The Use of Component Mode Synthesis Techniques for Large Finite Element Model Updating Using Dynamic Response Data

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ABSTRACT: This paper introduces a methodology that integrates a model reduction technique into a finite element model updating formulation. A Bayesian model updating approach based on a stochastic simulation method is considered in the present work. Stochastic simulation techniques require a large number of finite element model re-analyses to be performed over the space of model parameters. Substructure coupling techniques for dynamic analysis are proposed to reduce the computational cost involved in the dynamic re-analyses. The effectiveness of the proposed strategy is demonstrated with an identification application for a finite element building model using simulated seismic response data.

KEY WORDS: Bayesian updating; Component mode synthesis; Finite element; Model updating; Transitional Markov Chain Monte Carlo method.

1 INTRODUCTION

Model updating using measured system response has a wide range of applications in areas such as structural response prediction, structural control, structural health monitoring, and reliability and risk assessment [1], [2], [3], [4], [5]. For a proper assessment of the updated model all uncertainties involved in the problem should be considered. In this context a fully probabilistic Bayesian model updating approach provides a robust and rigorous framework for model updating due to its ability to characterize modeling uncertainties associated with the underlying structural system [6]. An efficient method called transitional Markov chain Monte Carlo is implemented in this work for Bayesian model updating [7]. This stochastic simulation method requires the solution of a large number of finite element dynamic re-analyses over the space of model parameters. Thus, the computational demands depend on the number of finite element re-analyses and the time required for performing each dynamic finite element analysis. The present work proposes to use an efficient model reduction technique to alleviate the computational burden involved in the implementation of a Bayesian technique for finite element model updating. Specifically, a class of model reduction techniques known as substructure coupling for dynamic analysis is considered here [8].

The organization of the paper is as follows. The mathematical background of the substructure coupling technique for dynamic analysis is outlined in Section 2. Basic aspects of Bayesian finite element model updating using dynamic data are presented in Section 3. The integration of the model reduction technique with the Bayesian model updating approach is discussed in Section 4. The effectiveness of the proposed scheme, in terms of computational efficiency and accuracy, is demonstrated with an identification application for a finite element building model using simulated seismic response data.

2 MODEL REDUCTION TECHNIQUE

A model reduction technique called substructure coupling or component mode synthesis is considered in this work [8]. Sub-structuring involves dividing the structure into a number of substructures obtaining reduced-order models of the substructures and then assembling a reduced-order model of the entire structure.

2.1 Basic Equations

In the present formulation it is assumed that the structural system satisfies the following equation of motion

\[ \mathbf{M}\ddot{u}(t) + \mathbf{C}\dot{u}(t) + \mathbf{K}u(t) = \mathbf{f}(t) \] (1)

where \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the \( N \times N \) mass, damping and stiffness matrices of the finite element model, respectively, \( \mathbf{u}(t) \) is the vector of dynamic displacements, and \( \mathbf{f}(t) \) is the excitation vector. The first step of the model reduction technique is the definition of a number of substructure modes. In order to define the set of substructure modes, the following partitioned form of the mass matrix \( \mathbf{M} \in \mathbb{R}^{n_s \times n_s} \) and stiffness matrix \( \mathbf{K} \in \mathbb{R}^{n_s \times n_s} \) of the substructure \( s, s = 1, \ldots, N \), is considered

\[ \mathbf{M}' = \begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{ib} \\ \mathbf{M}_{bi} & \mathbf{M}_{bb} \end{bmatrix}, \quad \mathbf{K}' = \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \] (2)

where the indices \( i \) and \( b \) are sets containing the internal and boundary degrees of freedom of the superstructure \( s \), respectively. The boundary degrees of freedom include only those that are common with the boundary degrees of freedom of adjacent substructures, while the internal degrees of freedom are not shared with any adjacent substructure. In this framework all boundary coordinates are kept as one set \( \mathbf{u}'(t) \in \mathbb{R}^b \) and the internal coordinates in the set \( \mathbf{u}_i(t) \in \mathbb{R}^{n'_i} \). The corresponding equation of motion of the undamped substructure \( s \) can be written as
where \( u^i(t) \) is the displacement vector (physical coordinates) of dimension \( n^i = n^i_1 + n^i_2 \) and the vector \( f^i(t) \) contains the externally applied forces as well as the reaction forces on the substructure due to its connection to adjacent substructures at boundary degrees of freedom.

