Modeling and propagation of stochastic linear viscoelastic material properties of asphalt mixtures in pavement structures

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ABSTRACT: Pavement structures of asphalt mixtures exhibit variability in their responses to loading and environmental conditions, such as heat and moisture. Thus, it is vital to account for this variability for robust predictions of the response and performance of pavement structures. This study addresses the stochastic modeling and propagation of macroscopic linear viscoelastic constitutive properties of asphalt mixtures within a parametric probabilistic framework. Experimental measurements of the creep compliance have been used to construct two types of stochastic models: spatially-invariant models and spectral stochastic spatially-dependent models of linear viscoelastic material properties. The latter methodology utilizes Karhunen-Loève expansion method for an optimally-reduced stochastic dimension. These models are propagated using a non-intrusive spectral projection method, for which the sparse grid method has been used to sample the cubature nodes. Accordingly, a spectral representation of the response of interest, strain, is constructed and used for statistical quantification. This approach could be appropriate to address various aspects of the performance of asphalt pavements.

KEY WORDS: Asphalt mixtures, linear viscoelastic materials, Non-intrusive spectral projection (NISP) approach, Sparse grid cubatures, random fields, Karhunen-Loève, polynomial chaos.

1 INTRODUCTION

Asphalt mixtures are complex heterogeneous materials composed of three phases (aggregates, asphalt binder, and air voids) with diverse material, geometric, bonding characteristics, and spatial positioning. Pavement structures of asphalt mixtures exhibit variability in their responses to loading and environmental conditions, such as heat and moisture [40]. This variability could be associated with the highly uncertain and variable microstructure, affected by the quality of the construction process, together with the uncertain operational environment. This, in turn, is reflected in the macroscopic behavior. The heterogeneity and variability associated with material properties, geometry, and spatial positioning of the constituents are considered one of the main contributors to the response variability. For instance, the materials, the geometry, and the spatial distribution of the aggregates, the size and the spatial distribution and connectivity of the air voids; and the cohesive and adhesive mechanical and chemical properties of the mastic, all contribute to such highly variable microstructure and corresponding macroscopic behavior.

Criteria and specifications are usually set to control the product quality of asphalt mixtures. However, even in the highly controlled production process, variability in the microstructure cannot be eliminated. For instance, there exist mixing and compacting methods that control the air void content in asphalt mixtures, nevertheless, selected specimens from the same mixture will comprise slightly different air void percentages, not to mention their spatial distribution and connectivity [21], [20], [16], [18], [17]. These same specimens, if tested, would not result in same measurements, rather, a certain level of scatter.

Asphalt mixtures composition and response to various boundary conditions could be well described as highly variable and uncertain. For a reliable modeling and robust prediction of this behavior, it is desirable to account for the possible sources of variability and uncertainties associated with their properties and behavior. To do so, sufficient data at various scales are required. For instance, stochastic modeling at finer scales of the structure is required to upscale towards the coarser scales, given that various sources of variability and uncertainties are associated with the microstructure of asphalt mixtures. The amount and quality of available relevant data dictates the thoroughness of the modeling. Advanced screening and testing equipment shows that heterogeneity and anisotropy are manifest in asphalt mixtures material properties [16], [19], [15], [18], [17] . Yet, these are modeled in most current applications as homogeneous and isotropic materials [22], [23], [24], [25], where measurements from two or three specimens are commonly averaged and considered as nominal properties.

Asphalt mixtures have been shown in several studies to exhibit time- and temperature-dependent viscoelastic and viscoplastic behavior [26], [27], [28], [29], [30]. However, they are commonly treated as elastic structures [24], [31]. Recent advanced works have been addressing linear and nonlinear viscoelastic, viscoplastic, etc., behavior of asphalt...
mixtures within deterministic frameworks. Yet, the majority of works addressing the stochastic modeling of material properties assumes homogeneous and elastic materials [32], [33], [34], [35], [36], [37]. Available studies on stochastic modeling of viscoelastic material properties are scarce [11], [38], [39], [14]. Hilton et al. [11] proposed to use a beta or a normal distribution function to represent the relaxation curve where upper and lower envelopes in the time domain were associated with ±3σ dispersion, for which the standard deviation was modeled as a function of time. Caro et al. [38] and Castillo et al. [39] proposed to model the viscoelastic material properties in terms of randomized air void fields. The spatial air void realizations were generated and used to infer associated realizations of viscoelastic properties in terms of the correspondence reported in literature. A recent work by Soize and Poloskov [14] proposed a time-domain formulation for linear viscoelastic media with model uncertainties and stochastic excitation based on nonparametric probabilistic approach and the random matrix theory.

