle plot and the associated wavelet transform analysis (performed with the Morlet mother wavelet), confirm the multi-frequency content of the IMFs. This circumstance shows clearly that pseudo-modes differ, in general, from modes. Nevertheless, the main three frequencies of the IMF$_{52}$ (123, 353, 699 Hz) are strictly related to the arch natural frequencies.
5 CONCLUSIONS

In this work a Pseudo-Mode Index (PMI) for structural damage identification was introduced. This pseudo-modal approach is based on the use of the Empirical Mode Decomposition (EMD) to derive a privileged basis for the projection of the structural response. More precisely, the EMD is applied to all the measured responses in order to extract a set of data-driven functions, known as Intrinsic Mode Functions (IMFs) or Orthogonal Intrinsic Mode Functions (OIMFs), if the Gram-Schmidt process is used. Then, this auto-adaptive bases are used to perform a comparison between undamaged and damaged structure, trying to detect and localize the damage.

The procedure was applied to the free dynamics of a parabolic steel arch to identify a damage represented by an abrupt reduction of the arch cross-section height. Only the undamaged structure was so far experimentally tested, thus the comparison was performed using numerical simulations calibrated on the undamaged arch. However, the method is response-based and, in principle, it doesn’t require a numerical model. The encouraging initial results show that the use of orthogonal version of EMD and the correct choice of the OIMF order, provided by an energy measure, are essential for a proper identification. Experimental tests on damaged states will be developed in order to confirm these numerical evidences.

Moreover, following the basic idea of analyzing the structural damage with a data-driven basis, a theoretical refinement of the introduced damage identification method is under investigation. The aim is to improve the damage localization process and to explore more challenging questions, such as the localization of multiple damages, as well as the identification of damage severity.

REFERENCES