### 2.2 Fixed-Interface and Constraint Normal Modes

The fixed-interface normal modes are obtained by restraining all boundary degrees of freedom and solving the eigenproblem

\[
M^iu^i(t) + K^iu^i(t) = f^i(t)
\]

(3)

where \( u^i(t)^T = [u_1^i(t), u_2^i(t)] \geq \mathbb{R}^{n^i} \) is the displacement vector and \( f^i(t) \) contains the externally applied forces as well as the reaction forces on the substructure due to its connection to adjacent substructures at boundary degrees of freedom.

\[\Phi^i_{ii}M^i_{ii}\Phi^i_{ii} = \lambda^i_{ii}\]

where the matrix \( \Phi^i_{ii} \) contains the complete set of \( n^i \) fixed-interface normal modes, and \( \Lambda^i_{ii} \) is the corresponding matrix containing the eigenvalues. The fixed-interface normal modes are normalized with respect to the mass matrix \( M^i_{ii} \)

\[
\Phi^i_{ii}T M^i_{ii}\Phi^i_{ii} = I^i_{ii}, \quad \Phi^i_{ii}T K^i_{ii}\Phi^i_{ii} = \Lambda^i_{ii}
\]

(5)

where \( I^i_{ii} \) is the identity matrix. On the other hand the constraint modes are obtained by setting a unit displacement on a specified degree of freedom. The set of interface constraint-modes \( \Psi^i \) employs a combination of fixed-interface normal modes and the boundary coordinates \( \hat{\Psi}^i = \Lambda^i_{ii}^{-1}\Phi^i_{ii} \)

\[
\Psi^i = \left[ \begin{array}{c}
\Psi^i_{bb} \\
I^b_{bb}
\end{array} \right] = \left[ \begin{array}{c}
\Lambda^i_{ii}^{-1}\Phi^i_{ii} \\
I^b_{bb}
\end{array} \right]
\]

(6)

where \( \Psi^i_{bb} \in R^{n^i_2 \times n^i_2} \) is the interior part of the constraint-mode matrix and \( I^b_{bb} \in R^{n^i_2 \times n^i_2} \) is the identity matrix.

### 2.3 Craig-Bampton Method

The Craig-Bampton method is used in the present formulation to define a set of generalize coordinates \[9\]. This method employs a combination of fixed-interface normal modes and interface constraint modes to define the following displacement transformation

\[
\{u^i(t)\} = \left[ \begin{array}{c}
\Phi^i_{bb} \\
\Psi^i_{bb} \\
I^b_{bb}
\end{array} \right] \{v^i(t)\} = \Psi^i v^i(t)
\]

(7)

where \( \Phi^i_{bb} \) is the interior part of the matrix \( \Phi^i_{ii} \) of the \( n^i_1 \) fixed-interface normal modes \( (n^i_2 \leq n^i_1) \). \( v^i(t) \) represents the substructure generalized coordinates composed by the modal coordinates \( v^i_j(t) \) of the kept fixed-interface normal modes and the boundary coordinates \( v^i_j(t) = u^i_j(t) \). \( v^i(t) \in R^{s^i \times 1} \) is the Craig-Bampton transformation matrix with \( s^i = n^i_2 + n^i_2 \) and all other terms have been previously defined. The substructure mass matrix \( \hat{M}^i \) is kept fixed-interface normal modes and the stiffness matrix \( \hat{K}^i \) is constraint-modes in generalized coordinates \( v^i(t) \) are given by

\[
\hat{M}^i = \Psi^iT\hat{M}^i\Psi^i, \quad \hat{K}^i = \Psi^iT\hat{K}^i\Psi^i
\]

(8)