This study addresses the stochastic modeling and propagation of macroscopic viscoelastic constitutive properties within a parametric probabilistic framework. Constitutive models of linear viscoelastic materials are usually modeled using rheological mechanical models which are also called prony series model [7]. A comprehensive accounting of randomness at finer scales is outside the scope of this study, rather, its effect that is conveyed to macroscopic behavior is considered. Due to the heterogeneous and chemical composition of asphalt mixtures, the material responds differently at different time zones or at different temperatures. This is governed by the terms of the prony series model. each term represents a phenomenon taking place at finer scales, which implies that the stochasticity at the macroscopic level is implicitly a result of random microstructure phenomena. The parameters of the prony series model, i.e., retardation times and creep coefficients, could be modeled either as spatially-invariant random variables or as spatially-dependent stochastic processes. In the latter approach, each time-dependent microstructure phenomenon will be dominant at other spatial locations, which in turns, suggests the presence of a certain form of spatial dependency. This spatial dependency is modeled using Karhunen-Loève expansion [4].

In the studies pertaining to infrastructure materials, it is customarily to test a very small number of samples, mostly two or three. Experimental data is needed to construct data-driven stochastic models. The proposed methodology is demonstrated using limited number of experimental measurements obtained from testing asphalt mixtures specimens designed to have 4% air void content. This number is insufficient to construct thoroughly stochastic models. Hence, some assumptions have to be made to compensate the lack of adequate information required to construct exhaustive stochastic models. As more data becomes available, these models could be updated. Nevertheless, the constructed models in this study conform perfectly to the available experimental measurements, i.e. creep compliance measurements[13]. Stochastic models of material properties are advantageous in terms that once propagated in a computational models, the uncertainty in the predicted response can be quantified. This is crucial to the design process or predictive studies of the highly uncertain and variable asphalt mixtures structures. A Non-intrusive spectral projection (NISP) approach is used to propagate the variability in the material properties [5]. To do so, the models are interfaced with an Abaqus finite element model of a 2D pavement structure via a user material subroutine UMAT. The coupled models are used to construct a stochastic representation of the strain response using the polynomial Chaos expansion [4][5]. This stochastic model has been used to generate random realizations which are used to statistically quantify the strain response.

2 MECHANICAL MODELS FOR THE RHEOLOGICAL BEHAVIOR OF LINEAR VISCOELASTIC MATERIALS

The constitutive equation, stress-strain relation, for uniaxial deformation of linear viscoelastic material under isothermal condition is given in the following form, according to the Boltzmann superposition integral [1],

$$\varepsilon(t) = D_0 \sigma(t) + \int_0^t D(t - \tau) \frac{d\sigma(\tau)}{d\tau} d\tau,$$  \hspace{1cm} (1)

where, $D_0$ and $D(t - \tau)$ are the instantaneous and transient creep compliance components. That is, total strain is composed of two components; an instantaneous component and a time-dependent component. The linear viscoelastic behaviour of materials is usually modeled using appropriate mechanical (rheological) models. These models comprise of ordered combination of linear elastic springs and linear viscous dashpots[1], [2]. Specifically, the so-called generalized Kelvin model, which comprises of a spring and several voigt units (spring and dashpot in parallel) connected in series, is usually used to approximate the retardation behavior. The general form of the stress-strain constitutive equation can be expressed in terms of linear differential operators as [2],

$$\sum_{n=0}^{N_p} b_n \frac{d^n \varepsilon(t)}{dt^n} = \sum_{m=0}^{M} a_m \frac{d^n \sigma(t)}{dt^n}$$  \hspace{1cm} (2)

The solution of this equation is usually obtained in the frequency domain and is dependent on the specific arrangement of the springs and dashpots in the model. The corresponding time domain expression of the solution, constitutive equation, known as the PSM [2], can be expressed as,

$$D(t) = D_0 + \sum_{n=1}^{N_p} D_n (1 - \exp(-t/\tau_n)),$$ \hspace{1cm} (3)

where $D_0$ is the instantaneous creep compliance, also known as the glassy compliance $D_g$ [7]. $D_n$ and $\tau_n$ are the retardation creep coefficient and the retardation time corresponding to the unit $n$ in the generalized kelvin model, respectively. Note that $D_n = 1/E_n$ and $\tau_n = \eta_n/E_n$, where $E_n$ and $\eta_n$ are the spring modulus and dashpot viscosity characterizing the unit $n$. Here, $n = (1, \cdots, N_P)$, where $N_P$ is the number of the units and consequently the number of terms in the PSM.

