Predictive modeling of large scale historic masonry monuments: uncertainty quantification and model validation

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ABSTRACT: This paper focuses on establishing confidence in model predictions through the validation of numerical models with a specific focus on large scale masonry structures. Through an overview of various case study applications, this presentation will discuss the relevant phases of model validation: numerical model development, field experimentation, model calibration and validation. The main goals of this paper are (i) to discuss where modeling errors and uncertainties originate and (ii) to present an approach to obtain model predictions that are statistically consistent with their respective measurements. Our approach considers all major sources of error and uncertainty, which originate from the numerical solutions of differential equations (numerical uncertainty), imprecise model input parameter values (parameter uncertainty), incomplete definitions of underlying physics due to assumptions and idealizations (bias error) and variability in measurements (experimental uncertainty). This paper provides an overview of the best-practices for developing finite element models for large-scale masonry monuments, designing and executing calibration experiments as well as calibrating numerical models under uncertainty.

Keywords: Model Updating; Model Calibration; Finite Element Modeling; Experimental Modal Analysis; Verification.

1 INTRODUCTION

Computational modeling of historic structures is a challenging task due primarily to the general absence of knowledge regarding the materials, construction history and support conditions. The finite element (FE) analysis of these historic structures has gained widespread acceptance on the basis of its ability to approximate geometrically-complex structures, predict a wide range of system responses and produce easily interpretable visuals of predicted results. Prediction uncertainty arises from a number of sources within FE modeling of historic monuments including simplifications and assumptions in the modeling process, imprecisely known input parameters (for instance, the material properties, boundary conditions and forcing function), and discretized and approximated solutions to the governing differential equations. Model verification and validation attempt to reduce these uncertainties and errors in the predictions via a systematic process of model development, experimentation and calibration. The process of developing validated numerical models of historic structures is overviewed in the following sections.

2 MODELING

The structural analysis of historic masonry has an elaborate history evolved from the traditional concepts discussed by Heyman [1]. However, most modern approaches employ the use of advanced computational techniques, most commonly finite element models [2]. The development and reliance upon these models demands the examination of two major sources of uncertainty and error: (i) the definition of model input parameters that govern the physical characteristics of the structure, i.e. geometry, material properties, external loads and other boundary conditions, and (ii) the formulation of the computer model with respect to the selection of suitable element types, mesh discretization, model simplifications and the solution algorithms. In the following subsections, each of the various facets comprising these two major sources of uncertainty will be discussed.

2.1 Modeling types

The accuracy of a numerical model depends on the accuracy with which the mechanical behavior of the heterogeneous composite masonry is reproduced. Indeed, the selected modeling type affects every subsequent step of the model development process as described in this section. Three plausible modeling strategies are available to model the masonry composite: (i) detailed micro-modeling, (ii) simplified micro-modeling and (iii) macro-modeling [3] [4]. Detailed micro-modeling provides the most realistic representation of the masonry assembly by modeling the units and mortar separately and assigning independent material properties to each. Detailed micro-modeling requires the highest effort in the development of the geometry as well as a very fine mesh, thus demanding the highest computational effort. In the simplified micro-modeling approach, each unit along with its surrounding mortar is approximated with idealized units each assigned homogenized material properties. Finally, macro modeling assumes the entire masonry composite as a homogenized continuum throughout the modeled structural member. Macro-modeling is the most commonly used modeling strategy owing to its geometric simplicity and relatively low computational effort. By comparison, micro-modeling is suggested only when local failure modes within structural elements are of interest.

In a comparative study of these three modeling approaches conducted on a model of a scaled masonry dome [3], detailed micro-modeling was found to be most accurate in the prediction of load-carrying capacity and stiffness while unsurprisingly macro-modeling was found to be the least accurate. More interestingly, the distribution of cracks was
more accurately predicted with detailed and simplified micro-modeling compared to macro-modeling. Furthermore, the detailed micro-modeling is reported in the study to be the least sensitive to variation within the input material parameters compared against simplified-micro and macro-modeling. It must be noted however that despite the proven superior accuracy and insensitivity to variation, the implementation of detailed- or simplified-micro modeling quickly becomes infeasible when involving actual masonry structures which often exhibit inordinate scales.

