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The identification of nonlinear damping of the selected components of MDOF complex vibratory systems

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ABSTRACT: The determination of dissipative properties of structural materials became a fundamental task in those cases, when a deformation velocity is very high. It is essential, especially in the process of piercing the ballistic shields in which the velocity of the projectile has initially a very high value (impact velocity) but decreases very quickly. In these cases, the proper modeling of dynamical material properties requires application of more complex models than the well-known linear Kelvin model. Therefore, the dissipative properties should be determined under the condition of dynamical loads, especially for impact loads or for periodical excitations at high frequencies. This paper presents two different methods for identification of the damping. These methods allow to determine the damping characteristic $F_d(v)$ of any nonlinear form. The first methods can be applied for impact loads while the second one can be used for periodical excitations. The first method is a typical nonparametric method and it depends on discovering the proper dissipative characteristic of the tested material. The second method belongs to the parametric methods. It assumes that the tested material element is a component of a multi-degree-of-freedom (MDOF) vibratory system of arbitrary complex form. The comparison of some of the results presented by both methods was carried out for nonlinear cubic form of damping by the numerical computer system. A special form of frequency response function, which depend only on the parameters of the selected component, have been applied.

KEY WORDS: Nonlinear dynamics; Ballistic impact; Behaviour of materials; Mathematical modelling; Dissaption energy.

1 INTRODUCTION

Elements of the machines that are subjected to the random shock loads are being protected with a specially constructed shields. Presently designed shields (e.g. ballistic shields) are constructed using lightweight composite materials (e.g. Kevlar, ballistic laminate, etc.) [1, 2, 3, 4, 5]. The primary objective of this type of the materials is to slow down the speed of the impacting object (e.g. projectile) while reducing the weight of the shield itself (e.g. bulletproof vests). To achieve high effectiveness of the process of slowing down the object, the material used in the construction of the shield should quickly dissipate the impact energy. Thus, the essential characteristics of these materials include their dissipative properties.

In the typical engineering applications of the machine dynamics, the linear theory is usually used. It assumes that a dissipative phenomena could be described by the single parameter of vibration damping associated with the viscous linear model. If the damping is small, then the linear model describes the dynamics of the real system accurately enough. In this scope we have the popular method of experimental modal analysis [6, 7, 8, 9, 10, 11]. However, in the case of lightweight composite materials deformed at high linear velocities, the model of viscous damping would be too great simplification. Models using to the description the non-classical, which structure and diversity result from the considered physical phenomenon [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] we can identify the specific characteristic of dissipation.

In such cases the important issue is to determine the shape of the model of dissipation characteristics. This applies to the non-linear function which describes the relation of the dissipation and the speed of deformation, as well as to checking if it does not depend on the level of deformation [22].

2 ASSUMPTIONS AND MODEL

The paper contains an original experimental method which allows to determine the shape of the function $S(x,v)$ that describes the impact of the material of the shield on the piercing mass (Figure 1).

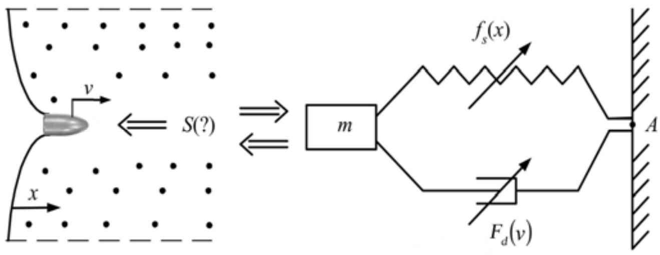

Figure 1. Conceptual model of the piercing process.

In this method it has been assumed that this force can be described by the function of the form:

$$S(?) = S(x,v) = f_s(x) + \lfloor \chi_0 + \chi(x) \rfloor v + g(v) \tag{1}$$

where:

$f_s(x)$ – function of the displacement x which describes the purely elastic impact,

$g(v)$ – function of velocity which describes the non-linear part of the dissipative impact,

$\chi(x)$ – impact factor of the level of deformation on the dissipative impact,

χ_0 – factor of the viscous linear damping.

If $\chi(x) = 0$ then the dissipative interaction does not depend on the position of the projectile in the shield, but only on its speed.

In such case, the overall impact $S(x,v)$ of the material of the shield could be modeled by the Kelvin type model [22] which assumes that the purely elastic interaction $f_s(x)$ works in parallel configuration with any nonlinear dissipative interaction $F_d(v)$ (Figure 1).