Thus, the strain in equation 1 can be estimated using the PSM by substituting $D(t - \tau)$ by $\sum_{n=1}^{N_p} D_n (1 - \exp(-t/\tau_n))$. The retardation times indicate how fast the material recovers
the strain while the creep coefficients indicate how much it is recovered over a specific time interval.

Of importance in many problems is the frequency-based expressions of the creep compliance, given that experimental measurements of the creep compliance are usually performed in the frequency domain. It is common to express the time-dependent creep compliance in the frequency domain in terms of the shear storage $D'(\omega)$ and loss $D''(\omega)$ compliances. These can be easily deduced from equation 3 [9] and will be used to model the viscoelastic properties, hereafter.

$$D'(\omega) = D_0 + \sum_{n=1}^{N_p} \frac{D_n}{1 + \omega^2 \tau_n^2}, \quad (4)$$

$$D''(\omega) = D_0 + \sum_{n=1}^{N_p} \frac{\omega \tau_n D_n}{1 + \omega^2 \tau_n^2}, \quad (5)$$

PSM is able to account for a range of temperature and time/frequency correspondence. That is, the number of terms in the PSM should be equal to the recovery times (decaying frequencies) covered by the experimental data, which is equivalent to a certain range of temperatures given that the asphalt mixtures are known to be thermo-rheological materials. The physical interpretation of this phenomenon is that due to the heterogeneous and chemical composition of asphalt mixtures, the material responds differently at different time zones or at different temperatures. This is governed by the different retardation times that correspond to the different recovery/decaying terms in the PSM. That is, the smallest retardation time and creep coefficient are associated with the first term in the PSM ($n = 1$) representing the first voigt unit. At the beginning, the contribution of this term would have the dominant contribution. As time passes, The contribution of this term reaches its maximum and becomes constant and equal to the associated creep coefficient ($D_1$) as the time-recovery (frequency-decaying) effect disappears. Concomitant to this, the contribution of the retardation time from the second term dominates the response and acts as a correction to the first one. This process continues until the effect of the largest retardation time diminishes and the corresponding creep compliance reaches $D'' = D_0 + \sum_{n=1}^{N_p} D_n$. The effect on the creep compliance at later times will be governed by more terms of the PSM. Thus, each retardation time and creep coefficient can be assumed to correspond to a different phenomenon taking place at the micro-level. At a certain time/temperature, the dominant phenomenon will be also dominant at different locations. Therefore each retardation time or creep coefficient at one point can have the same or dependent effect at another location because it reflects the retardation of the same phenomenon, while, the associated creep coefficient represents the importance of this phenomenon. This concept is significant as stochastic models of these parameters will be developed in the following section.

3 STOCHASTIC MODELING: A SPECTRAL APPROACH

In order to account for the uncertain and variable nature of asphalt mixtures composition and hence behavior, stochastic models of the linear viscoelastic properties will be introduced in this section. Generally speaking, the stochastic models could assume that the retardation times and creep coefficients will be modeled as either spatially-invariant random variables (RVs) or spatially-dependent stochastic processes (SPs). According to the discussion in the previous section, each parameter in the PSM will be modeled as an independent random quantity. In the case of spatially-invariant variables, the parameters can be modeled as a second-order random variable, $R$, associated with a probability distribution function $P_R$. Since the parameters of the PSM can be strictly assigned positive values, candidate distributions could be a lognormal distribution $logN(\lambda_R, \sigma_R^2)$, or alternatively a uniform distribution $U(a,b)$, or a Beta distribution $Beta(r, \alpha, \beta)$, in case bounded distributions are sought [6]. Alternatively, if none of the known probability distributions fit the data well, an arbitrary random distribution can be constructed using spectral expansion methods such as Polynomial Chaos (PC) expansion [4].

