An investigation into the effects of resonances on the time delay estimate for leak detection in buried plastic water distribution pipes

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ABSTRACT: In water distribution systems, old metallic pipes have been replaced by plastic pipes due to their deterioration over time. Although acoustic methods are effective in finding leaks in metallic pipes, they have been found to be problematic when applied to plastic pipes due to the high damping within the pipe wall and the surrounding medium. This is responsible for the leak signal not traveling long distances. Moreover, the leak energy in plastic pipes is generally located at a narrow frequency range located at low frequencies. However, the presence of resonances can narrow even more this frequency range. In order to minimise the influence of background noise and resonances on the calculation of the time delay estimate, band-pass filters are often used to suppress undesirable frequency components of the noise. The objective of this paper is to investigate the influence of resonances in the pipe system (pipe, valves, connections and hydrants), on the time delay estimate calculated using acoustic signals. Analytical models and actual leak data collected in a bespoke rig located in the United Kingdom are used to investigate this feature.

KEY WORDS: leak detection, plastic pipes, resonances

1 INTRODUCTION
Buried water pipelines are susceptible to leakage. To repair these pipe-systems, holes have to be dug to access the pipe section. In the UK, up to 4 million holes are dug every year to repair buried infrastructure, causing significant financial loss [1]. Acoustic methods are often used to detect and locate water leaks, and correlators have been used for this purpose for more than 30 years [2]. These devices calculate cross-correlation functions, which are then used to detect and locate a leak. Although these methods work well for metallic pipes [3-4], there are some issues in plastic pipes, which need to be taken into account. The two main issues are related to the propagating leak noise, which is heavily attenuated by the damping in the pipe and, the speed at which such noise propagates along the pipe, which is heavily influenced by the pipe properties [5-6].

In leak detection, the most widely used correlators utilise the so-called basic cross correlation (BCC) function. The BCC may be performed by taking the inverse Fourier transform of the cross spectral density (CSD) function of two leak signals, measured either side of the leak. However, there are other options in some types of commercial correlators [7-8]. One of them uses the phase transform (PHAT) as discussed by Gao et al [9]. The PHAT is used to sharpen the peak in the cross-correlation function and to suppress other additional peaks not related to the time delay information. In this process the modulus of the CSD between the signals is “flattened” or “whitened” prior to the transformation to the time domain. In this way, only the phase information is used to determine the time delay estimate.

As mentioned above, two issues are related to problems with leak detection in plastic pipes. However, extra features can be present in the leak data which may further jeopardize the use of correlators. The aim of this paper is to investigate an additional feature in leak signals that may introduce errors in the estimation of the time delay using the BCC and PHAT correlators. This feature is the presence of phase changes in the CSD which are not related to the time delay, but are due to resonances in the pipe system which are frequently observed in practice.

2 LEAK DETECTION OVERVIEW
The noise generated by a leak can be used to detect it and to determine its position along a pipe. Vibrations and/or acoustic sensors can be placed at two different positions (access points), typically hydrants or valves either side of a suspected leak. A typical measurement set-up used for leak detection in a buried water pipe is depicted in Figure 1. The distance between the sensors is $d = d_1 + d_2$, where $d_1$ and $d_2$, are the respective distances between the leak and the access points. The measured signals are $x_1(t)$ and $x_2(t)$.

The peak in the cross correlation function, which is a measure of similarity between the two leak signals [10], occurs at the time delay $T_0$, and the, distance of the leak from sensor 2, $d_2$, is given by [11],

$$d_2 = \frac{d - cT_0}{2},$$

where $c$ is the speed of propagation of the leak noise.
THE BLITHFIELD PIPE RIG – DESCRIPTION AND INSTRUMENTATION USED

The Blithfield pipe rig was provided by South Staffs Water plc, a company located in the UK. Figure 2 shows the schematic of the pipe rig highlighting the distances between the access points, along with the positions where the leak was generated and, measurements taken. The pipe extremity close to Position 1, where the sensor 1 is placed, is connected to the mains water distribution pipe, which supplies water at a pressure of about 6 bar. At the other extremity the pipe is terminated with a blank.

