Shape optimization under vibroacoustic criteria in the mid-high frequency range using gradient-based approach

R. Troian\textsuperscript{1,a}, S. Besset\textsuperscript{1,b}, and F. Gillot\textsuperscript{1,c}

\textsuperscript{1}Laboratoire de Tribologie et Dynamique des Systèmes, UMR CNRS 5513,
Ecole Centrale de Lyon, 36 avenue Guy de Collongue 69134 Ecully Cedex, France
\{\textsuperscript{a}Renata.Troian, \textsuperscript{b}Sebastien.Besset, \textsuperscript{c}Frederic.Gillot\}@ec-lyon.fr

ABSTRACT: Shape optimization issues under vibroacoustic criteria within the specific mid high frequency range are under consideration in the present paper. The main objective of this research is to develop an adjoint formulation for the shape optimization using Simplified Energy Method (MES). This will allow to implement the method into numerical modeling of engineering applications, such as noise reduction, aeronautical domain, etc., with large number of variables and reasonable computational cost. The adjoint method was developed to minimize the energy density in the cavity by changing its geometry parameters. We use the Simplified Energy Method (MES), which gives a solution that only depends on the cavity shape, not on the material properties and doesn’t need fine meshes. Firstly, the proposed shape optimization method aims at avoiding the remeshing during the optimization process and it has to model the acoustic cavity surface exactly. To achieve this goal, we rely on a transformation function which maps a 3D cavity surface on a 2D domain. Hence, the optimization is conducted on this function directly. Secondly, as for realistic applications the number of design variables is very high, so the gradient based methods are in demand. In the present contribution the optimization process is based on adjoint calculation of the gradient that leads to an analytical expression of the directional derivatives without additional computational cost. To prove the versatility of the method, we apply it on a rectangular cavity shape modeled with patches of Bezier surfaces. The results of shape optimization are presented and the robustness of the method is shown.

KEY WORDS: energy method; shape optimization; vibroacoustic criteria; gradient.

1 INTRODUCTION

In the new developing domain of design optimization, approaches taking into account vibroacoustic criteria are in demand for many engineering applications, for example reducing noise of engines in car cabins at the positions of the driver and the passengers, controlling the noise in industrial rooms where people are working at certain locations among noisy machinery or protecting electronic equipment on which sound waves can have an influence. An important issue in such optimization problems is the frequency range. When considering vibroacoustic shape optimization in the high-mid frequency range we meet the serious drawback of using kinematic description of the displacement field, because it needs fine meshes to obtain the robust results, so the use of FEM-BEM is justified only for low frequencies. In the opposite, the energy methods chosen for the present study do not need fine meshes and are expected to offer many advantages. Among these methods, the most widespread remains the Statistical Energy Analysis (SEA) (1), which provides the mechanical energy of complex built structures. It is well adapted to medium and high-frequency situations and yield smaller matrices and fast optimization processes. The energy flow variables in the SEA model are well suited for use in optimization algorithms because of their smooth frequency response functions. De Jong (2) gives the examples of the use of SEA in optimizing the noise reduction and the sound quality of a machinery enclosure and a vehicle body. Chavan and Manik (3) present the optimization of an automobile model for cabin noise reduction by calculating the design sensitivity vector for damping loss factors of subsystems using the transmission path approach.

In this paper we use a local energy formalism, proposed by Nefske and Sung (4) and improved after by many authors (5) (6) (7). This energy method was named the Simplified Energy Method (MES in French). MES has already been evaluated and validated for such elastic media such as membranes and plates (5) (9), beams (5) (8) and acoustic cavities (8). This method has also been considered in both the transient and stationary cases (6), (8). The optimization of the vibroacoustic characteristics using MES was presented by Besset and Ichchou (10), regarding the optimizing absorption coefficients at the boundaries of an acoustic cavity.

The main objective of this contribution is to show the advantages of the adjoint formulation for the shape optimization under vibroacoustic criteria using MES. This will allow implementing the method into numerical modeling of engineering applications with large number of variables and reasonable computational cost. The results presented here are obtained with the shape optimization method that is based on a transformation function mapping 3D cavity surface on a 2D domain. The optimization process directly relies on this function and thus avoid remeshing of the geometry. The method allows to describe the geometry through Bezier, B spline and NURBS parametrization. The projection function approach has been presented in (11)

The article is organized as follows. First, the Simplified
Energy Method is presented and its formulation in curvilinear coordinates adapted to the adjoint-based optimization is obtained. The optimization problems is then formulated that makes calculation of the derivatives and construction of the adjoint equation possible. To prove the versatility of the method, we apply it on a rectangular cavity shape parametrized with patches of Bezier surfaces. The results of shape optimization are presented for several initial data and the robustness of the method is shown.