The PSM was shown in the previous section to cover the material behavior at different time zones through the retardation times in the exponential terms. Each term, corresponding to one retardation time and weighted by a creep coefficient, governs one time interval and thus can be associated with a different phenomenon taking place in the microstructure. At this certain time interval, the dominant phenomenon will be also dominant at different spatial locations. This suggests the presence of certain form of spatial dependency associated with each phenomenon. To model the macroscopic effect of these phenomena, each retardation time or creep coefficient can be modeled as stochastic process (SP). Let $\mathbf{sp}$ denotes a second-order SP, $\mathbf{sp}(x, \theta)$ is defined such that [4],

$$\mathbf{sp} : (x, \theta) \in \Lambda \times \Theta \rightarrow \mathbf{sp}(x, \theta) \in \mathbb{R}, \quad (6)$$

where $\Lambda$ is a spatial bounded domain, $d$ is the dimension of the domain ($d = 1$ for unidimensional domain, i.e. $x = x$, and $d = 2$ for two-dimensional domain, i.e. $x = (x, y)$), and $\theta$ is an event $\in \Theta$, the space of all random events. As a continuous function of space, an accurate representation of the SP requires an infinite number of correlated RVs which is computationally intractable. It is customarily to utilize discretization methods to approximate the SP in terms of denumerable RVs. The efficiency of the discretization methods are usually measured by the number of RVs required to achieve a certain error level [12][8]. Spectral discretization method, specifically, the Karhunen-Loève expansion method [4] is the most efficient and has been used widely in literature. This is due to the efficient reduction of the stochastic dimension (number of the RVs representing the SP) for a given accuracy. In addition, the random and spatial dependencies can be separated in terms of a set of uncorrelated RVs and a deterministic correlation function of the spatial domain. The stochastic process can be written in terms of a deterministic and a random components $\mathbf{sp}(x, \theta) = \mathbf{sp}(x) + \mathbf{sp}(x, \theta)$. The first component represents the mean (mathematical expectation) of the SP while the second component represents a zero-mean SP characterized with a correlation kernel $C(x, x')$, $x, x' \in \mathbb{R}$. $C(x, x')$ is bounded symmetric, and positive definite that accepts a spectral decomposition in the form $C(x, x') = \sum_{i=1}^{\infty} \lambda_i f_i(x) f_i(x')$ [4]. Here, $\lambda_i$ and $f_i(x)$ are the $i^{th}$ eigenvalue and eigenfunction of
\( C(x, x') \) and can be obtained by solving:

\[
\int_{\Lambda} C(x, x') f_i(x) \, dx = \lambda_i f_i(x), \tag{7}
\]

in which the normalized eigenfunctions form an orthonormal set, i.e., \( f_i(x) f_j(x) = \delta_{ij}, \forall i \) and \( j \). Accordingly, \( sp(x, \theta) \) can be discretized as,

\[
sp(x, \theta) = \tilde{sp}(x) + \sum_{i=1}^{N_{KL}} \sqrt{\lambda_i} \xi_i(\theta) f_i(x), \tag{8}
\]

where \( \xi_i(\theta), i = (1, \ldots, \infty) \) is a set of uncorrelated centered second-order RVs such that, \( E[\xi_i] = 0 \) and \( E[\xi_i \xi_j] = \delta_{ij} \).

This KL expansion can be approximated by considering a finite number of terms \( N_{KL} \), which follows from an error minimizing property [4]. The truncated expression of the SP has the following form,

\[
\tilde{sp}(x, \theta) = \tilde{sp}(x) + \sum_{i=1}^{N_{KL}} \sqrt{\lambda_i} \xi_i(\theta) f_i(x), \tag{9}
\]

The truncation order of the KL expansion depends on the decaying rate of the eigenvalues which form a decreasing sequence of positive numbers. That is, for \( N_{KL} \) determined such that \( \sum_{i=1}^{N_{KL}} \lambda_i / \sum_{i=1}^{\infty} \lambda_i \sim 1 \), the mean-square-error due to the truncation, i.e., \( E[|sp - \tilde{sp}|^2] = \sum_{i=N_{KL}}^{\infty} \lambda_i \), is minimized. The variance function associated with the truncated series can be estimated as,

\[
Var[sp(x, \theta)] = \sigma_x^2 \sum_{i=1}^{N_{KL}} \lambda_i f_i(x)^2 = \sigma_x^2 \sum_{i=1}^{N_{KL}} \lambda_i \leq \sigma_x^2, \tag{10}
\]