The procedure developed by the authors of this study includes:

- pre-determination of the shape of the functions χ(x) and $g(v)$,
- using the parametric method of identification to determine the coefficients of the function $g(v)$ that has been determined in the step (1).

In both phases of this procedure, it is assumed that the tested material functions as the last elastic-dissipative element in the complex dynamic system of any form (Figure 2). This is an important facilitation in the construction of a testing stand because the spot A of the attachment of the tested element to the complex system doesn't have to be fixed (see Figure 1 and Figure 2).

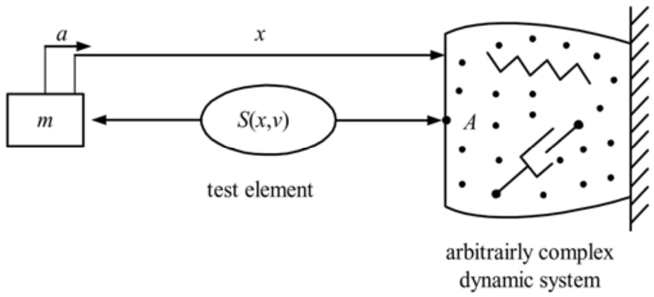

Figure 2. Schematic presentation of the location of the tested element aiming at its identification on the test stand.

In this approach, using any exciting force $p_i(t)$ to the complex system, differential equation of the motion of the concentrated mass m has the form:

$$ma + S(x, \dot{x}) = 0 \tag{2}$$

where: x – displacement of the mass m in relation to the point A of the complex system, a – absolute acceleration of the mass m.

This paper presents this procedure on the example of a nonlinear system in which the function $g(v)$ has the form:

$$g(v) = \chi_3 v^3 \tag{3}$$

Experimental verification of this procedure has been initially carried out for the system of the linear characteristics of elasticity $f_s(x) = cx$ and the lack of influence of the level of deformation, that is $\chi(x) = 0$.

3 NONLINEAR COMPONENT OF THE DISSIPATIVE CHARACTERISTICS

The conception of the method used to determine the shape of the functions $g(v)$ and $\chi(x)$ has been already shown in the previous papers of the authors [23, 24]. This is a non-parametric method [25, 26] which involves measuring the signals of velocity and acceleration at the selected points of time t_i for which the displacement x is equal to zero. For the moments t_i the relation between these values is as follows:

$$ma_i = \chi_0 v_i + g(v_i) = 0 \tag{4}$$

where $a_1 = a(t = t_i)$, $v_i = v(t_i)$.

The above relation results from the equation (2) and from the adopted form of the function $S(x,v)$, if the following conditions are satisfied for $x = 0$:

$$f_s(0) = 0, \qquad \chi(0) = 0 \tag{5}$$

If the mass m is known, we can calculate the values:

$$B_i = -ma_i \tag{6}$$

According to the equation (4) the relationship $B_i(v_i)$ must have the form:

$$B_i(v_i) = \chi_0 v_i + g(v_i) \tag{7}$$

This implies that the determination of the form of the function $g(v)$ can be performed on the basis of the experimentally determined dependence $B_i(v_i)$.

4 FUNCTION OF THE INFLUENCE OF THE LEVEL OF DEFORMATION

Similar procedure leads to determining the shape of the function $\chi(x)$ which describes the influence of the level of deformation on the dissipation. For this purpose, one should select the moments of time t_j for which the acceleration a is equal to zero. It may be noted that the relation including other variables x, v for the moment t_j would take the form:

$$[\chi_0 + \chi(x_j)]v_j + g(v_j) + f_s(x_j) = 0 \tag{8}$$

where $x_j = x(t_j)$, $v_j = v(t_j)$.

If the functions $f_s(x)$ and $g(v)$ are known, then based on the equation (8) we can calculate the values of z_j using the formula:

$$z_j = \frac{-g(v_j) - f_s(x_j)}{v_j} \tag{9}$$

Equation (8) shows that the dependence $z_j(x_j)$ must have the form of:

$$z_j = \chi_0 + \chi(x_j) \tag{10}$$

This implies that the determination of the form of the function $\chi(x)$ can be performed on the basis of the experimentally determined dependence $z_j(x_j)$.