Next, the vector of generalized coordinates for all the \( N_s \) substructures

\[
v(t)^T = [v_1^1(t)^T, ..., v_{N_s}^1(t)^T] > R^{n_s}
\]

(9)

where \( n_s = \sum_{i=1}^{N_s} n^i_1 \) is introduced. Based on this vector, a new vector \( q(t) \) that contains the independent generalized coordinates consisting of the fixed-interface modal coordinates \( v^i_j(t) \) for each substructure and the physical coordinates \( v^i_k(t), l = 1, ..., N_b \) at the \( N_b \) interfaces is defined as

\[
q(t)^T = [v^1_1(t)^T, ..., v^1_{N_s}(t)^T, v^2_1(t)^T, ..., v^2_{N_s}(t)^T, ..., v^N_1(t)^T, ..., v^N_{N_s}(t)^T] > R^{n_q}
\]

(10)

where \( n_q = \sum_{i=1}^{N_s} n^i_1 + \sum_{i=1}^{N_s} n^i_2 \), and \( n^i_q \) is the number of degrees of freedom at the interface \( l = 1, ..., N_b \). These two vectors are related by the transformation

\[
v(t) = Tq(t)
\]

(11)

where the matrix \( T \in R^{n_q \times n_q} \) is a matrix of zeros and ones that couples the independent generalized coordinates \( q(t) \) of the reduced system with the generalized coordinates of each substructure. The assembled mass matrix \( \hat{M} = \sum_{i=1}^{N_s} \hat{M}^i \) and the stiffness matrix \( \hat{K} = \sum_{i=1}^{N_s} \hat{K}^i \) for the independent reduced set \( q(t) \) of generalized coordinates take the form

\[
\hat{M} = T^T \left[ \begin{array}{ccc}
\hat{M}^1 & 0 & 0 \\
0 & ... & 0 \\
0 & 0 & \hat{M}^N_s
\end{array} \right] T,
\]

(12)

\[
\hat{K} = T^T \left[ \begin{array}{ccc}
\hat{K}^1 & 0 & 0 \\
0 & ... & 0 \\
0 & 0 & \hat{K}^N_s
\end{array} \right] T
\]

(13)

where the matrices \( \hat{M}^i, \hat{K}^i, s = 1, ..., N_s \) are defined in Eq. (8).

### 2.4 System Response

The dynamic response of the finite element model of the original system is obtained by modal solution in the present formulation. In this approach it is assumed that the dynamic response can be represented by a linear combination of the mode shapes as

\[
u(t) = Y\eta(t)
\]

(14)

where \( \nu(t) \) is the displacement vector of the original structure, \( Y \) is the matrix of mode shapes associated with the eigen-problem of the undamped equation of motion of the original system, and \( \eta(t) \) is the vector of modal response functions. The natural frequencies of the original unreduced model \( \omega_r, r = 1, ..., m \), where \( m \) is the number of modes considered are obtained by solving the eigen-problem of the reduced-order system model

\[
(\hat{K} - \omega^2\hat{M})\nu_{rr} = 0, \quad r = 1, ..., m
\]

(15)

where \( \nu_{rr}, r = 1, ..., m \) are the mode shapes of the reduced-order system. Introducing a constant matrix \( T \in R^{n_q \times n_q} (n_q = \sum_{i=1}^{N_s} n^i_q) \) to map the vector
of the physical coordinates for all substructures to the independent physical coordinates $u(t)$ of the original structure, the physical mode shapes $u_r$ of the structure can be recovered from the mode shapes $u_{qr}$ as

$$u_r(T) = \sum_{q=1}^{N_r} u_{qr}^T v_q$$

where the matrix $\Psi \in R^{n_u \times n_r}$ is a block diagonal matrix defined in terms of the Craig-Bampton transformation matrices of all substructures, that is, $\Psi = \text{blockdiag}(\Psi_1, \ldots, \Psi_{N_r})$.