### 2.2 Geometry

Construction drawings, if available, may be used to recreate the solid geometry. However, these drawings rarely match the on-site geometry owing to permanent deformations commonly exhibited amongst aging structures. On the other hand, on-site measurements which ensure the recreation of the as-is geometry, can be time-consuming, especially for a large structure with inaccessible components. 3D photogrammetry [5] supplies a fast and accurate technique that can facilitate the visualization of a structure to account for existing cracks and other structural degradation within the masonry. However, 3D photogrammetry only provides information on the surface of the masonry. The thickness of the members beneath the surface must be obtained through either direct measurement, when possible, or with analysis of the construction drawings.

Another alternative is 3-D laser scanning [6], a fast and accurate technique that is gaining increasing popularity. The laser scans typically produce a cloud of points, which can be reduced in a CAD modeling software to specific keypoints preserving the structurally significant features of the monument. If both surfaces of a structural component, such as a wall or a vault are accessible, the thickness of members can be obtained by stitching the scan data. If the rear region of the walls and vaults is inaccessible with no readily available holes to measure the depth, non-invasive techniques such as impact-echo [7], impulse radar [8], sonic tomography [9] or electromagnetic conductivity [10] may be used to determine the thickness.

In micro-modeling, the unit dimensions and mortar joint depth are typically assumed to be perfectly uniform throughout the structural member, thus disregarding the minor variations due to uneven units and variable workmanship. In macro-modeling, minor material degradations and geometric deformations as well as non-structural elements are typically disregarded. While making the process of geometry recreation and meshing much easier, such minor simplifications only negligibly affect the model predictions.

### 2.3 Material properties and material models

Accurately defining material properties is challenging because of the spatial and temporal variability of material properties typical of historic masonry structures. At best, one may obtain samples from the site in the form of cores or loose material. However, when samples are collected from the site, large sample-to-sample variability is likely to be observed [11]. Material properties obtained from tests on samples may be used as nominal values before being subject to calibration in a process known as fine-tuning. If material tests are sufficiently extensive as to provide information about the distribution of mechanical property values, such distributions may be used in the calibration process. More on this topic, as well as calibration and fine-tuning, is provided within section 4.

Masonry material behavior is characterized by a small tensile capacity and relatively large compressive strengths. Additionally, masonry may be viewed as a composite material with rigid blocks bound by relatively soft mortar. However, in the analysis of more realistic, full-scale constructions, the macro-modeling approach is frequently used and masonry is treated as a homogeneous material where homogenized properties of the masonry assemblage are obtained either from tests on the assembly or using one of a number of homogenization techniques [12] [3].

Constitutive models, which describe a material body’s stress-strain characteristics, must be implemented within the larger model. Although masonry behavior is largely nonlinear under substantial loads due to the formation of extensive micro-cracking, a simple linear elastic treatment of the material may be justified under small loads, such as self-weight, to understand the stress distributions and to identify regions with high tensile stresses. It is recognized that masonry’s primary mode of failure, the brittle cracking which may result from these tensile forces, occurs most prominently in the mortar joints [13]. Consequently, several yield criterion may be considered in the formation of constituent models for historic masonry materials [14] [15].

The Drucker-Prager yield criterion is used for ductile materials that are weak in tension and exhibit volumetric plastic strain. This criterion requires specification of the friction angle and cohesion which can be adjusted such that the analytical tensile and compressive strength matches the desired values. The Mohr-Coulomb yield criterion is the elastic-perfectly plastic implementation of the Drucker-Prager yield criterion specialized for brittle materials in which the tensile strength is low relative to its compressive strength. William-Warnke [16] failure criteria may be considered a combination of the Mohr-Coulomb and Drucker-Prager yield criteria and can be implemented to model historic masonry materials [17]. This failure criterion is capable of predicting the cracking and crushing indicative of the material via a smeared crack analogy. The failure surface is be defined by three parameters, namely its material’s elastic (Young’s) modulus, ultimate uniaxial tensile strength and compressive strength. The material exhibits failure if either of the tensile or compressive principal stresses lie outside the failure surface.

### 2.4 Boundary conditions

Computer models focus on the structural components of interest and thusly, require the definition of structural behavior at the boundaries between modeled and unmodeled components. Defining the structural behavior at these boundaries is a challenging undertaking. Theoretically fixed or free boundary conditions that are readily available in most structural analysis software packages are only approximations of the complex, semi-rigid behavior of such boundaries within practical masonry systems. The
restraining forces at these boundaries are dependent on the masonry material properties and the configuration of the adjacent members. Unlike material properties, non-destructive test methods which estimate such boundary conditions are limited.