2 SIMPLIFIED ENERGY METHOD. GENERAL DESCRIPTION

The Simplified Energy Method method operates two continuous fields to describe the energy transfer inside the medium in the mid-high frequency range. The first energy quantity is the total energy density \( W \) defined as the sum of the potential energy density and the kinetic energy density. The second energy variable \( I \) is the energy flow. In order to derive the energy density equations, a wave description of vibrational-acoustical behavior is definitely adopted. The Simplified Energy Method has been exposed in details in (7) (9) (10) (12). Here we recall the main quantities that will be used further.

In the considered problem symmetrical propagating disturbances in a medium are taken into account.

\[
W = W_{\text{dir}} + W_{\text{rev}}. \tag{1}
\]

This approach considers the reverberated field as the result of secondary sources (fictitious sources) located at boundaries (Figure 1). Thus the energy density inside the cavity can be expressed as a function of the primary sources and fictitious sources (12):

\[
W(P) = \int_{\partial \Omega} \Phi(M) \, \mathbf{u}_{PM} \cdot \mathbf{n}(M) \, G(M) \, d\partial \Omega + \int_{\partial \Omega} \sigma(M) \, \mathbf{u}_{PM} \cdot \mathbf{n}(M) \, G(M) \, d\partial \Omega, \tag{2}
\]

where \( P \) is a point inside the cavity where \( W \) is measured, \( M \) is a point of integration on the cavity surface, \( G(r) = \frac{1}{4\pi r} \). We use the term “boundary source“ to denote the sources located on the cavity boundary (which may be due, for example, to external excitations).

For every point \( M_0 \) of the boundary \( \partial \Omega \), \( \sigma(M_0) \) depends on the absorption coefficient \( \alpha \), acoustic boundary sources of the system \( \Phi \) and fictitious boundary sources in all other points of \( \partial \Omega \): \n
\[
\sigma(M_0) = (1 - \alpha) \int_{\partial \Omega} \sigma(M) \, \mathbf{u}_{M_0M} \cdot \mathbf{n}(M) \, G(M) \, d\partial \Omega + (1 - \alpha) \int_{\partial \Omega} \Phi(M) \, \mathbf{u}_{M_0M} \cdot \mathbf{n}(M) \, G(M) \, d\partial \Omega. \tag{3}
\]

Energy variables are given as a solution of a Fredholm equation, corresponding to an energy balance at the boundary of the domain.

3 OPTIMIZATION PROBLEM

The subject of the present research is the shape optimization of the acoustic structure applicable for engineering applications. For this the proposed shape optimization method need to meet several criteria: to avoid the remeshing during the optimization process, to model the acoustic cavity surface exactly and not to be computationally expensive.

The geometric parametrization plays an important role in this process and the adequate boundary description is essential to satisfy two first declared demands. Different boundary representations, such as polynomial, Bezier, Bspline and non-uniform rational B-Splines (NURBS) descriptions that proved to be very effective because of their smoothness and boundary regularity appeared for the geometric description. The energy methods chosen for the present study do not need fine meshes and are suitable for such geometric parametrization of the acoustic structure.

Though we didn't aim ourselves in the developing code for an engineering applications it is clear that proposed method have also to deal with realistic problems, which need to consider the complex cavities and high number of design variables. To model such surfaces, as it was mentioned before, one can use B-spline and NURBS technique, and treating the problems with high number of design variables demands using quick and robust optimization strategies, such as gradient-based methods using adjoint formulation.