In the absence of sufficient experimental data, the correlation function \( C(x, x') \), is approximated by an exponential correlation function; typically assumed for the case of a rectangular domain, which has the advantage to express equation 7 analytically [4]. For a rectangular domain \( \Lambda \) with \( x \in [-a, +a] \) and \( y \in [-b, +b] \) and corresponding correlation length \( l_x \) and \( l_y \), the exponential correlation function can be defined for unidimensional (1D) domain or two-dimensional (2D) domain as,

\[
\rho(x, x') = \begin{cases} 
\sigma_x \exp \left( -\frac{|x-x'|}{l_x} \right) & \text{for 1D domain,} \\
\sigma_x \exp \left( -\frac{|x-x'|}{l_x} - \frac{|y-y'|}{l_y} \right) & \text{for 2D domain,}
\end{cases} \tag{11}
\]

where, \( \sigma_x = \sigma_x, \sigma_y \).

For 2D domain and assuming that the eigenfunctions are separable, i.e., \( f(x, y) = f(x)f(y) \) and given that \( C(x, x') = C(x-x', y-y') = \sigma_x \exp \left( -\frac{|x-x'|}{l_x} \right) \exp \left( -\frac{|y-y'|}{l_y} \right), \) the solution of the eigenvalue problem, equation 7, can be given as,

\[
f_i(x) = f_p(x), f_q(y), \quad \lambda_i = \lambda_p \lambda_q, \tag{12}
\]

where, \( p = (1, \ldots, N_{KL}^p), q = (1, \ldots, N_{KL}^q), f_p(x), f_q(y), \lambda_p, \) and \( \lambda_q \), the eigenfunctions and eigenvalues in \( x \) and \( y \) directions, are estimated depending on the indices \( p \) or \( q \) be it odd or even [4]. The truncation order of the KL expansion depends on the decaying rate of the eigenvalues which form a decreasing sequence of positive numbers. Thus, \( \lambda_i \) estimated from equation 12 need to be sorted in descending order. Then, \( N_{KL} \) can be determined such that \( \sum_{i=1}^{N_{KL}} \lambda_i / \sum_{i=1}^{\infty} \lambda_i \sim 1 \). The corresponding eigenfunctions of the retained terms in KL expression should be considered in the model, accordingly.

4 STOCHASTIC MODELING OF LINEAR VISCOELASTIC PROPERTIES OF ASPHALT MIXTURES

Asphalt mixtures are thermoreologically simple material, i.e., the temperature-dependent curves of the measured dynamic modulus can be superimposed [Pellinen 2003]. This is usually performed by shifting the data horizontally by the so-called time-temperature shifts factors determined with respect to a predefined reference temperature; so that they are laid on a master curve corresponding to this reference temperature. Measurements of the dynamic creep compliance \( D' \) and corresponding phase angle \( \theta \) in dynamic modulus tests [13], are used to construct the master curves of the storage compliance \( D' = D' \sin \theta \) and the loss compliance \( D'' = D' \cos \theta \). In turn, these are used to identify the parameters of the associated PSMs, eqs. 4 and 5, according to the procedure introduced in [13]. The data adopted for this study corresponds to specimens S1AV4Ag0 and S2AV4Ag0; in which, (S1, S2) refer to the specimen, (AV4) refers to the air void percentage, and (Ag0) refers to the aging time.

According to eq. 9, a PSM associated with the mean of all possible realizations is of importance. The available measurements from specimens S1 and S2 are used to update the parameters of a PSM representing the mean model. The order of the PSM defined in equations 3-5 is determined so that the decaying terms cover the (reduced) frequency range of the available experimental data. Thus, The PSM is defined for a prony order equal to 10, that is, the retardation times are assumed such that \( \tau_n = (10^{-0.3}, 10^{-2}, 10^{-0.1}, 10^{0}, 10^{1}, 10^{2}, 10^{3}, 10^{4}, 10^{5}) \) given that \( t = 2\pi/\omega_0 \).

\[
D(t) = D_0 + \sum_{n=1}^{N_p=10} D_n \left( 1 - \exp(-t/\tau_n) \right), \tag{13}
\]