Figure 2. The pipe rig used for the experimental work.

All the access points are set in concrete to provide rigid supports for the pipe connections, while the pipe sections are buried at a depth of about 0.8 m. Figure 3a shows one of the access points. Leak noise was generated by opening a secondary valve fitted to a blanking piece attached to the hydrant as shown in the sketches in Figs. 3b and c. The main valve shown in these figures allowed the water contained in the buried pipe to enter the hydrant. The leak was 30 metres away from position 1. A pressure gauge was also attached to the hydrant as shown in Figure 3c. Details of the transducers at Positions 1 and 2, and the instrumentation used are given in Tab. 1.

Table 1. Transducers and instrumentation used in the experiments.

<table>
<thead>
<tr>
<th>Device</th>
<th>Manufacturer</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer</td>
<td>Bruel and Kjaer</td>
<td>4384</td>
</tr>
<tr>
<td>Charge Amplifier</td>
<td>Bruel and Kjaer</td>
<td>2635</td>
</tr>
<tr>
<td>Acquisition System</td>
<td>Prosig</td>
<td>5680</td>
</tr>
</tbody>
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Measured vibration data collected from the pipe rig are used to illustrate the presence of a resonance in the pipe. The modulus and phase of the CSD, and the coherence from typical strong leak noise-signals measured between the leak position and position 1 using two accelerometers are shown in Figs 4a(i), 4b(i) and 4c(i), respectively. It is observed that there is quite a broad bandwidth over which the unwrapped phase has linear behaviour. This region, where the phase can be unwrapped, contains information about the difference in the arrival times of the leak noise at the two sensors (time delay). In practice the signals are passed through band-pass filters to attenuate the signals outside the frequency range of interest [11], suppressing undesirable noise effects.

Figs 4a(ii), 4b(ii) and 4c(ii) depict the measured modulus and phase of the CSD, and the coherence between two signals measured at position 1 and position 2. As observed in Fig. 4b(ii), there is a phase shift at about 88 Hz in addition to the phase shift associated with the time delay. This additional phase shift is related to the presence of resonance in the pipe system. Hence, as the phase spectrum is affected by the presence of resonance, the time delay can also be affected by it.
is the phase between

\[ \omega = \frac{1}{2 \pi} \int \omega S_{\Psi_1, \Psi_2}(\omega) e^{i\phi(\omega)} d\omega , \]

where \( F^{-1} \{ \} \) is the inverse Fourier transform, \( \Lambda(\omega) \) is the frequency weighting function for the different correlators, and

\[ S_{\Psi_1, \Psi_2}(\omega) = \int \Omega S_{\Psi_1, \Psi_2}(\omega) e^{i\phi(\omega)} , \]

is the CSD between the signals \( x_i(t) \) and \( x_j(t) \), where \( i = \sqrt{-1} \) and \( \phi \) is the phase between \( x_i(t) \) and \( x_j(t) \). In the case where there is a pure time delay between the arrival of the leak noise at sensors 1 and 2, then \( \phi = \omega T_0 \), where \( T_0 \) is the difference between the arrival times between the two sensors.

The weighting function applied to the CSD can take various forms [9] but in this work, only two are considered. They are

\[
\begin{align*}
\text{BCC: } & \quad \Lambda(\omega) = 1 \\
\text{PHAT: } & \quad \Lambda(\omega) = \frac{1}{S_{\Psi_1, \Psi_2}(\omega)}.
\end{align*}
\]

When the frequency weighting function is given by Eq. (4a), the correlator uses the so-called BCC as described by Knapp and Carter [12]. If the frequency weighting function is given by Eq. (4b) then the correlator uses the phase transform (PHAT). In this case the cross-correlation function is only a function of the phase of the CSD.