The mathematical formulation of a structural shape optimization problem reads as (13):

\[
\begin{align*}
\min_S & \quad f(X, Y(X)) ; \quad X \in \mathbb{R}^n \\
\text{such that} & \quad x_i \leq x_j \leq x_f, \\
& \quad h_j(X, Y(X)) = 0, \\
& \quad g_k(X, Y(X)) \leq 0, \\
\text{with} & \quad i = 1..n, \\
& \quad j = 1..m, \quad \text{of equality constraints,} \\
& \quad j = 1..n, \quad \text{of inequality constraints,}
\end{align*}
\tag{4}
\]

Figure 1. MES formulation: direct and reverberated fields.
where \( f \) is the objective function and \( X \) are the design variables with \( n \) components which control the geometry. The state variable \( Y \) describes the structural response. The functions \( h_j \) and \( g_k \) represent behavioral constraints on quantities describing the characteristics of the structure.

3.1 MES formulation in curvilinear coordinates.

When studying the vibroacoustical behavior of the 3D cavity using MES, only the internal 3D surface of the cavity need to be modeled to calculate the energy quantity inside. (see 2). This 3D cavity can be described by parametrized functions of two variables, for example mentioned above Bezier, Bspline and NURBS. The shape of the surface may be controlled by number of function parameters: when they change, the shape of the surface changes as well. Therefore, we associate the design variables \( x_i \) with those function parameters that are allowed to move during the iteration process.

\[
\mathbf{P_M} = [x(\xi_1, \xi_2) - b_1; y(\xi_1, \xi_2) - b_2; z(\xi_1, \xi_2) - b_3],
\]

so that the vectors
\[
\mathbf{R}_\alpha = \frac{\partial \mathbf{r}}{\partial \xi_\alpha}, \quad \alpha = 1, 2,
\]

are linearly independent for all points \( \xi = (\xi_1, \xi_2) \in S \). These two vectors define the tangent plane to the surface \( \partial \Omega \) at the point \( \mathbf{r}(\xi) \). Next, we define the normal vector:

\[
\mathbf{n}(\mathbf{M}) = \mathbf{n}(\xi_1, \xi_2) = \frac{\mathbf{R}_1 \times \mathbf{R}_2}{|\mathbf{R}_1 \times \mathbf{R}_2|}.
\]

Then, the point \( \mathbf{r}(\xi) \) and three vectors \( \mathbf{R}_1, \mathbf{R}_2, \mathbf{n} \) define a local reference system for the surface \( \partial \Omega \).

In new coordinate system we have:

\[
M = \mathbf{r}(\xi) = (x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)),
\]

The functions \( \Phi \) and \( \sigma \) can be represented by functions \( \mathbf{r}(\xi) \). That is why we take the coordinates \( (b_1, b_2, b_3) \) of it as constants and obtain:

\[
\Phi_{\text{curv}} = (\Phi \circ \mathbf{r})(\xi_1, \xi_2),
\]

\[
\sigma_{\text{curv}} = (\sigma \circ \mathbf{r})(\xi_1, \xi_2).
\]

The energy quantities \( \Phi \) and \( \sigma \) changed, but for the sake of simplicity we will keep the usual notation. After substituting (8)-(11) to (2) with \( (\xi = (\xi_1, \xi_2)) \) we have:

\[
W(b_1; b_2; b_3) = \int_S \Phi \mathbf{n}(\xi_1) \cdot \mathbf{n}(\xi_2) \; G(\xi) \; \|\mathbf{a}_1(\xi_1) \times \mathbf{a}_2(\xi_2)\| \; d\xi
\]

\[
+ \int_S \sigma \mathbf{n}(\xi_1) \cdot \mathbf{n}(\xi_2) \; G(\xi) \; \|\mathbf{a}_1(\xi_1) \times \mathbf{a}_2(\xi_2)\| \; d\xi.
\]

\[
\sigma(\xi_{10}, \xi_{20}) = (1 - \alpha) \int_S \sigma \mathbf{n}(\xi_1) \cdot \mathbf{n}(\xi_2) \; G(\xi) \; \|\mathbf{a}_1(\xi_1) \times \mathbf{a}_2(\xi_2)\| \; d\xi
\]

\[
+ (1 - \alpha) \int_S \Phi \mathbf{n}(\xi_1) \cdot \mathbf{n}(\xi_2) \; G(\xi) \; \|\mathbf{a}_1(\xi_1) \times \mathbf{a}_2(\xi_2)\| \; d\xi.
\]

Such formulation of MES in curvilinear coordinates can be easily used for the cavities of complex geometry, when surface is modeled with the help of Bezier curves, Bsplines and NURBS.