In this study, the retardation times are fixed while the creep coefficients are updated. This implies that the ratio between the viscous parameter of the dashpot and the modulus of the spring in the corresponding Voigt unit is constant according to the definition of the retardation time, \( \tau_n = D_n \cdot \eta_v = const \). The identified PSMs of the creep compliance associated with measurement data of S1 and S2, as well as of the mean of measurement data at each experimental frequency \( \mu_{S1,S2} \), are plotted in both the frequency and time domains in figure 1. These models will be used in the construction of the stochastic models hereafter. Figure 1 shows that the variation range is significant at low frequency and reduces with the increase in the frequency. That is, the variation becomes more significant as time passes. Creep coefficients can be modeled either as RVs or as SPs. In the absence of available data needed to examine the dependency among these RVs/SPs, they are assumed statistically independent. Given that the PSM parameters are strictly positive quantities, the underlying
stochastic processes could be modeled such that the KL RVs are modeled as lognormal variables, or rather, as uniform or Beta-distributed variables is bounded supports are desired. In this work, uniform distributions are selected to model the KL RVs with appropriate support [a, b] selected so that \( E \{ \xi^2 \} = 1 \). The standard deviation of each process is proposed with regard to the experimental PSM parameters obtained for the specimens S1, S2. The corresponding truncated Karhunen-Love decomposition for each SP can be expressed as,

\[
D_0^0(x, \theta) = \mu_{D_0}(x) + \sum_{i=1}^{m_0} \sqrt{\lambda_i^0} f_{D_0,i}(x) \xi_{D_0,i}(\theta), \quad (14)
\]

\[
D_i^j(x, \theta) = \mu_{D_i}(x) + \sum_{j=1}^{n_j} \sqrt{\lambda_j^i} f_{D_i,j}(x) \xi_{D_i,j}(\theta), \quad (15)
\]

where, \( m_0 \) and \( n_j \) are the truncation orders of the SPs. \( \mu_{D_0}(x) \) and \( \mu_{D_i}(x) \) are the mean values of the SPs. These are assumed constant and equal to the mean values of the creep coefficients \( \bar{\mu}_{S1,S2} \) identified from S1 and S2, given that spatial measurements are not available. \( \lambda_i^0, \lambda_j^i, f_{D_0,i}(x), \) and \( f_{D_i,j}(x) \) denote the eigenvalues and eigenfunctions of the correlation functions associated with the SPs. \( \xi_{D_0,i}(\theta) \) and \( \xi_{D_i,j}(\theta) \) are uncorrelated random variables of the KL expansions. The exponential shape correlation function, equation 11 is considered. Assuming a 2D pavement structure, the eigenvalue problem defined in equation 7 accepts an analytical solution [4]. A key parameter in the discretization of SPs is the correlation length. Thus, future experimental investigations are required to identify appropriate physical values of the correlation lengths associated with asphalt mixtures in pavement structures.

5 APPLICATION TO A NUMERICAL CREEP TEST

A numerical creep test is used to demonstrate the effect of variability in the spatially-invariant model of the PSM on the uniaxial strain using equation 1. A constant stress with a magnitude of \( \sigma_0 = 500 kPa \) is applied. Since, the strain can be expressed analytically, Monte Carlo method has been used to quantify the strain using a set of \( N_t = 10^8 \) realizations of the PSM (where each creep coefficient is modeled using a uniform random variable). A selected set of the corresponding time-dependent strain are plotted in left-side figure 3 (fine lines) together with the sample mean of the strain \( \mu_{S1,S2} \) (thick black line). The strain responses associated with specimens S1 and S2 are also plotted (thick lines) together with the upper and lower confidence bounds of 95% (thick dashed black curves) and of 90% (thick dashed red curves). Moreover, the 95% (black lines) and the 90% (red lines) confidence intervals are also plotted in the left-side figure at selected time intervals. The probability density functions (pdfs) of the ensemble of strain realizations, estimated at the same selected time intervals, are plotted in the right-side figure. The spread of these pdfs reflects the corresponding variance of the strain response, e.g., a sharper pdfs and smaller strains are associated with earlier times, the strains and their associated variance increase as time passes, which corresponds to the proportional increase in the creep compliance.