4.2 Pipe model for leak detection

An analytical model of the cross-correlation function was derived by Gao et al [11], and it is briefly reviewed here. The frequency response function (FRF) between a leak and the pressure in the pipe at a distance \( x \) from the leak is given by

\[ H(\omega, x) = e^{-\beta x} e^{-i\omega x/c}, \]

where \( \beta \) is the attenuation factor given by [11]

\[ \beta = \frac{1}{c_f} \frac{\eta B a / E_h}{1 + (2B_a/E_h)^{1/2}}. \]

The CSD between signals \( x_i(t) \) and \( x_j(t) \) is then given by

\[ S_{\Psi_1, \Psi_2}(\omega) = S_{\Psi}(\omega) \Psi(\omega) e^{i\omega T_0}, \]

where \( S_{\Psi}(\omega) \) is the power-spectral density of the leak signal \( l(t) \), which is the acoustic pressure at the leak location; and \( \Psi(\omega) = H_l(\omega, d_1) H^*_l(\omega, d_2) = e^{-\omega^2 \gamma^2} \). Combining Eqs. (2) and (7) the BCC function can be determined by setting \( \Lambda(\omega) = 1 \) to give

\[ R_{\Psi_1, \Psi_2}(\tau) = F^{-1} \{ \Lambda(\omega) S_{\Psi_1, \Psi_2}(\omega) \} = R_\Psi(\tau) \otimes \psi(\tau) \otimes \delta(\tau - T_0). \]

where \( \otimes \) denotes convolution, \( R_\Psi(\tau) \) is the auto-correlation of the leak signal, and \( \psi(\tau) = F^{-1}[\Psi(\omega)] = \beta d / \pi (\beta d)^2 + \tau^2 \).

Equation (8) shows that the delta function is broadened by the frequency weighting function \( \Psi(\omega) \) and the leak characteristics. The characteristics of the leak have been studied by Papastefanou et al [13], but for the purpose of the analysis in this work, it is assumed to be white noise over the frequency bandwidth of interest so that \( S_{\Psi}(\omega) = S_0 \) is constant, which is consistent with [11]. The cross-correlation is, therefore, only a function of the distance between the sensors, the pipe properties, and frequency. As mentioned previously, in practice band-pass filters are used to suppress the signals outside the frequency range of interest. For the simple case where an ideal band-pass filter is applied to remove the noise, the frequency response of the filter is given by

\[ G(\omega) = 1 \quad \omega_2 \leq |\omega| < \omega_1 ; \]

\[ = 0 \quad \text{otherwise.} \]
and then Eq. (8) becomes
\[ R_{tx}(\tau) = g(\tau) \otimes S_\text{o}(\tau) \otimes \delta(\tau-T_0), \]
(10)
where
\[ g(\tau) = F^{-1}\{G(\omega)\} = \frac{\Delta \omega \sin(\Delta \omega \tau/2)}{\pi} \cos(\omega \tau), \]
(11)
in which \( \Delta \omega = \omega_1 - \omega_0 \) is the bandwidth of the band-pass filter and \( \omega_0 = (\omega_1 + \omega_2)/2 \) is its centre frequency. Using the model of the pipe system described in [11], Eq. (10) can be written as
\[ R_{tx}^{\text{BCC}}(\tau) = A \left[ \cos(\omega_0(\tau-T_0)+\theta) - e^{-\Delta \omega \beta \tau} \cos(\omega_1(\tau-T_0)+\theta) \right], \]
(12)
where \( A = S_\text{o} e^{-\alpha \beta \tau} \sqrt{\frac{1}{\pi(\beta \delta)^2 + (\tau-T_0)^2}} \) and \( \theta = \tan^{-1}((\tau-T_0)/\beta \delta) \). For the PHAT correlator, the weighting function in Eq.(2) is then \( \Lambda(\omega) = 1/(S_\text{o}(\omega)\Psi(\omega)) \), so the equation for the PHAT is a special case of Eq. (10), and is given by
\[ R_{tx}^{\text{PHAT}}(\tau) = g(\tau) \otimes \delta(\tau-T_0). \]
(13)
It can be seen that the PHAT correlator is simply a time-shifted version of the auto-correlation function of the ideal band-pass filter. Using Eq.(11), Eq.(13) can be written as
\[ R_{tx}^{\text{PHAT}}(\tau) = \frac{\Delta \omega \sin(\Delta \omega(\tau-T_0)/2)}{\pi} \Delta \omega(\tau-T_0)/2 \cos(\omega_0(\tau-T_0)). \]
(14)