One suitable technique used to approximate boundary conditions is substructuring. This process reduces the adjacent structure to its interface at the shared degrees of freedom to form superelements. This approach is advantageous for structures with repetitive geometries such as the naves of gothic cathedrals, casemates of 19th century forts, etc. since the same FE model may be used to generate the substructures of adjacent assemblies.

Another technique widely used to approximate the boundary conditions is the application of translational and rotational springs at the boundary with assumed (but unknown) spring stiffnesses. These stiffness constants must then be calibrated using non-destructive test data (as discussed in Section 4). A parametric analysis, using spring stiffness as the primary variable, may be completed to define a range of stiffness values that lead to a semi-rigid connection. This range can be identified by observing the response of interest of the system at varying levels of spring stiffness before selecting the range in which the response of interest varies between an upper and lower limit.

2.5 Element type and meshing

In addition to the material model, an element type is required to represent masonry which can approximate the material’s behavior. Many available FE software packages commonly offer dedicated element types such as the SOLID65 element provided by ANSYS, originally designed to emulate concrete [18]. To account for cracking and crushing, the SOLID65 element uses the Willam-Warnke failure criterion discussed above and accounts for the material behavior via a smeared crack analogy. The element introduces a plane of weakness in the direction of the failure by modifying the elastic modulus at the element face to a near-zero value, thus replicating the cracking behavior while maintaining mesh continuity. Consequently, this and other elements have been extensively applied to model the unique material properties and geometric irregularities of historic masonry structures [19] [20] [21].

The meshing of the FE model must achieve a suitable balance between solution accuracy and computational time. A coarse mesh can degrade solution accuracy while an overly fine mesh can result in excessive computation. A mesh refinement study may be performed by predicting the response of interest at varying mesh sizes followed by an extrapolative calculation of a reference solution. The reference solution is an approximation of the solution corresponding to an infinitesimal element size yielding a theoretically-exact solution. A mesh size must be sought that yields an error with respect to the reference solution that is less than the expected measurement error, which is indicated to be around 10% for large scale civil structures under normal operational conditions [22].

3 EXPERIMENTATION

Vibration testing offers a non-destructive solution for the rapid condition assessment of historic structures. Vibration testing serves three main purposes in the assessment of historic structures: (i) to obtain vibration features such as natural frequencies and mode shapes in support of numerical model development by providing calibration and validation data [23] as well as to facilitate seismic evaluation of the structure [24]; (ii) to facilitate a rapid condition assessment after a severe loading event such as an earthquake [25] or after repair/retrofitting activities to assess the efficacy of the repairs; and (iii) to continuously monitor the structural health of the system in order to facilitate condition-based maintenance [26]. The use of vibration features in structural health monitoring is based on the notion that changes in a system’s structural stiffness and mass produce variation in a structure’s vibration characteristics which, if accurately measured, can allow the engineer to draw inference of the system’s structural condition.

The experimental techniques for vibration analysis of historic structures differ from modern constructions. The complex geometries and boundary conditions typical of historic structures make the estimation of expected modal displacements and natural frequencies a complex and unintuitive task. Severe non-linearities in the form of cracks, ubiquitously seen in virtually all historic masonry structures, cause nonlinear behavior and degrade the accuracy of the identified modal parameters. This is due to the fact that modal analysis assumes the structure to produce a linear response at the recorded vibration amplitudes. Furthermore, due to the highly dissipative nature of masonry, extracting vibrations far from the excitation source becomes difficult.

Owing to these unique features of historic masonry structures, the planning and execution of vibration testing requires special consideration discussed within the following subsections.

3.1 Reconnaissance and preliminary tests

Before embarking on a full-scale vibration test, preparatory studies including reconnaissance trips, preliminary FE models and preliminary in-situ tests can be applied to effectively strategize the implementation of sensing equipment helping to determine such aspects as sensor location, mounting strategy and equipment logistics. The issues typically addressed by a reconnaissance survey are as follows [27]: the accessibility of the measurement locations, means to transport equipment to the measurement locations, layout of power sources, required length of sensor cables (for wired sensors), range of wireless data transfer (for wireless sensors), appropriate mounting techniques, condition of the vibration surface (rough or smooth; dirty or clean), geometry of the vibration surface (curved/inclined; note that special mounts must be designed to mount sensors vertically or horizontally on curved surfaces), excitation sources present at the structure (carillon bells, pipe organs etc.) and safety concerns (slippery surfaces, edges of the roof etc.).

