Stochastic seismic load model for push-over analyses

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ABSTRACT: Nonlinear time history analyses are usually required to build fragility curves for seismic reliability analyses. However, this becomes time consuming for large FE model and a great number of runs required to obtain statistical stable results. Alternatively, the nonlinear response of structures subjected to earthquakes can be sufficiently analysed by pushover analysis. For this purpose a stochastic seismic load model in the frequency domain is required, which is presented in this paper. The first part of the model describes the lateral load pattern while the second part comprises the target displacement. The model enables seismic reliability analysis by pushover analysis, which is more efficient than commonly used time history analysis. Furthermore, a direct relationship between the response spectrum at a site and the expected uncertainties in the seismic loading is given. The application of the model is shown in a case study of a moment resisting steel frame.

KEYWORDS: reliability, seismic, pushover analysis, stochastic load model.

1 INTRODUCTION

1.1 Seismic risk analysis

In seismic risk analysis the performance of the structure is usually analysed separately from the seismic hazard at a site.

The structural performance is represented by the fragility curve \( \mathcal{F} \), which describes the conditional failure probability of the structure as a function of the seismic intensity. It is built by non-linear time history analyses, where uncertainties of the seismic input at a specific intensity level, uncertainties of the structural properties and uncertainties of the capacities are considered. The scattering of the seismic input is accounted for by using different accelerograms representative for the earthquake characteristic at a site, which are e.g. defined by a response spectrum. The uncertainties of the structural properties are included by using a new set of samples for the random variables at each time-step analysis with a new accelerogram. The samples are generated by Monte-Carlo simulation or more efficient stratified simulation techniques (e.g. Latin Hypercube sampling).

The occurrence of earthquake with a specific intensity at a site is determined in probabilistic seismic hazard analyses (PSHA). The result is described by the hazard curve \( \mathcal{H} \) in discrete or closed form. The total failure probability \( P_f \) of a structure at the site is obtained by combining the fragility curve with the hazard curve and integration over the seismic intensities.

\[
P_f = \int_{0}^{\infty} f(im) \cdot \mathcal{H}(im) \, id \, im \tag{1}
\]

where \( f \) is the pdf of the fragility curve, \( \mathcal{H} \) the cdf of the hazard curve and \( im \) the seismic intensity measure.

The disadvantages of this procedure are the time consuming non-linear time history analyses. The combination of a large number of samples to obtain stabilized results and costly analysis methods for large FE models becomes inefficient.

1.2 Pushover analysis

Beside time history analysis nonlinear static (pushover) analysis procedures are an appropriate method to investigate the inelastic performance of structures subjected to seismic loading. Lateral loads are increased monotonically up to a target displacement of roof drift, while gravity loads are kept constant. The lateral load pattern over the height of the building is chosen proportional to mass distribution and consistent with the fundamental mode shape. However, additionally a constant lateral load pattern consistent with uniform response acceleration should be applied to consider higher mode affects [1]. The target displacement is determined on the basis of the elastic displacement response spectrum of an equivalent SDOF system. At the end, the plastic mechanism as well as local deformation demands at the target displacement can be assessed in relation to the deformation capacities of members and connections. The simplicity of this inelastic seismic analysis procedure paved the way in many current seismic codes and guidelines, e.g. [1], [2], [3].

Improvements of the basic procedure are published in recent years [4]. It comprises improvements for the determination of the target displacement as well as the consideration of higher mode effects, strength degradation and soil-structure interaction.

However, pushover analyses are applicable only for deterministic seismic analysis until now, as a stochastic seismic load model is not available for this procedure. The most comprehensive and most realistic description of seismic loadings is by recorded or artificial generated ground motion histories expected at a site. For nonlinear static analysis however, the seismic loading has to be described in the frequency domain, e. g. by response spectra. In this paper a
stochastic seismic load model in the frequency domain is presented, which is applicable in nonlinear static analysis. It aims to obtain a similar scattering of the seismic loading in the frequency domain like in the time domain. This enables the application of the more efficient nonlinear static analysis method within seismic risk analysis. Furthermore, a direct relation between the deterministic characterisation of the seismic loading (e.g. response spectra) and the expected scattering of the seismic loading is given. The stochastic model comprises formulas to describe the lateral load pattern as well as to describe the target displacement; both are derived and presented in the following sections.