### 3 BAYESIAN FINITE ELEMENT MODEL UPDATING

#### 3.1 Problem Formulation

Consider a parameterized finite element model class $M$ of a structural system by a set of model parameters $\Theta \in \Theta \subset R^p$. The plausibility of each model within a class $M$ based on data $D$ is quantified by the updated joint probability density function $p(\Theta | M, D)$ (posterior probability density function). By Bayes’ Theorem [6], [10] the posterior probability density function of $\Theta$ is given by

$$p(\Theta | M, D) = \frac{p(D | \Theta, M) p(\Theta | M)}{p(D | M)}$$

where $p(D | M)$ is the normalizing constant which makes the probability volume under the posterior probability density function equal to unity, $p(D | \Theta, M)$ is the likelihood function based on the predictive probability density function for the response given by model class $M$, and $p(\Theta | M)$ is the prior probability density function selected for the model class $M$. In what follows it is assumed that $D$ contains input dynamic data and output responses from measurements on the system. Specifically let $u_n(t_j, \Theta)$ denotes the output at time $t_j$ at the $n^{th}$ observed degree of freedom predicted by the proposed structural model, and $u^{*}_n(t_j)$ denotes the corresponding measured output. The prediction and measurement errors

$$e_n(t_j, \Theta) = u^{*}_n(t_j) - u_n(t_j, \Theta), n = 1, \ldots, N_o, j = 1, \ldots, N_t$$

where $N_o$ denotes the number of observed degrees of freedom and $N_t$ denotes the length of the discrete time history data, are modeled as independent and identically distributed Gaussian variables with zero mean [3], [11]. Using the above probability model for the prediction error it can be shown that the likelihood function $p(D | \Theta, M)$ can be expressed in terms of a measure-of-fit function $J(\Theta | M, D)$ between the measured response and the model response at the measured degrees of freedom. Such function is given by [11], [12]

$$J(\Theta | M, D) = \frac{1}{N_o N_t} \sum_{n=1}^{N_o} \sum_{j=1}^{N_t} (u^{*}_n(t_j) - u_n(t_j, \Theta))^2$$

For a large number of available data ($N_t N_o$ is large) it has been found that the most probable model parameters are obtained by minimizing $J(\Theta | M, D)$ over all parameters in $\Theta$ that it depends on [12]. Under the previous assumption the posterior probability density function $p(\Theta | M, D)$ is in general concentrated in the neighborhood of a lower dimensional manifold in the parameter space [13]. In general the problem of estimating the model parameters is potentially ill-posed, that is, there may be more than one solution. To solve this problem a simulation-based Bayesian model updating technique is adopted in this study. Such technique, which efficiently generates samples asymptotically distributed as the posterior probability density function, is described in the following section.

#### 3.2 Simulation-Based Approach

An efficient method called transitional Markov chain Monte Carlo is implemented in this work for Bayesian model updating [7]. Validation calculations have shown the effectiveness of this approach in a series of practical Bayesian model updating problems [14]. The method can be applied to a wide range of cases including high-dimensional posterior probability density functions, multimodal distributions, peaked probability density functions, and probability density functions with flat regions. The method iteratively proceeds from the prior to the posterior distribution. It starts with the generation of samples from the prior distribution in order to populate the space in which also the most probable region of the posterior distribution lies. For this purpose a number of non-normalized intermediate distributions $p_j(\Theta | M, D), j = 1, \ldots, J$, are defined as