Figures 5a and 5b show the normalized BCC \( \hat{R}_{tx}^{\text{BCC}} \) and PHAT \( \hat{R}_{tx}^{\text{PHAT}} \) with respect to its maximum value, calculated using Eq.(12) and Eq.(14), respectively. As the intention is to illustrate the different shapes of the cross-correlation functions, the time delay \( T_0 \) has been neglected for simplicity. Moreover, the lower and upper limits of the band pass filter were set at 10 Hz and 150 Hz, respectively. These limits were chosen based on the fact that below 10 Hz there is generally only background noise, and no leak energy is generally found above 150 Hz [4]. Note that the peak in the PHAT correlator is sharper than the peak in the BCC, so that is easier to detect the time delay estimate in the PHAT correlator.

Figure 5. Cross-correlation functions calculated using Eqs.(12) and (14) normalized by the maximum values and, neglecting the shift due to the time delay \( T_0 \). (a) The Basic Cross-correlation (BCC); (b) The Phase Transform (PHAT).

4.3 Phenomenological model of the resonance behaviour

As seen in Fig. 4, distortions caused by resonances might be present in the CSD of leak signals. An additional phase shift at about 88 Hz is observed in Fig.4b(ii) which indicates the presence of a resonance or some other dynamic effect unrelated to the time delay associated with the propagating leak noise. Moreover, the modulus of the CSD reduces rapidly just above this frequency as seen in Fig.4a(ii), even though the coherence is not greatly affected by this feature as shown in Fig.4c(ii).

In this section, the influence of these resonances on the estimate of the time delay calculated by the BCC and PHAT correlators is investigated using a phenomenological model involving a resonator attached to a hydrant. Figure 6 shows a schematic diagram of the phenomenological model. It is assumed that the resonator has no effect on the pipe vibration and also that the measurements are made on the mass of the resonator. The leak induces vibration of the pipe at the access point, which excites the resonator through its base.
The frequency response between the base \( Y(\Omega) \) and the mass \( X(\Omega) \) is given by [14]

\[
H_{\text{res}}(\Omega, \zeta) = \frac{X(\Omega)}{Y(\Omega)} = \frac{1 + i2\Omega \zeta}{1 - \Omega^2 + i2\Omega \zeta},
\]

where \( \Omega \) is the ratio \( \omega/\omega_n \), in which \( \omega_n \) the natural frequency of the resonator and, \( \zeta \) is the damping in the resonator.

The total FRF between the pressure generated by the leak and the sensor signal with a resonator attached to the pipe is given by

\[
\begin{align*}
H(\omega, x) &= \frac{1 + i2\Omega \zeta}{1 - \Omega^2 + i2\Omega \zeta} e^{-i\omega \beta x} e^{-i\omega \tau}.
\end{align*}
\]

where \( H(\omega, x) \) is the FRF between a leak and the pressure in the pipe at a distance \( x \) from the leak given by Eq. (5), and it is repeated here for convenience. This expression is used to simulate the cross-correlation functions of cases where one resonance is present such as in Fig. 4. Equation (16) gives the FRF between the pressure at the leak position and the displacement of the resonator mass. However, the acceleration response which is often measured is given by this expression multiplied by \( 2\omega \) [15]. Thus, the cross-spectrum for acceleration signals is given by

\[
S_{y,x}(\omega) = \omega^4 S_y(\omega)\Psi(\omega)H^*_{\text{res}}(\omega, \zeta) e^{i\omega \tau},
\]

where * denotes the complex conjugate.