2 STOCHASTIC MODEL OF LATERAL LOAD PATTERN

2.1 Fundamentals and assumptions

Seismic loading is characterized by the probability of occurrence of an earthquake with certain intensity and the variation of the ground motion history (frequency content, duration, etc.). Therefore, the lateral load pattern can be divided in a dominating part consistent with the fundamental mode shape and deviations in respect to fundamental mode shape due to higher mode effects. These deviations are the basis of the model to describe the uncertainties in the lateral load pattern.

The model was derived from a number of elastic and inelastic time history analyses of moment resisting steel frames, which were designed according to EN 1998-1. It could be shown that the plastic hinge pattern in an elastic structure (where plastic hinge is defined as a moment capacity ratio greater than one) is highly correlated with the plastic mechanism in the corresponding inelastic structure. Therefore, it is assumed that the lateral load pattern for elastic and inelastic structural behaviour is very similar (what is already the basic assumption for every elastic static seismic analysis procedure, too). Furthermore, there a high correlation between the time step of the maximum roof drift and the time step, in which the plastic mechanism occur, could be observed.

Hence, the model for the lateral load pattern was derived from the lateral displacement shape of elastic structures at maximum roof drift in time history analysis. The advantage is that for elastic structures a direct relationship between displacement shape and the lateral load pattern exist via the stiffness matrix. Additionally, principles of the linear random process theory are applicable for linear structural behaviour.

2.2 Evaluation of displacement and forces

In Fig. 1 the lateral displacement shapes at maximum roof drift are shown for a 5 storey moment resisting steel frame subjected to 20 different accelerograms. The accelerograms meet sufficiently the same target spectrum \( \dot{a} = 0.25 \text{ g} \), spectrum type 1, soil class C acc. to EN 1998-1) and are scaled in such a way that the spectral value at the fundamental period is equal to the target spectrum (conditioned spectrum). Obviously, the deformation is dominated by the fundamental mode and the mean value of all samples is equivalent to the fundamental mode shape. The coefficient of variation (COV) of lateral displacements is between 0.02 and 0.05 depending on the storey. In Fig. 2 displacements caused by higher mode shapes are separated by subtracting the fundamental mode shape scaled to mean value of roof drift.
shape of the total deformation. For this it is assumed that the lateral displacement of higher modes is zero at the node of 2nd eigen-mode. This assumption is only exact for the 2nd mode shape, but not for higher mode shapes. However, the error is low as the displacement amplitude of higher modes is very small. In Fig. 2 the 2nd and 3rd mode shape can be clearly identified. But it should be emphasized that the displacement amplitudes of higher modes are small (~0.01 m) in relation to the displacement caused by the fundamental mode (~0.40 m).

The corresponding lateral loads are determined with the condensed stiffness matrix for the total displacement (Fig. 3) and displacements caused by higher modes (Fig. 4). The COVs of the lateral forces are significant larger than for the displacements; displacements in higher mode shapes require higher forces (more energy) than the displacements in the fundamental mode shape. Hence, the influence of higher modes is more obvious for forces than for displacements.

2.3 Maximum displacement of fundamental mode

The contribution of each mode shape to the total displacement was evaluated by modal superposition of equivalent SDOF. For the fundamental mode could be observed that its maximum displacement occurs close to the time step of the maximum roof drift of the whole structure (Fig. 5). In general this is the case, if the total displacement is dominated by the fundamental mode shape and displacement amplitudes of higher modes are relatively small. However, this is true for moment resisting steel frames designed acc. to EN 1998-1. The maximum displacement of the fundamental mode is given by the ordinate of the displacement response spectra at this period $S_d(T_1)$. Finally, the mean value of the lateral displacement of the structure is determined by multiplication of $S_d(T_1)$ with the participation factor $\Gamma_1$ and the eigenvector $\phi_1$ of the fundamental mode:

$$\mu(u) = u_{1,\text{max}} = q_{1,\text{max}} \cdot \phi_1 \quad (2)$$

$$q_{1,\text{max}} = \left| \Gamma_1 \right| \cdot S_d(T_1, \xi_1) = \left| \Gamma_1 \right| \cdot \frac{T_1^2}{4 \pi^2} \cdot S_a(T_1, \xi_1) \quad (3)$$

where $\phi_1$ and $\Gamma_1$ are the Eigenvector and participation factor of the fundamental mode; $S_d$ and $S_a$ are the spectral values of the displacement respectively the acceleration response spectra.