6 STOCHASTIC PROPAGATION OF LINEAR VISCOELASTIC PROPERTIES USING NON-INTRUSIVE SPECTRAL PROJECTION APPROACH

Non-intrusive Spectral Projection (NISP) approach [5] is used to propagate the stochastic linear viscoelastic model through a pavement structure. Based on this approach, a stochastic surrogate model could be constructed to approximate the response of a numerical complex model; pavement structure. In other words, using this approach, it is possible to construct a spectral stochastic model of the response as a function of the stochastic properties associated with the structure. This is done by projecting the stochastic model output on a finite-dimensional stochastic space through an orthogonal projection.
To do so, a mapping of the output $\varepsilon(x,t)$ to a probability space, $\Theta$, can be expressed in terms of a truncated polynomial chaos (PC) expansion as,

$$\varepsilon(\xi) = \sum_{j=0}^{N_{pc}} \alpha_j \Psi_j(\xi), \quad (16)$$

where, $\Psi_j(\xi)$, PC basis, is a set of multi-dimensional orthogonal polynomials in $\xi^{N_{rv}}$, $\xi^{N_{rv}}$ is the collection of all the independent random variables, introduced in the previous section, that define the random quantities in the PSM. Here, $\alpha_j$ are the projection coefficients and $N_{pc} + 1$ is the dimension of the PC terms, which can be defined in terms of the PC order $q$ and the stochastic dimension $N_{rv}$. The orthogonality condition can be expressed in terms of the inner product, defined on the stochastic space $\Theta$, as,

$$\langle \Psi_j(\xi), \Psi_k(\xi) \rangle = \delta_{jk}, \quad (17)$$

where,

$$\langle u, v \rangle = \int_{\Theta} u(\xi) v(\xi) \rho(\xi) \, d\xi, \quad (18)$$

Accordingly, the projection coefficients $\alpha_j$ can be expressed, given the orthogonality of the basis, in terms of the following inner products,

$$\alpha_j = \frac{\langle \varepsilon(\xi), \Psi_j(\xi) \rangle}{\langle \Psi_j(\xi), \Psi_j(\xi) \rangle}, \quad j = 0, 1, ..., N_{pc}. \quad (19)$$

In this work, a sparse grid cubatures approach has been used to estimate $\langle \varepsilon(\xi), \Psi_j(\xi) \rangle$. According to this approach, $N_{sgc}$ sets of $\xi^{N_{rv}}$ realizations and associated weights are generated; $\xi^{(j)}$ and $w_i$, $i = 1, \cdots, N_{sgc}$ (http://dakota.sandia.gov/). The dimension of each set $N_{sgc}$ is a function of the stochastic dimension and the level of the sparse grid cubatures $l$. Thus,

$$\alpha_j = \sum_{i=1}^{N_{sgc}} w_i \varepsilon(\xi^{(j)}) \Psi_j(\xi^{(j)}) , \quad j = 1, \cdots, N_{pc}. \quad (20)$$

Once the PC coefficients are identified, the spectral projection defined in eq. 16 is fully defined. The variance of the projected response can be estimated, given the orthogonality of the basis, as,

$$\text{Var}[\varepsilon(\xi)] = \sum_{j=1}^{N_{pc}} \alpha_j^2. \quad (21)$$

7 APPLICATION TO A PAVEMENT STRUCTURE

A three layer asphalt pavement structure consisting of a 100 mm asphalt layer, 300 mm base layer, and 400 mm of subgrade is constructed. Figure 4 shows the 2D axisymmetric FE mesh along with the applied load and geometry of the simulated pavement structure. A uniform contact stress of 0.88 MPa is applied over a circular contact area with the effective radius of 160 mm to represent a 71 kN wheel load applied to the structure from a wide base tire type 425 [41]. The used element type in this simulation is an axisymmetric four node element with reduced integration (CAX4R). According to the convergence studies, $12.5 \times 12.5 mm^2$ elements are used under and close to the load. The element size is then increased using transition elements for the areas far from the load. As shown in Figure 4, infinite elements are used in these simulations to eliminate the boundary condition effect. Both base and subgrade layers are assumed to be linear elastic with the stiffness moduli of 100 MPa and 50 MPa, respectively. The asphalt concrete layer is modeled using the stochastic spatially-dependent linear viscoelastic model introduced in section 4. Linear viscoelastic constitutive relationship is implemented into the well-known FE software (Abaqus2013) via the user material subroutine UMAT. Random realization of spatially-dependent viscoelastic material are generated and assigned to the elements in the asphalt layer according to their spatial location. That is, each integration point is assigned a different realization of the PSM according to equations 13, 14, and 15. The common practice in assigning different material properties to each integration point is to partition the FE model to numerous sections such that each element is defined as a different section with its specified material properties. However, this approach is cumbersome and does not have the flexibility to easily change the geometry and mesh density of the FE model. In this study, the asphalt layer is modeled as a single section. The UMAT is modified to access the connectivity matrix of the FE model at the beginning of the analysis. The modified UMAT reads the connectivity matrix and stores material properties associated with each integration point at the beginning of the analysis (i.e., increment zero). For the next increments, UMAT recalls the material properties associated with each integration point.