5 PARAMETRIC IDENTIFICATION OF THE DISSIPATIVE CHARACTERISTICS

If the shape of the function of dissipation $g(v)$ is defined, one can move to the stage of parametric identification. This section presents an original method for determining the parameters of vibration damping, built on the basis of the criterion of resonance. This method assumes that the dissipative characteristics should clearly define the experimentally determined relation between the amplitude of velocity V_r and the resonance frequencies ω_r for the various (random) amplitudes of excitation forces. This method has been shown below on the example in which:

- function $g(v)$ has the form:

$$g(v) = \chi_3 v^3 \tag{11}$$

- the influence factor of the level of deformation on the dissipation is equal to zero and the function of elasticity $f_s(x)$ has a linear form of:

$$f_s(x) = cx \tag{12}$$

In the case of the functions $g(v)$ and $f_s(x)$ of the form (11) and (12) the equation (2) takes the form:

$$ma + cx + \chi_0 \dot{x} + \chi_3 \dot{x}^3 = 0 \tag{13}$$

Taking into account the fact that the total acceleration a is the sum of the acceleration a_A of the point A and the relative component $\ddot{x}$, so:

$$a = a_A + \ddot{x} \tag{14}$$

equation (13) can be written in the form:

$$m\ddot{x} + cx + \chi_0 \dot{x} + \chi_3 \dot{x}^3 = -ma_A \tag{15}$$

Underslung system of the mass m and the deformation x can be therefore considered as a system with one degree of freedom which is under the influence of the exciting force $w(t)$ of the form:

$$w(t) = -ma_A \tag{16}$$

If the complex system (Figure 2) is approximately linear and is subjected to the harmonic force $p(t)$ then the force $w(t)$ of the form (16) can be approximately described by the function:

$$w(t) = 2W \cos \omega t \tag{17}$$

where: $2W$ – amplitude of the force $w(t)$, ω – frequency of the force.

The differential equation (15) will then take the form:

$$m\ddot{x} + cx + \chi_0 \dot{x} + \chi_3 \dot{x}^3 = We^{j\omega t} + We^{-j\omega t} \tag{18}$$

($j = \sqrt{-1}$). Using the approximate harmonic balance method [27] results in obtaining the solution with accuracy of the first harmonic form:

$$x(t) = \underline{X} e^{j\omega t} + \underline{X}^* e^{-j\omega t} = 2X \cos(\omega t + \alpha) \tag{19}$$

($\underline{X} = Xe^{j\alpha}$) wherein the relation $X(\omega)$ is described by the function of the form:

$$X^2 = \frac{W^2}{\left(c - m\omega^2\right)^2 + \left(\chi_0 \omega + 3\chi_3 X^2 \omega^3\right)^2} \tag{20}$$

Relation (20) would define the amplitude-frequency characteristics of the underslung element if the amplitude W of the force $w(t)$ will not depend on the frequency ω and was fixed at the certain level, so:

$$W(\omega) = const. = W_i \tag{21}$$

It can be achieved by increasing or decreasing the amplitude P of the force $p(t)$ during the slow (stable) changes of the value of the frequency ω. Obtained characteristic of $X(\omega)$ will be similar to the characteristic of the system with one degree of freedom. Using designation:

$$\Omega = \omega^2 \tag{22}$$

relation (20) can be written in the form:

$$\left[(c - m\Omega)\cdot(-m) + \left(\chi_0 + 3\chi_3 X^2 \Omega\right)^2\right] \cdot X^2 = W^2 \tag{23}$$

Differentiating the above equation by the variable Ω we have:

$$\left[2(c - m\Omega)\cdot(-m) + \left(\chi_0 + 3\chi_3 X^2 \Omega\right)^2 + 2\Omega\left(\chi_0 + 3\chi_3 X^2 \Omega\right) 3\chi_3 \left(X^2 + \Omega \frac{dX^2}{d\Omega}\right)\right] \cdot X^2 + \frac{dX^2}{d\Omega} \cdot [\cdots] = 0 \tag{24}$$

Introducing the frequency Ω_r for which the square of the amplitude X of the displacement $x(t)$ reaches the extreme, that is:

$$\frac{dX^2}{d\Omega}(\Omega = \Omega_r) = 0 \tag{25}$$

we obtain:

$$2m(m\Omega_r - c) + \left(\chi_0 + 3\chi_3 X_r^2 \Omega_r\right)^2 + 6\chi_3 \Omega_r X_r^2 \cdot \left(\chi_0 + 3\chi_3 X_r^2 \Omega_r\right) = 0 \tag{26}$$

Observing that:

$$X_r^2 \cdot \Omega_r = V_r^2 \tag{27}$$

where V_r is the amplitude of the velocity corresponding to the frequency Ω_r. After the elementary transformations we obtain the simple linear relation $\Omega_r(V_r)$ of the form:

$$\Omega_r = l_0 - l_1 V_r^4 \tag{28}$$

where constants l_0, l_1 are equal to:

$$l_0 = \frac{c}{m} - \frac{\chi_0^2}{2m^2}, \qquad l_1 = 9\frac{\chi_3^2}{2m^2} \tag{29}$$

Experimental determination of the dependence $\Omega_r(V_r^4)$ and its approximation by the linear function of the form (28) allow to obtain the optimal values of the constants l_0, l_1 and consequently – to calculate the value of χ_0 and χ_3 .