As conditioned spectra are used in this study (see section 2.2) the scattering of the displacement response spectra at the fundamental period is zero. However, this is true for the stochastic seismic load model in general. Response spectra of (recorded or artificial generated) ground motions, which do not exactly meet the target spectrum, are an artificial introduced scattering due to non-perfect time histories and do not represent the real scattering of the ground motions.

2.4 Displacement of higher modes

In the previous section an analytical formula for the mean value of the lateral displacement was derived. Now the scattering due to higher mode effects is investigated. The critical time step is defined at the maximum roof drift, which is approximately equal to the time step of the maximum displacement of the fundamental mode. However, if the critical time step is fixed by the fundamental mode, the displacement of higher modes at this time is random (Fig. 6). In general, the displacement history of a SDOF subjected to a ground motion can be described by a stationary Gauss process with zero mean [5]. Hence, the displacement at a specific time step is normally distributed with mean value equal to zero. (This is absolutely correct for stochastically independent modes; however, for moment resisting steel frames this can usually be assumed). The standard deviation of the Gauss process is connected with the maximum displacement by the peak factor $r$. Hence, the standard deviation of the displacement of higher modes $\sigma(\eta_n)$ is equal to the ordinate of the displacement response spectrum at the corresponding period divided by the peak factor $r$:

$$S_d(T_n) = \mu(\eta_n) + r \cdot \sigma(\eta_n) = r \cdot \sigma(\eta_n) \quad (4)$$

$$\sigma(\eta_n) = \frac{S_d(T_n)}{r_n} \quad (5)$$

The peak factor is a function of the spectral values of the displacement time history, the duration of the stationary part of the earthquake and the probability of exceedance. For earthquakes $r$ is approximately 3 [5]. Finally, the standard deviation of the lateral displacement can be determined for modes each mode $n > 1$ with Eqn. (4) and its participation factor $\Gamma_n$ as well as its eigenvector $\phi_n$. The total standard
deviation due to all higher modes is calculated with the SRSS-rule:
\[
\sigma(u_n) = q_{n,\sigma} \cdot \phi_n
\]
(6)
\[
q_{n,\sigma} = \left| \Gamma_n \right| \cdot \frac{S_d(T_n \cdot \xi_n)}{r_n} = \left| \Gamma_n \right| \cdot \frac{T_n^2}{4 \cdot \pi^2} \cdot \frac{S_d(T_n \cdot \xi_n)}{r_n}
\]
(7)
\[
\sigma(u) = \sqrt{\sum_{n=2}^{n\max} \sigma(u_n)^2}
\]
(8)
where \( \phi_n \) and \( \Gamma_n \) are the Eigenvector and participation factor of the nth mode; \( S_d \) and \( S_a \) are the spectral values of the displacement respectively the acceleration response spectra.

2.5 Stochastic parameter of lateral loads

Based on the analytical formulas for mean value and standard deviation of the lateral displacement, the stochastic parameter of lateral load pattern can be determined with the condensed stiffness matrix. Alternatively, these can be obtained directly with the acceleration response spectrum:
\[
\mu(F) = s_1 \cdot S_a(T_1 \cdot \xi_1)
\]
(9)
\[
\sigma(F) = s_n \cdot \frac{S_a(T_n \cdot \xi_n)}{r_n}
\]
(10)
\[
\sigma(F) = \sqrt{\sum_{n=2}^{n\max} \sigma^2(F_n)}
\]
(11)
\[
s_n = \left| \Gamma_n \right| \cdot M \cdot \phi_n
\]
(12)
where \( s_i \) is the excitation vector and \( M \) the mass matrix.

The analytical formulas of the stochastic load model (Equ. (1) and (5)) were compared with stochastic parameter obtained from statistical evaluation of time history analysis in several case studies. The model corresponds very well with the empirical data.

3 STOCHASTIC MODEL OF TARGET DISPLACEMENT

3.1 Mean value of maximum roof drift

As shown in the previous sections, the maximum roof drift of an elastic structure is given by the displacement response at the fundamental period plus scattering due to higher modes. The maximum response of structures with linear behaviour is proportional to the seismic intensity (e.g. PGA or \( S_a(T_1) \)). It can be determined directly by the elastic displacement response spectra:
\[
u_{D,el} = S_d(T_1) \cdot \Gamma_1
\]
(13)
The maximum roof drift of inelastic structure, which is equal to the target displacement in a pushover analysis, is additionally affected by uncertainties in the inelastic response (e.g. accumulative plastic deformations). The nonlinear response of structures with moderate to high eigen-periods \( T_1 > T_C \) (typical for moment resisting steel frames), is shown schematically in Fig. 7. After the first yielding occurs, the roof drift \( u_0 \) increases under proportionally, as seismic energy is dissipated by plastic deformations. For significant larger seismic intensities the roof drift grows disproportionally high due to P-\( \Delta \)-effects; finally the structure collapse.