Figures 7a and 7b shows the modulus and phase of the CSD simulated using the phenomenological model given by Eq. (17). This case simulates the actual data given in Fig. 4(ii). (a) modulus. (b) phase. ----- \( \zeta : 0.02; \) --- \( \zeta : 0.05; \) ---- \( \zeta : 0.2. \)

Assuming that \( S_y(\omega) = S_0 \), the BCC function is determined by applying the inverse Fourier transform to Eq.(20), to give

\[
R_{1/2}(\tau) = S_0 h(\tau) \otimes h_{\text{res}}(\tau) \otimes g(\tau) \otimes \delta(\tau - T_o),
\]

where \( h(\tau) \) the cross-correlation function of the pipe characteristics and, the conjugate of the resonator impulse response, respectively.

To investigate the influence of a resonance on the shape of the BCC and PHAT correlators, simulations are carried out for the case shown in Fig. 4(ii). Figure 8a shows the normalized BCC function (\( BCC_{21} \)) with respect to its maximum value, calculated using Eq. (18) where the term \( h_{\text{res}}(\tau) \) is neglected, even though there is no resonator attached to the pipe. Figure 8b depicts the time reverse impulse response of the resonator \( h_{\text{res}}(\tau) \). Figure 8c shows the final normalized BCC function (\( BCC_{21} \)) with respect to its maximum value, calculated using Eq. (18) where the term \( h_{\text{res}}(\tau) \) is neglected, even though there is no resonator attached to the pipe. Figure 8b depicts the time reverse impulse response of the resonator \( h_{\text{res}}(\tau) \). Figure 8c shows the final normalized BCC function (\( BCC_{21} \)) with respect to its maximum value, calculated using Eq. (18). The band pass filter used has lower and upper frequency limits of 10 Hz and 150 Hz, respectively. The damping ratio of the resonator is set now to 0.05 in order to achieve the best fit between the actual and theoretical phase spectrum as will be shown in the next section. The other parameters, such as the natural frequency and damping of the resonator and attenuation factor are set to 88 Hz and \( 2.9 \times 10^{-5} \) s/m, respectively.
Figure 8. The effects of a resonance on the shape of the simulated BCC for the case shown in Fig. 4. (a) The normalized basic cross correlation with respect to its maximum value, without any resonator attached to the pipe. (b) The conjugate of the resonator impulse response. (c) The normalized BCC function with respect to its maximum value.

As observed in Fig. 8, the presence of the resonance affects the shape of the cross-correlation function, shifting the main positive peak, where the time delay is located, away from the actual time delay and also introducing another peak, which is negative. Thus, the effect of a resonance within the bandwidth in which the time delay is calculated, can be significant.

As discussed previously, the PHAT correlator is a special case of the BCC where the modulus of the cross-correlation is flattened, such that only phase information is used to estimate the time delay. In this case Eq. (18) becomes

$$\hat{R}_{y,x}^{\text{PHAT}}(\tau) = \phi_{\text{res}}(\tau) \otimes g(\tau) \otimes \delta(\tau - T_0),$$

where $\phi_{\text{res}}(\tau) = F^{-1}\{e^{-j\phi_{\text{res}}(\omega)}\}$ in which $\phi_{\text{res}}$ is the phase of the resonator. Figure 9a shows the normalized PHAT correlator with respect to its maximum value, given by Eq.(19) without the term $\phi_{\text{res}}(\tau)$ corresponding to the case when there is no resonator attached to the pipe. Figure 9b shows the PHAT correlator given by Eq.(19) with the resonator now attached to the pipe as before. It can be seen that the influence of the resonance on the shape of the PHAT correlator is similar to that with the BCC correlator.

Figure 9. The effects of a resonance on the shape of the simulated PHAT correlator of the case shown in Fig. 4. (a) The normalized PHAT with respect to its maximum value, without any resonator attached to the pipe. (b) The normalized PHAT with respect to its maximum value.