In order to construct the spectral PC representation of the strain response defined in equation 16, the PC (projection) coefficients need to be identified according the methodology introduced in the previous section, specifically, equation 20. To this end, $N_{sgc}$ response realizations associated with FE models are estimated. Each FE model is assigned a PSM estimated at each of the $N_{sgc}$ realizations of the random variables $\xi^{N_{rv}}$ (also called the PC germs) generated according to the sparse grid cubatures approach [5]. A python script is developed to automatically run the FE analysis and extract the desired output for the $N_{sgc}$ realization of the material properties. Once the PC coefficients are estimated, the stochastic representation of the strain response at any spatial location and any time point is fully identified. The element located at the bottom of asphalt layer at the axis of symmetry of the applied load, element...
is chosen to demonstrate the stochastic quantification of the response, for instance, the axial strain $\varepsilon_{11}$. According to the stochastic propagation methodology introduced in the previous section, two convergence studies are carried out. In other words, the truncation order of the PC representation $q$ and the level of the sparse grid cubatures $sgc_{level}$. To illustrate the convergence of the PC representation, three values of the PC order $q = 1, 2, \text{and } 3$ are used to construct PC representations of $\varepsilon_{11}$ for each of $sgc_{level} = 1, 2, \text{and } 3$. The corresponding probability density functions (pdfs) at selected time points are plotted in the first three sub-plots in figure 5.

![Figure 5](image)

**Figure 5.** Convergence of the PC representation for the strain at the bottom of the asphalt layer below the load

It is clear from these plots that $sgc_{level} = 1$ is not appropriate while a second-order PC representation is an appropriate truncation order. For this order, the pdfs associated with $sgc_{level} = 2, \text{and } 3$ are plotted in the fourth sub-plot, which shows that the nodes generated according to $sgc_{level} = 2$ are appropriate. Thus, it can be deduced that the spectral representation of the strain can be adequately constructed for $q = 2$ and $sgc_{level} = 2$. As a result, the strain response is analytically constructed in terms of the random variables used to define the stochastic material properties of the asphalt layer in the pavement structure. This model can be used to statistically quantify the strain response at any time point. This is depicted in figure 6, where the strain curves are plotted with respect to time in the left sub-figure. The thick black line represents the mean of the strain response, the dashed black line represents one standard deviation range, the red and blue dashed lines represent the confidence intervals associated with 90% and 95% significance levels. The red and blue error bars represent the corresponding confidence intervals at selected time points. The pdfs of the strain response at these time points are plotted in the right sub-figure.

![Figure 6](image)

**Figure 6.** Statistical quantification for the PC representation of the strain response

It is worth noting that the stochastic representation can be used to generate random samples of the strain response by simply generating random samples for the PC germs $x_0^{N'}$. Some generated samples are plotted in figure 6. This is advantageous in terms that the FE model will be no longer required for this structure. This formulation can be used to statistically analyze and quantify the behavior of pavement structures; e.g., fatigue, damage.

8 CONCLUDING REMARKS

The behavior of asphalt mixtures materials is influenced by their highly uncertain and variable nature upscaled from the finest scales up towards the macroscopic scale and structure scale. Hence, they are well-suited to be modeled as random media. For low-strain applications, asphalt mixtures exhibit linear viscoelastic behavior. This study is devoted to introducing stochastic macroscopic constitutive models appropriate for linear viscoelastic materials. These models can be interfaced easily with any uncertainty propagation analysis, which in turn, enables a complete statistical quantification of the response. This is demonstrated through a simple numerical creep test and a more complex application where random quantities are propagated through a two-dimensional pavement structure model in order to quantify time-dependent strain response. The efficiency of the spatially-dependent stochastic model introduced in this paper is advantageous in uncertainty quantification analyses, both forward propagation problems and inverse identification problems. The spectral representation of the response can be readily implemented in statistical quantification and reliability analyses of pavement structure behavior such as failure due to fatigue, moisture damage, etc.; regardless of its complexity, the FE model will no longer be required for such studies.

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