6 EXPERIMENTAL VERIFICATION

The verification tests of the presented method has been conducted using the computer simulation technique. For the purpose of this study the dynamic system has been constructed which is presented in the Figure 3 and is characterized by the following data:
k_1 = 68 Ns/m, k_2 = 0 Ns/m, k_3 = 45 Ns/m, c_1 = 2000 N/m, c_2 = 3500 N/m, c_3 = 1700 N/m, c = 900 N/m, m_1 = 16 kg, m_2 = 40 kg, m = 15 kg, χ_0 = 10 Nm/s, χ_3 = 80 Ns/m.

This system has been built using Simulink software according to the following differential equations of motion:

$$\begin{cases} m_1\ddot{x}_1 + k_1\dot{x}_1 + c_1x_1 - k_2(\dot{x}_2 - \dot{x}_1) - c_2(x_2 - x_1) = 0 \\ m_2\ddot{x}_2 + k_2(\dot{x}_2 - \dot{x}_1) + c_2(x_2 - x_1) + k_3\dot{x}_2 + c_3x + \\ - f_s(x) - \chi_0\dot{x} - g(\dot{x}) = p(t) \\ ma + f_s(x) + \chi_0\dot{x} + g(\dot{x}) = 0 \end{cases} \tag{30}$$

where: x_1, x_2 – absolute displacement of the mass m_1, m_2; x – relative displacement of the mass m to the mass m_2; $p(t)$ – exciting force (applied to the mass m_2).

The block diagram of the computer system has been shown in the Figure 4.

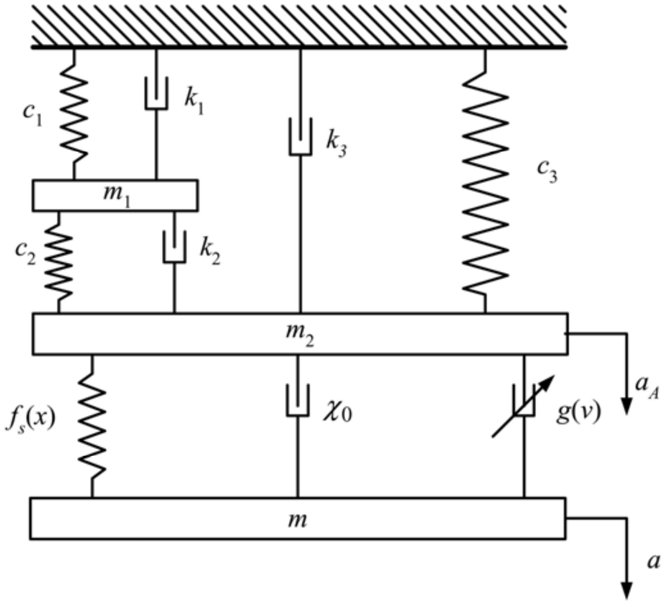

Figure 3. Scheme of the tested system.