3.2 Optimization procedure.

- First one has to specify the geometry of the cavity \( \Omega \) with bounding surface \( \partial \Omega \) and the function of transformation \( \mathbf{r}(\xi_1, \xi_2) \) defined on \( S \) to describe the cavity shape.
- After, we define the design variables \( x_i \) as characteristics of transformation function \( \mathbf{r}(\xi_1, \xi_2) = (x(\xi_1, \xi_2), y(\xi_1, \xi_2), z(\xi_1, \xi_2)), \) i.e. the parameters of the functions \( x(\xi), y(\xi) \) and/or \( z(\xi) \).
• The parts of the cavity surface where primary point sources are situated remain fixed during the optimization process to avoid the effect of respiration.

• An objective function has to be specified. The Simplified Energy Method (MES) has been described in Section 2, and its implementation leads to a matrix formulation of the problem that can be written as follows:

\[ \mathbf{W}(\mathbf{x}_i) = \mathbf{M}(\mathbf{x}_i) \mathbf{\Phi}. \] (16)

The energy density vector \( \mathbf{W} \) depends on the geometry of the cavity, and thus on the design variables \( x_i \).

Now we can define the quantity \( F \) to be minimized:

\[ F = \| \mathbf{W}(\mathbf{x}_i) \|. \] (17)

• The structural response \( \mathbf{Y} \) from (4) is given by function \( \sigma \) ((3)) or (15) in curvilinear coordinates) that describes fictitious sources of the system, so (3) itself gives the equality constraint:

\[ h(X,\sigma(X)) = \sigma - (1 - \alpha) \left[ \int S \sigma u \cdot n G \| \bar{a}_1 \times \bar{a}_2 \| d\xi \right] + \int S \Phi u \cdot n G \| \bar{a}_1 \times \bar{a}_2 \| d\xi ] \]

Hence the optimization problem can be formulated as follows:

\[ \min_{\mathbf{X}} \| \mathbf{W}(\mathbf{X}) \|; \quad \mathbf{X} \in \mathbb{R}^n \]

such that \[ \underline{x}_i \leq x_i \leq \bar{x}_i, \]

\[ h(X,\sigma(X)) = 0, \]

with \[ i = 1..n. \]

The gradient of the objective function can be calculated by several means, such as the finite difference method, adjoint-based calculation, etc. The finite difference method is a natural approach but it results in an approximate evaluation of the gradient, and the cost of it increases with the number of design variables. We concentrated on adjoint-based calculation of the gradient because it leads to an analytical expression of the directional derivatives without additional computational cost. This property makes the use of adjoint equations popular in the field of shape optimization with high number of design variables.

In the present study we follow the procedure of deriving the adjoint equations described in (15).

4 NUMERICAL EXAMPLE

The aim of the paper is to develop a gradient based optimization method allowing realistic industrial issues to be solved. By using the MES approach, acoustic cavity of any shape can be modeled by dividing the whole volume on several subvolumes with simple geometries. In the following we study a numerical example (Fig. 3) with six patches of Bezier surfaces as presented in Fig. 3. The function of transformation for face \( i \) of the parallelepiped is given as:

\[ r_i(\xi_1, \xi_2) = [\Xi_1][N][B_i][N]^T[\Xi_2]. \] (19)

Let’s consider a parallelepiped that takes an area \( \Omega = \{ x \in [0;4]; y \in [0;2]; z \in [0;2] \} \) (Fig. 3). The surface of the parallelepiped is considered to be assembled with six patches of Bezier surfaces. Every patch is determined by \( 4 \times 4 \) control vertices as presented in Fig. 3. The function of transformation for face \( i \) of the parallelepiped is given as:

\[ r_i(\xi_1, \xi_2) = [\Xi_1][N][B_i][N]^T[\Xi_2]. \] (19)
where
\[
[\Xi_1] = [\xi_1^3 \xi_2^1 \xi_1^1],
\]
\[
[\Xi_2] = [\xi_2^3 \xi_2^2 \xi_2^1],
\]
\[
[N] = \begin{bmatrix}
-1 & 3 & -3 & 1 \\
3 & -6 & 3 & 0 \\
-3 & 3 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}.
\]

[Bi] - control vertices to define the face i of the parallelepiped, \(\xi_1 \in [0; 1]\) and \(\xi_2 \in [0; 1]\) for every patch. Bezier surfaces provide the \(C^1\) continuity for all the points of the surface and thus the vectors \(a_\alpha\) exist (see (6)). Control vertices on the boundary between the patches coincide and the \(C^0\) continuity is thus provided. Due to the MES formulation we do not need the \(C^1\) continuity on the boundary between the patches, so no extra constrains have to be imposed.