Preliminary modal tests can be performed with a limited number of sensors to get estimates for the quality of the acquired data, ranges of frequency and vibration amplitudes,
dissipation of the vibration from the excitation source and measurement noise. These estimates allow better pre-planning of the main experiment in deciding the equipment to be used and signal conditioning to be applied. Preliminary modal testing may also be used to improve the preliminary FE model for better estimation of higher modes of the structure that were missed in the preliminary tests.

3.2 Sensors

Accelerometers are the most widely used sensors for vibration measurement of stationary systems because of their high dynamic range, small size and ease of installation when compared to velocity and displacement transducers. Since the first natural frequency of historic masonry is typically low, the accelerometer must be capable of detecting frequencies lower than 3 Hz. Atamturktur et al. [27] have tested the vaults of five Gothic Cathedrals and found the first 10 modes to fall under 30 Hz. Typical amplitudes of ambient vibration encountered in historic masonry structures are to the order of 0.02 - 0.05% g. Thus, the accelerometer must have sufficiently high sensitivity to low amplitude vibration in that an output voltage of 1V/g typically yields sufficient sensitivity.

If the modal analysis data is to be used for the calibration of the computer model, it is most convenient to have the direction of vibration sensors aligned with the global Cartesian co-ordinate system employed in the model. The curvature of the structural components makes the mounting of the sensors in the horizontal or vertical directions difficult, making it necessary to cast or fabricate mounting plates with adjustable screws. Mounting the accelerometers directly on the masonry surface using modeling clay, sticky tack or double-sided tape can also provide satisfactory measurement quality [27] [28].

3.3 Sensor location

While conducting vibration testing with the intent to identify the modal parameters, the placement of the sensors must be aimed at effectively identifying the first few mode shapes of the structure with the available number of sensors. Placing sensors at nodal points of the desired modes must be avoided as at these locations the modal displacements corresponding to that mode are negligible. Furthermore, special attention must be given to avoid spatial aliasing of desired modes, in which higher order modes appear to be of lower order. Prabhu and Atamturktur [29] have demonstrated the use of the Modified Effective Independence Method in the selection of optimal locations for a given number of sensors on a masonry cathedral. The method selects sensor locations such that maximum linear independence between the measured mode shape vectors is obtained while maintaining a certain minimum distance between the sensors. The method requires a numerical model of the structure and the sensor locations obtained for a masonry cathedral were found to be fairly robust with respect to variations in the model input parameters.

3.4 Excitation source

For traditional modal analysis, the most common sources of forced excitation are shaker, vibrodyn, mechanical actuator or instrumented hammer. The choice of excitation source depends on various factors including the accessibility of the structure, the available budget, etc. Shakers, vibrodynes and actuators capable of producing sufficient energy to excite large structures are typically difficult to transport. Instrumented impact hammers, which are relatively light weight and inexpensive by comparison, are more flexible in their application. Due to the inherent damping in masonry structures, the impact hammer must have a sufficiently high peak force rating to generate vibration energy sufficient to excite all measurement locations. At the same time, the system response must remain in the linear range with the vibration amplitudes not overloading the sensors nearest the excitation location. Typically, a soft hammer tip must be used that can excite frequencies up to 100 Hz. Atamturktur et al. [27] provide detailed guidelines for using impact hammers as an excitation source in testing the vaults of masonry cathedrals.

Operational modal analysis, commonly known as ambient vibration testing, is another widely-implemented technique. This output-only method utilizes the ambient operational vibrations of wind, waves, traffic etc. as an excitation source. Since no artificial source of excitation is required, operational modal analysis is generally less time consuming and more economical than traditional modal analysis. Atamturktur et al. [22] have compared traditional and operational modal analysis techniques on an historic structure.