$$p_j(\Theta | M, D) \propto p(D | \Theta, M)^{a_j} p(\Theta | M)$$

where the parameter $a_j$ increases monotonically with $j$ such that $a_0 = 0$ and $a_J = 1$. The parameter $a_j$ can be interpreted as the percentage of the total information provided by the dynamic data which is incorporated in the $j^{th}$ iteration of the updating procedure. The first step ($j = 0$) corresponds to the prior distribution and in the last stage ($j = J$) the samples are generated from the posterior distribution. The idea is to choose the values of exponents $a_j$ in such a way that the change of the shape between two adjacent intermediate distributions be small. This small change of the shape makes it possible to efficiently obtain samples from $p_{j+1}(\Theta | M, D)$ based on the samples from $p_j(\Theta | M, D)$. The value of the parameter $a_{j+1}$ is chosen such that the coefficient of variation for $p(D | \Theta, M)^{a_{j+1}-a_j}, k = 1, \ldots, N_j$ is equal to some prescribed target value. The upper index $k = 1, \ldots, N_j$ in the previous expression denotes the sample number in the $j^{th}$ iteration step ($\Theta^k_j, k = 1, \ldots, N_j$). Once the parameter $a_{j+1}$ has been determined the samples are obtained by generating Markov chains where the lead samples are selected from the distribution $p_j(\Theta | M, D)$. Each sample of the current stage is generated by applying the Metropolis-Hastings algorithm [15]. The lead sample of the Markov chain is a sample from the previous step that is selected according to the probability equal to its normalized weight

$$w(\Theta_j^k) = \frac{w(\Theta_j^{k-1})}{\sum_{j=1}^{N_j} w(\Theta_j^j)}$$

where $w(\Theta_j^j)$ represents the plausibility weight which is given by

$$w(\Theta_j^j) = p(D | \Theta_j^j)^{a_{j+1}-a_j}$$
For a detailed implementation of the transitional Markov chain Monte Carlo method, including proofs concerning the statistical properties of the estimators the reader is referred to [7], [16].

4 MODEL UPDATING PROCESS

In the present formulation it is assumed that the stiffness matrix of the original system depends linearly on the model parameters \( \theta \). This is the case encountered in many practical applications such as model updating, structural optimization and damage detection techniques. This linear dependence implies that at the substructure level the stiffness matrix as well as its partitions admit a similar characterization. In particular the following two cases are considered here. In the first case it is assumed that the stiffness matrix of a substructure \( s \) does not depend on the model parameters. In this case the stiffness matrix is written as \( K^s = K^0_s \). The corresponding normal and constraint modes are computed once for the corresponding model. In the second case it is assumed that the stiffness matrix of a substructure \( s \) depends only on one model parameter, say \( \theta_j \) (\( j \)-th component of \( \theta \)). Specifically, let \( S_j \) be the set of substructures that depends on \( \theta_j \). Then the stiffness matrix of a substructure \( s \in S_j \) can be written as \( K^s = \hat{K}^s \theta_j \), where the matrix \( \hat{K}^s \) is independent of \( \theta_j \). Of course the partitions of the stiffness matrix \( K^s \) admit the same parametrization. From this representation it is easy to show that the eigenvalues and eigenvectors associated with the fixed-interface normal modes are given by

\[
\Lambda^s_i = \hat{\Lambda}^s_i \theta_j, \quad \Phi^s_i = \hat{\Phi}^s_i
\]

(24)

where the matrices \( \hat{\Lambda}^s_i \) and \( \hat{\Phi}^s_i \) are the solution of the eigenproblem

\[
\hat{K}^s \hat{\Phi}^s_i = M^s \hat{\Phi}^s_i \hat{\Lambda}^s_i
\]

(25)

where the matrices \( \hat{\Phi}^s_i \) and \( \hat{\Lambda}^s_i \) are independent of \( \theta_j \). In addition, the interface constraint modes are also independent of \( \theta_j \) since

\[
\Psi^s_{ib} = -\hat{K}^s_{ib}^{-1} K^s_{ib} = -\hat{K}^s_{ib}^{-1} \hat{K}^s_{ib}
\]

(26)

Therefore a single substructure analysis is required to provide the exact estimate of the normal and constraint modes for any value of the modal parameter \( \theta_j \) [17]. Based on the previous results it is simple to verify that the reduced stiffness matrix of the substructure \( s \), considering the full set of interface degrees of freedom, takes the form

\[
\hat{K}^s = \hat{K}^s \theta_j, \quad \hat{K}^s = \Psi^T \hat{K}^s \Psi
\]

(27)

Thus, it is clear that the reduced matrix \( \hat{K}^s \) is a constant matrix independent of the model parameters \( \theta \). Consequently the stiffness matrix of the Craig-Bampton reduced system can be written as

\[
K = K_0 + \sum_{j=1}^{N_\theta} \hat{K}_j \theta_j
\]