The nonlinear behaviour of moment resisting steel frames in incremental dynamic analysis is very similar, if the results are scaled to the ductility \( \mu \) (x-axis) respectively the displacement ductility \( \mu_d \) (y-axis) (Fig. 8): Until \( \mu = 1.0 \) the relation between \( \mu \) and \( \mu_d \) is linear; after first yielding up to \( \mu = 8.0 \) the relation is under proportional. This can be approximately described by following linear equations:
\[
\mu_d = \mu \quad \text{for} \quad \mu \leq 1.0
\]
\[
\mu_d = 0.7 \cdot \mu + 0.3 \quad \text{for} \quad 1.0 < \mu \leq 8.0
\]
(14)
Hence, the mean value of maximum roof drift for structures with nonlinear material behaviour respective the target displacement can be determined with Equ. (12) and Equ. (13):
\[
u_{D,max} = u_{D,max,el} \cdot \frac{\mu}{\mu_d} = S_d(T_1) \cdot \Gamma_1 \quad \text{for} \quad \mu \leq 1.0
\]
(15)
\[
u_{D,max} = u_{D,max,el} \cdot \frac{0.7 \cdot \mu + 0.3}{\mu} = S_d(T_1) \cdot \Gamma_1 \quad \text{for} \quad 1.0 < \mu \leq 8.0
\]
Figure 7. Maximum roof drift of MRF steel frame in an IDA (schematic).

Figure 8. Displacement ductility vs. ductility of steel frames based on IDAs (mean values of 20 accelerograms).
3.2 Scattering of maximum roof drift

The scattering of the maximum roof drift is partially influenced by the scattering of elastic higher mode effects (see section 2). However, the main part comes from uncertainties in the inelastic response (Fig. 9). The scattering due to inelastic material behaviour is determined by statistical evaluation of IDAs. The COV of the maximum roof drift (elastic part is subtracted) vs. ductility for three moment resisting steel frames is shown in Fig. 10: Obviously, for \( \mu < 1.0 \) the COV is zero; for \( \mu > 6.0 \) the COV keeps rather constant at a level of 0.3. The relation between COV and ductility can be roughly described by following bi-linear function:

\[
\begin{align*}
\nu(u_{D,\text{max, pl}}) &= 0 \quad \text{for} \quad \mu \leq 1.0 \\
\nu(u_{D,\text{max, pl}}) &= 0.06 \cdot \mu - 0.06 \quad \text{for} \quad 1.0 < \mu \leq 6.0 \\
\nu(u_{D,\text{max, pl}}) &= 0.30 \quad \text{for} \quad 6.0 < \mu \leq 8.0
\end{align*}
\]

The standard deviation of the maximum roof drift results from the aforementioned relation multiplied by the mean value of the maximum roof drift (Equ. (14)):

\[
\sigma(u_{D,\text{max, pl}}) = \nu(u_{D,\text{max, pl}}) \cdot u_{D,\text{max}}
\]

![Figure 9. Coefficient of variation of maximum roof drift based on IDA's (with elastic part, 20 accelerograms).](image)

3.3 Influence of material scattering

In the stochastic model for the target displacement (Equ. (14) to (16) scattering of the nonlinear response only due to ground motions but not due to uncertainties of the material strength is considered. However, the real material strength affects the plastic resistance of the structure and therefore the start of yielding. As shown in Fig. 11, structural overstrength leads to a slight increase of roof drift up to a ductility of \( \mu = 8.0 \). This affects also the standard deviation, see Equ. (16). The influence of uncertainties in the structural strength can be considered in Equ. (15) and (16), if \( \mu \) is replaced by \( \mu / \Omega_{ov} \):

\[
\begin{align*}
\sigma(u_{D,\text{max}}) &= S_d (T_1) \cdot \Gamma_1 \cdot \frac{0.7 \cdot \mu + 0.3 \cdot \Omega_{ov}}{\mu} \quad \text{for} \quad \mu \leq 8.0 \cdot \Omega_{ov} \\
\sigma(u_{D,\text{max}}) &= 0 \quad \text{for} \quad \mu \leq 1.0 \cdot \Omega_{ov} \\
\sigma(u_{D,\text{max}}) &= u_{D,\text{max}} \cdot \left( 0.06 \cdot \mu - 0.06 \right) \quad \text{for} \quad 6.0 \cdot \Omega_{ov} > \mu \end{align*}
\]