4 MODEL CALIBRATION

The precision and accuracy of numerical model predictions rely on the uncertainty and accuracy of the model input parameters and the assumptions established in the formulation of the numerical model. Model calibration (also known as model updating) refers to adjusting the model input parameters and correcting for the model bias to ultimately obtain a calibrated and bias-corrected model which better represents the behavior of the structure. The general process of model calibration, which involves systematic comparisons between model predictions and experimental measurements is outlined in Figure 1 and discussed in the following paragraphs.

![Figure 1: General process flow of model calibration.](image-url)
4.1 Selection of comparative feature

The comparative features are the low dimensional signatures extracted from both the model predictions and experimental measurements. As their name indicates, they serve as a link between test and analysis. Hence, feature extraction is essentially a process of data reduction where a large number of raw data points are reduced to a much smaller vector or a scalar quantity. Comparative features must be selected such that they are sensitive to the selected calibration parameters. A variety of comparative features which can be extracted via on-site dynamic measurements, such as temporal moments and regression characteristics of the time history data, are discussed in [30].

Features most commonly implemented during calibration are the modal parameters including natural frequencies and mode shapes. It must be noted however that modal parameters only allow calibration of the linear parameters in a model, such as elastic modulus, density and linear boundary conditions. For calibration of non-linear parameters such as tensile strength, destructive experiments would be needed [31] [32]. Thus, the process of selecting comparative features and calibration parameters is intertwined and depends on the calibration goals and available experimental data.

It should be emphasized that the use of modal parameters in model calibration mandates special attention to the correct ordering of the modes (also known as pairing of test and analysis modes); as calibration parameters are perturbed (typically iteratively through inverse analysis algorithms used in model calibration), modes may swap order, appear/disappear or become combinations of each other. Hence, assuring a correct pairing of modes is of the utmost importance in utilizing modal parameters within calibration.

4.2 Selection of calibration parameters (uncertainty propagation, effect screening)

The selection of a model input parameter as a calibration parameter depends on its sensitivity to the model prediction of interest as well as the uncertainty regarding the parameter’s precise value. These two factors of sensitivity and uncertainty are assessed in combination in the Phenomenon Identification and Ranking Table (PIRT) [33], based on which, a decision may be made on the selection of calibration parameters.

The uncertainty in the parameters typically originates from a lack of knowledge owing to the unavailability of a large number of experiments on the parameter values. There are a number of different ways to represent this uncertainty, such as through probabilistic methods, fuzzy sets or convex modeling. In the absence of sufficient experimental data, one common approach is defining parametric uncertainty using expert opinion on the minimum and maximum bounds and assuming a uniform distribution within these bounds.

Prior to the selection of calibration parameters, a sensitivity analysis must be conducted. The first goal of sensitivity analysis is to ensure that the comparative feature is sufficiently sensitive to the selection of calibration parameters. The second objective of sensitivity analysis is to assess the interactions between the parameters and their combinatorial effect on the comparative feature. If the parameters are strongly interdependent, calibrating one parameter may compensate for the inaccuracy in another parameter and ultimately lead to an unsatisfactory model calibration. The effect of interdependency or correlation between parameters, once identified, may be resolved either by holding one of the correlated parameters at its nominal value (assuming that reliable information regarding the nominal value is available) or by performing co-ordinate transformation on correlated parameters to obtain new uncorrelated parameters.

4.3 Test analysis correlation and associated metrics

As the name suggests, test-analysis correlation involves systematic correlations between the comparative features obtained from model predictions and experimental measurements. For such comparison however, a suitable metric that quantifies the agreement (or lack thereof) in the comparative feature must first be defined. The definition of this metric closely depends on the nature of the comparative features. A select few examples of such metrics include the Euclidian distance, i.e. the absolute geometric distance between two points [34]; the Mahalanobis distance, i.e. the weighted distance between a point and a population that considers the correlations [35]; and the Bhattacharyya distance, i.e. the weighted distance between two populations that also considers the correlations [36].

4.4 Calibration procedure

The goal of calibration is to adjust the parameters such that the test-analysis correlation metric is improved. The two common approaches to parameter calibration are (i) optimization-based techniques [37], which often treat the model predictions and experimental measurements in a deterministic manners and (ii) probabilistic techniques [38], which acknowledge the inherent uncertainties in the model predictions and experimental measurements. A notable probabilistic approach is Bayesian inference, which has recently received attention from those involved in the modeling and simulation of masonry monuments [39].