(28)

where \( N_\theta \) is the number of independent model parameters and the matrices \( K_0 \) and \( \hat{K}_j, j = 1, \ldots, N_\theta \) are defined as

\[
\hat{K}_j = T^T \begin{bmatrix}
\hat{K}_{0j} \delta_{0j} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \hat{K}_j \delta_{nj}
\end{bmatrix} T,
\]

(29)

\[
\hat{K}_j = T^T \begin{bmatrix}
\hat{K}_{1j} \delta_{1j} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \hat{K}_j \delta_{nj}
\end{bmatrix} T
\]

(30)

where \( \delta_{0j} = 1 \) if the substructure \( s \) does not depend on the model parameters \( \theta \) and \( \delta_{0j} = 0 \) otherwise, \( \delta_{nj} = 1 \) if the substructure \( s \) depends on the parameter \( \theta_j \) and \( \delta_{nj} = 0 \) otherwise, and all other terms have been previously defined. Note that the assembled matrices \( \hat{K}_j, j = 1, \ldots, N_\theta \) are independent of the value of \( \theta \) and therefore these matrices are computed and assembled once. This results in substantially savings since there is no need to define these matrices during the identification process.

5 NUMERICAL EXAMPLE

5.1 Problem Description

The structural system shown in Figure (1) consists of a ten floors three-dimensional reinforced concrete building. Material properties of the reinforced concrete structure have been assumed as follows: Young’s modulus \( E = 2.34 \times 10^{10} \) N/m²; Poisson ratio \( \nu = 0.3 \); and mass density \( \rho = 2500 \) kg/m³. The height of each floor is 3.5 m leading to a total height of 35.0 m for the structure. The floors are modeled with shell elements with a thickness of 0.3 m and beam elements of 35.0 m for the structure. The floors one to five and 0.25 m of dimension 0.8 m.

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Each floor is supported by 48 columns of rectangular cross section of dimension 0.3m × 0.6m from floors 1 to 5 and 0.25m × 0.5m from floors 6 to 10. Each floor is supported by 48 columns of rectangular cross section of dimension 0.8m × 0.9m. The corresponding finite element model has approximately 40,000 degrees of freedom. A 2% of critical damping for the modal damping ratios is introduced in the model.

Figure 1. Isometric view of the finite element model.
For the identification application considered in this example the structure is divided into a number of substructures and it is assumed that a stiffness reduction is concentrated in one or more substructures producing changes in the stiffness characteristics of those substructures. In particular the structural model is subdivided into six substructures. Substructures 1, 3 and 5 are composed by the column elements of the first, second and third floor, respectively. Substructures 2 and 4 correspond to the slabs and beam elements of the first and second floor, respectively, and substructure 6 contains the upper floors structural components (columns, beams and slabs). For illustration purposes a model class \( M \) is introduced to monitor the updated process, which is defined in terms of a stiffness reduction of 20\% of the modal stiffness value in the \( x \) direction of the columns of the first floor. The model class \( M \) contains two parameters \( (\theta_1, \theta_2) \) associated with the stiffness of the columns of the first and second floor, respectively. The model class is characterized from the unchanged or nominal structure \( (\theta_1 = 1, i = 1,2) \) corresponding to the reduced-order model.

The identification process is based on simulated data. To this end the original unreduced finite element model is excited horizontally (in the \( x \) direction) with the Santa Lucia ground-motion record recorded during the 2010 Chilean earthquake. The input ground acceleration time history is shown in Figure (2). It corresponds to a ground motion of moderate intensity. The measured response is simulated by first calculating the absolute acceleration response of the modified original unreduced structure at floors 1, 2, 3 and 4 (in the \( x \) direction) and then adding Gaussian discrete white noise with standard deviation equal to 10\% of the root-mean square value of the corresponding absolute acceleration time histories. The responses are computed at the center of mass of the floors. One hundred seconds of data with sampling interval \( \Delta t = 0.05 \) s are used, given a total of \( N_t = 2000 \) data points.