Each face of the parallelepiped was discretized and the single mesh for the whole surface was created. For numerical computations in classical isogeometry analysis the loop through the patches is used first. After that, for every patch the code enters the loop through all the elements of the patch (14). For the MES formulation all the elements of the surface have to be considered simultaneously in order to calculate the reverberated field (see (3)). Nevertheless, the advantages of non-remeshing, exact surface approximation and smoothness of the obtained shape are still valid.

4.2 Optimization problem statement.

An acoustic source is applied on the surface (Fig. 3, marked with cross); the test point inside the cavity with coordinates (1.33; 0.6; 0.6) was chosen to compute energy density vector. One control vertice was chosen to be design parameter (Fig. 3, marked with triangles). The coordinate of the vertice perpendicular to the patch plane is under consideration, so the optimization problem depends on one design variable \(x\). Two test cases were considered, with different locations of optimization parameters (Table 1). Test point and the acoustic source positions remain the same for both cases.

The cavity surface is discretized as follows: \((24 \times 12 \times 12)\) for faces that remain plane during the optimization process and \((36 \times 18 \times 18)\) for the curved ones.

The formulation of the optimization problem takes the form (18). The optimization algorithm is performed using a gradient method based on adjoint formulation. Validity of the approach is shown in the next subsection.

4.3 RESULTS VALIDATION.

To verify the robustness of the proposed method the results are compared with the ones obtained by direct calculation of the energy vector. The physical phenomena of the vibrations inside the acoustic cavity provides the decrease of the vibrations’ level with the growth of the cavity volume, i.e. the value \(W\) decreases as the value of design variable \(x\) increases (Fig. 4, a). This fact is recognized by the adjoint method (see Fig. 4, b); the gradient is negative on the whole research interval meaning the decrease of the objective function.

For the chosen geometry and design variables the objective function \(f\) (see 18) has no local minimum, it is decreasing on the whole definition domain. To study the capabilities of the proposed adjoint method in finding the local minimum the objective function was modified. The important criteria of the cavity surface area \(S\) that reflects the quantity of the material used for the cavity construction was added with the coefficient \(\alpha\) that reflects the weight of the surface criteria in the objective function. The modified formulation of the optimization problem reads:

\[
\min_x ||W(X)|| + \alpha S; \quad X \in \mathbb{R}^n
\]

such that \(x_i \leq x_i \leq \bar{x}_i\),

\[
h(X, \sigma(X)) = 0,
\]

with \(i = 1..n\).

Figure 4. Objective function \(f = W\) vs cavity geometrical parameter \(x\) for each case (see Fig. 3)

Figure 5. Objective function \(f = W + \alpha S\) vs cavity geometrical parameters. Direct calculation.
In the present paper the coefficient $\alpha = 0.06$ was chosen empirically regarding the correlation between the values of $W$ and surface area of the cavity. Considering the optimization problem (20) by the direct calculation of $f = W + \alpha S$ the local minimum for the first test case is found when $x = 1.51$ (see Fig. 5). The adjoint method gives the minimum when $x = 1.51$ as well (Fig. 6). The optimized shape is presented in Fig. 7. Changing of the color reflects the change in the coordinate normal to the side of parallelepiped.

5 CONCLUDING REMARKS.

The present contribution is devoted to the new shape optimization method under vibroacoustic criteria that is based on adjoint calculation of the gradient. Due to the analytical calculation of the gradient the method is suitable for the problems with large number of design variables without additional computational cost. Simplified energy method used for the calculation of the system total energy doesn’t demand coarse mesh for robust calculations. Implementation of the curvilinear coordinates and function of transformation for the description of the cavity geometry aims at avoiding the remeshing during the optimization process and models the acoustic cavity exactly. All these advantages make the proposed method perspective for the realistic engineering applications in different domains.

The developed optimization method was validated for the case of parallelepiped cavity, which was modeled with Bezier surfaces. It was shown the capability of the method to find the minimum for several types of the objective functions.

References