It must be emphasized however that calibrating parameters of a model against experiments neglects any potential biases that might be present in the model due to unavoidable model imperfections. Such biases, which may result from simplifying assumptions established during the development of the geometric model or through the use of a homogenized, macro-model representation of the heterogenous masonry and mortar assembly, are commonplace in modeling masonry monuments. Bias in model predictions can be approximated through an independently developed error model. Such a model, once trained, can also be used to bias-correct the model predictions. Of course, training of this error model must be completed simultaneously with the fine tuning of calibration methods. Recently, methods have been developed to simultaneously fine tune calibration parameters and train a model to represent bias [40]. Another future area of study may lie in the development of methods for the determination of predictive maturity among numerical models to establish for the user a more quantified level of confidence in model predictions [41].
5 VALIDATION

A widely accepted definition of model validation is “the process of determining the degree to which a model and its associated data is an accurate representation of the real world from the perspective of the intended uses of the model” [42]. A numerical model that is calibrated against experiments becomes conditioned on the set of experiments used in calibration. Therefore, validation of the calibrated numerical model implies a demand for separate experiments that were not used in model calibration. Validation of a model also requires a definition of a validation metric along with a well-defined sufficiency criterion. The sufficiency criterion must be determined taking into account the manifestations of errors in the experimental process which arise at various stages of model development and experimentation such as natural variability between repeated experiments, imperfect sensor alignment, mismatch in the locations of sensors and corresponding nodes in the numerical model, geometric or construction imperfection as well as undocumented repairs that were not taken into account in model development.

The concept of model validation must also be closely tied to a specific domain of applicability. This domain defines the operational conditions of the system of interest for which the model is expected to deliver predictions. The calibration domain however represents the domain which is contained by the available experiments for calibration. For masonry monuments, such a domain can be defined for various load levels, in that the experiments representing the behavior of the structure within a certain range of force levels would yield a model that is validated with a domain bounded by this range. Hence, if the model is executed to predict outside of this range, the predictions should not be considered validated. Ideally, the calibration domain and the domain of applicability overlap to prevent extrapolative predictions.

6 CONCLUSION

Numerical models are being implemented at an increasing rate to improve our understanding of the behavior of historic masonry monuments. Indeed, the use of such tools is becoming commonplace by the practicing engineering community, warranting a thorough discussion on the best practices for obtaining models of historic masonry monuments that are rigorously validated against experimental measurements with quantified uncertainty bounds on their predictions.

The model development process requires a series of decisions made with established assumptions for achieving an accurate representation of the model. Although not discussed here in the level of detail it deserves, there is an established literature on the verification of numerical models along with mature tools for quantification of numerical uncertainties. It is the responsibility of the model developer to deliver a numerical model verified to yield converged predictions within the domain of applicability.

In the model development process, many of the simplifying assumptions established at this stage lead to systematic biases in the predictions. Lack of knowledge about the input parameters needed to solve the equations of interests lead to uncertainty in the predictions. Recent methods available for model calibration can simultaneously handle these two sources of errors and uncertainties.

The calibration and validation of numerical models invariably requires the availability of experimental measurements describing the performance of the structure under well-known conditions. Conducting on-site experiments can be demanding on resources (cost, personnel, equipment and time) and thus, utmost care must be given to ensure the experiments are carefully designed and executed to maximize the information gained for model calibration.

The use of a numerical model may entail making predictions regarding the current condition of a structure (i.e. current world) or investigating what-if scenarios to predict the behavior of the structure under a possible load configuration that may occur in the future. For the latter, validation of the model becomes a challenging undertaking due to the absence of experimental measurements for a hypothetical scenario. Thus, model validation becomes a process of compiling evidence regarding the credibility of model predictions. Obviously, a larger body of evidence would lead to a higher confidence in a model’s ability to represent reality.

This paper presents a thorough discussion on the validation and uncertainty quantification of numerical models of historic monuments. The reality of practical applications of the procedures presented herein is that the availability of resources (cost, personnel, equipment and time) will dictate both the scope and implementation of a model validation campaign. We must emphasize that neglecting model validation altogether and using numerical models that are not validated can deliver incorrect predictions for the management of structural heritage, and can ultimately increase the probability of failure.

The discussions presented herein are specifically focused on masonry monuments. This focus however does not imply any restriction in expanding these concepts to other structural systems and to other fields.

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