\( \Omega_{ov} \) is a random variable for the structural over strength; a similar distribution than for material strength can be chosen. The scattering of the maximum roof drift due to uncertainties of the structural strength are small in comparison to the ground motion and will be neglected (see Fig. 9 and 12).

![Figure 11. Displacement ductility vs. ductility of steel frames in IDAs: nominal and mean value of 100 material samples.](image)

![Figure 12. Coefficient of variation of maximum roof drift vs. ductility (100 material samples).](image)
4 APPLICATION EXAMPLE

The seismic load model presented in section 2 and 3 is applied in pushover analysis to build fragility curves for a 5 storey moment resisting steel frame [6]. The structure is designed acc. to EN 1998-1 for a seismic loading of $a_g = 0.25 \, \text{g}$, spectrum type 1 and soil class C. The maximum rotation capacity and the joint capacity are analysed as limit state functions.

Besides the stochastic seismic load model the yield over strength ($\gamma_{ov} = \text{LN}(1.18;0.07)$), the model uncertainty of rotation capacity ($\theta_{av,cyc} = \text{LN}(1.00;0.30)$) as well as of joint capacity ($m_R = \text{LN}(1.38;0.14)$) are considered as random variables. The failure probability is determined at several seismic intensities by adaptive Monte-Carlo simulations with the open source reliability analysis tool FERUM 4.1 [7]. System failure is assumed, if failure of all beams in one storey or failure of one column occurs.

The limit state function of the rotation capacity check at one plastic hinge is given by following formula:

$$ g_{\theta}(x) = \theta_{av,cyc,i} - \theta_{pl,i} = 0 \quad (20) $$

where $\theta_{av,cyc,i}$ is the available cyclic plastic rotation capacity and $\theta_{pl,i}$ the maximum plastic rotation in one plastic hinge.

The limit state function to evaluate the joint capacity is written as follow:

$$ g_{m}(x) = 1.1 \cdot \gamma_{ov,design} \cdot m_R - m_{pl} = 0 \quad (21) $$

where $m_R$ is the joint capacity and $m_{pl}$ the connection force normalized to $M_{pl,nom}$ of a structural member; $\gamma_{ov,design} = 1.25$ is the design overstrength factor acc. to EN 1998-1.

In Fig. 13 and Fig. 14 the fragility curves for rotation capacity check and joint failure are shown. In the first case the fragility curve follows the typical behaviour of deformation oriented limit states and the discrete failure probabilities can be described by a lognormal distribution function. This is not possible in the latter case. At a certain limit the failure probability does not increase with the seismic intensity. This corresponds with the seismic intensity, where the plastic capacity of the structural members next to joints is reached. Hence, further increase of seismic intensity does not increase the failure probability.

![Figure 13. Fragility function of 5 storey moment resisting steel frame for rotation capacity check.](image)

![Figure 14. Fragility function of 5 storey moment resisting steel frame for joint failure.](image)

5 CONCLUSIONS

In seismic reliability analyses incremental dynamic analyses are usually performed to build fragility curves. However, this becomes time consuming for large FE model and large number of runs required to obtain statistical stable results. Alternatively, the nonlinear response of structures subjected to earthquakes can be sufficiently analysed by pushover analysis. However, for this purpose a stochastic seismic load model in the frequency domain is required.

Such a model is presented in this paper. The first part of the model describes the lateral load pattern. The mean value corresponds to the fundamental mode shape, while the scattering of the loads results from higher modes. The scattering was derived on the basis of the linear random process theory. The required input data are obtained from the response spectra at the site of the structure. The second part comprises the stochastic parameter of the target displacement. Again, the basic data are achieved from the response spectra, while uncertainties due to nonlinear structural response are considered by empirical formulas.

The model enables seismic reliability analysis with pushover analysis, which is more efficient than commonly used time history analysis. The application of the model is demonstrated in two seismic reliability analyses on a MRF steel frame. Furthermore, a direct relationship between the response spectrum at a site and the expected uncertainties in the seismic loading is given.

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