5 RESULTS – THE INFLUENCE OF THE RESONANCE ON THE TIME DELAY ESTIMATE

In this section a comparison is made between the theoretical model and actual leak data. Figures 10a and 10b show the simulated modulus of the CSD and its phase, respectively, overlaid with the measured data. In Fig. 10b it is observed that the additional phase given by the phenomenological model does not match exactly the measured data. At frequencies greater than the resonance frequency, the difference is about 1.5 rad. Although there is this discrepancy the model qualitatively describes the general characteristics.
To investigate the effects of a resonance present in the pipe system, two frequency ranges are considered, which are shown in Fig. 10: The first is below the additional phase shift in which the limits are 10 Hz and 70 Hz, and the second that includes the additional phase shift in which the limits are 10 Hz and 150 Hz. An analysis is conducted by calculating the BCC and PHAT correlation functions using the data filtered over each frequency region.

Figures 11a(i) and a(ii) show the respective BCC and PHAT calculated in the frequency range below the additional phase shift. The time delay estimates given by the largest peak in the cross-correlation function for the BCC and PHAT correlators are 23 ms and 23.2 ms, respectively. Figures 11b(i) and b(ii) show the BCC and PHAT correlators calculated over the frequency range that includes the resonance within its limits. In this case, the correlators have two distinct peaks, one being positive and one being negative. The largest peaks in both the BCC and PHAT correlators are negative and correspond to a time delay estimate of 14.2 ms and 13.2 ms, respectively. The largest positive peak corresponds to a time delay of 22.8 ms and 24.4 ms for BCC and PHAT, respectively. The wavespeed estimate given by the largest positive peak is about 440 m/s, which is close to values found in the literature (360 m/s up to 512 m/s [4, 16]). Hence, choosing the largest positive peak gives a better time delay estimate.

Figure 12 shows the way in which the normalized time delay estimate given by the largest peak and largest positive peak changes with bandwidth. The normalization is with respect to the actual time delay given by the positive peak in the cross-correlation function performed over the frequency range 10-70 Hz. The time delay estimates are calculated by fixing the lower limit of the filter bandwidth at 10 Hz and increasing the upper frequency from 70 Hz to 150 Hz. The time delay given by the peak of the BCC within this bandwidth is 23 ms. As observed, both estimates given by the largest peak (black dashed line) and largest positive peak (blue solid line) in the BCC correlator behave in a similar way. When the resonance is included within the bandwidth, the time delay estimates start to deviate from the actual value of 23 ms by up to 2% using the largest positive peak in the BCC, and up to 6% using the largest peak in the BCC. As expected, the estimates given by the PHAT correlator fail when the resonance is included within the bandwidth chosen for analysis. Here the largest peak, which is negative, is much larger than the largest positive peak. If the modulus of the PHAT cross-correlation function is taken, then a large error in estimating the actual time delay will lead to a wrong location of the leak.
Figure 12. The normalized time delay estimate given by the largest peak and largest positive peak in the BCC and PHAT correlation function as a function of the bandwidth. The time delay is normalized with respect to the time delay estimate of 23 ms calculated using the largest positive peak in the BCC performed over the frequency range 10-70 Hz. BCC largest positive peak; BCC largest peak; PHAT largest positive peak; PHAT largest peak.

6 CONCLUSIONS

In this paper, the BCC function and the PHAT correlation function, which are often used for leak detection in plastic pipes, have been introduced. An analytical model has been proposed to investigate their features. This model was developed to simulate cases where resonances are present within the frequency bandwidth of interest. Simulations have been compared with accelerometer-measured leak signals, where there was distortion in the phase of the cross-spectral density due to resonances in the system.

For plastic pipes, it was found that the BCC is the most suitable for leak detection. The modulus of the CSD decays rapidly above the resonance frequency and so the resonance frequency does not have a strong influence on the time delay estimate from this correlator. The PHAT correlator, however, is sensitive to phase changes, such as additional shifts due to the presence of a resonance. Hence, the time delay estimate calculated using this correlator is affected by the presence of resonances.

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