The simulated response data provides the data for the model updating process. The actual implementation of this process is carried out by using a reduced-order system model which is defined as follows. For each substructure of the finite element model it is selected to retain all fixed-interface normal modes that have frequency less than a given cut-off frequency. The cut-off frequency is set proportional to the 12\% modal frequency of the original unreduced finite element model. Validation calculations show that retaining a total of 365 internal degrees of freedom for all substructures is adequate in the context of this application. In fact, with this number of generalized coordinates the fractional error (in percentage) between the modal frequencies using the complete finite element model and the modal frequencies computed using the reduced-order model falls bellow 0.1\% for the lowest 12 modes. Then, a total of 365 interior degrees of freedom, corresponding to the fixed interface generalized coordinates, out of 39,136 of the original model are retained for the all substructures. The number of interface degrees of freedom is equal to 864 in this case. The total number of degrees of freedom of the reduced model represents a 97\% reduction with respect to the unreduced model. Thus, a significant reduction in the number of generalized coordinates is obtained with respect to the number of the degrees of freedom of the original unreduced finite element model.

5.2 Results

The model updating is performed using the transitional Markov chain Monte Carlo method with 1000 samples per stage. The prior probability density function for the model parameters \( \theta_i, i = 1,2 \) are independent uniform distributions defined over the interval \([0.5, 1.5]\). The reference structure (unchanged) is characterized in terms of the model parameters with values equal to \( \theta_i = 1, i = 1,2 \). Figures (3) and (4) show the histograms defined by the posterior samples of the model parameters. In addition, the values of the nominal system parameters are also indicated in the figures.

![Figure 3. Posterior histogram of model parameter \( \theta_1 \). Mean estimate: \( \bar{\theta}_1 = 0.79 \)](image)

It is seen that the agreement between the actual system and the model characterized by the posterior samples is excellent. The model parameter \( \theta_1 \) is distributed around the value \( \bar{\theta}_1 = 0.8 \). In fact the mean estimate of this parameter is equal to \( \bar{\theta}_1 = 0.79 \). On the other hand it is observed that the parameter related to the stiffness of the columns of the second floor is distributed around its actual value \( \bar{\theta}_2 = 1.0 \). This is reasonable since these columns are unchanged. To get insight into the identification process Figures (5-7) show how the samples in the \( \theta_1 - \theta_2 \) space converge. From the different steps or stages of the transitional Markov chain Monte Carlo method it is observed that the data is strongly correlated along a certain direction in the parameter space. Such correlation shows that an increase in the stiffness of the columns of the first floor is compensated by a decrease in the stiffness of the columns of the second floor during the...
identification process, which is consequent from a structural point of view. Thus, all points along that direction correspond to structural models that have almost the same response at the measured degrees of freedom.

The number of finite element runs required for the identification process depends on the number of transitional Markov chain Monte Carlo stages which in this case is equal to 8. The total computational time for the identification using the original unreduced finite element model is expected to be of the order of 3 days. In contrast, for the reduced-order model the computational demand is reduced to less than 4 hours. Thus, a drastic reduction in computational efforts is achieved without compromising the predictive capability of the proposed identification methodology.

6 CONCLUSIONS

A methodology that integrates a model reduction technique into a finite element model updating formulation using dynamic response data has been presented. In particular, a method based on fixed-interface normal component modes plus interface constraint modes is considered in this work. In general, the method produces highly accurate models with relatively few component modes. The finite element model updating is carried out by using a simulation-based Bayesian model updating technique. Specifically, the transitional Markov chain Monte Carlo method is implemented in the present formulation. It is demonstrated that the fixed-interface normal mode of each component and the characteristic interface modes are computed only once from a reference finite element model. In this manner the re-assembling of the reduced-order system matrices from components and interfaces modes is avoided during the updating process. Results show that the computational effort for updating the reduced-order model is decreased drastically by two or three orders of magnitude with respect to the unreduced model, that is, the full finite element model. Furthermore, the drastic reduction in computational efforts is achieved without compromising the predictive capability of the proposed identification methodology.

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