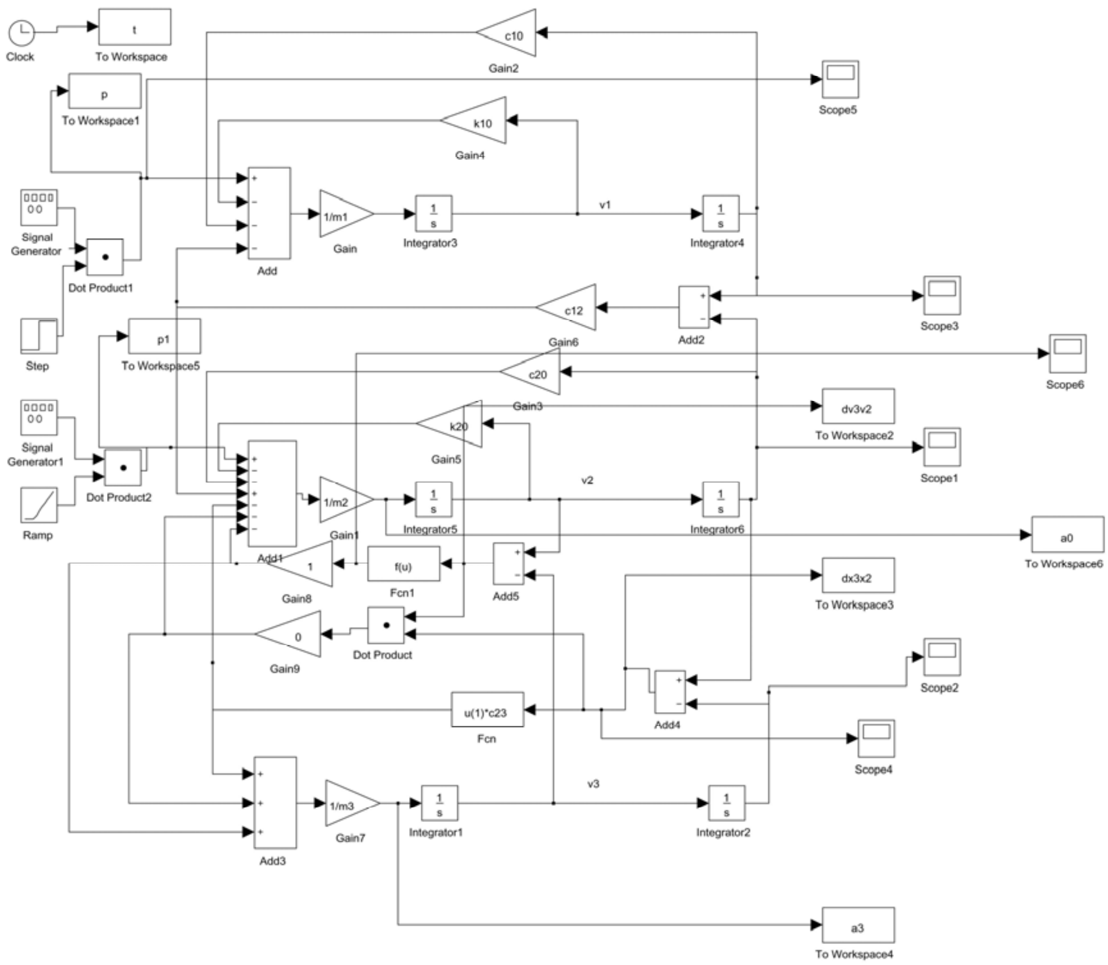

Figure 4. Block diagram of the measuring system.

To estimate the shape of the dissipative function $g(v)$ the shock loads have been applied to the mass m_2, which had a form shown in the Figure 5.

The maximum values (amplitudes) of these forces were random. For each amplitude, the free vibrations of the mass *m* have been recorded. Its examples in the form of dependences $x(t)$, $v(t)$, $a(t)$ are shown in the Figure 6.

After defining the values of a_i and v_i on the basis of the obtained courses of the free vibrations for which $x = 0$, the value of B_i have been calculated according to the formula (6). Then the dependence $B_i(v_i)$ have been obtained in the form of n = 24 points shown in the Figure 7. As it is clearly seen, these points are situated exactly on the curve of the third degree. In the result the approximating function of the characteristics $g(v)$ can be adopted in the form (3). In order to estimate the influence of the level of deformation, the specific values of x_j and v_j have been chosen, for which the acceleration *a* of the mass *m* was equal to zero.

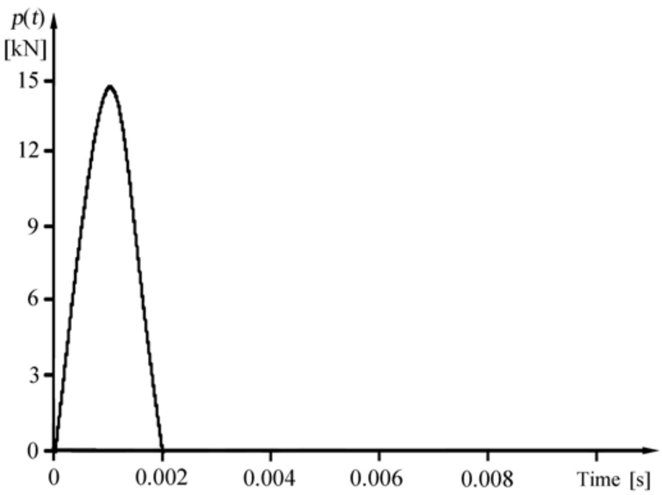

Figure 5. Example of the applied shock loads.

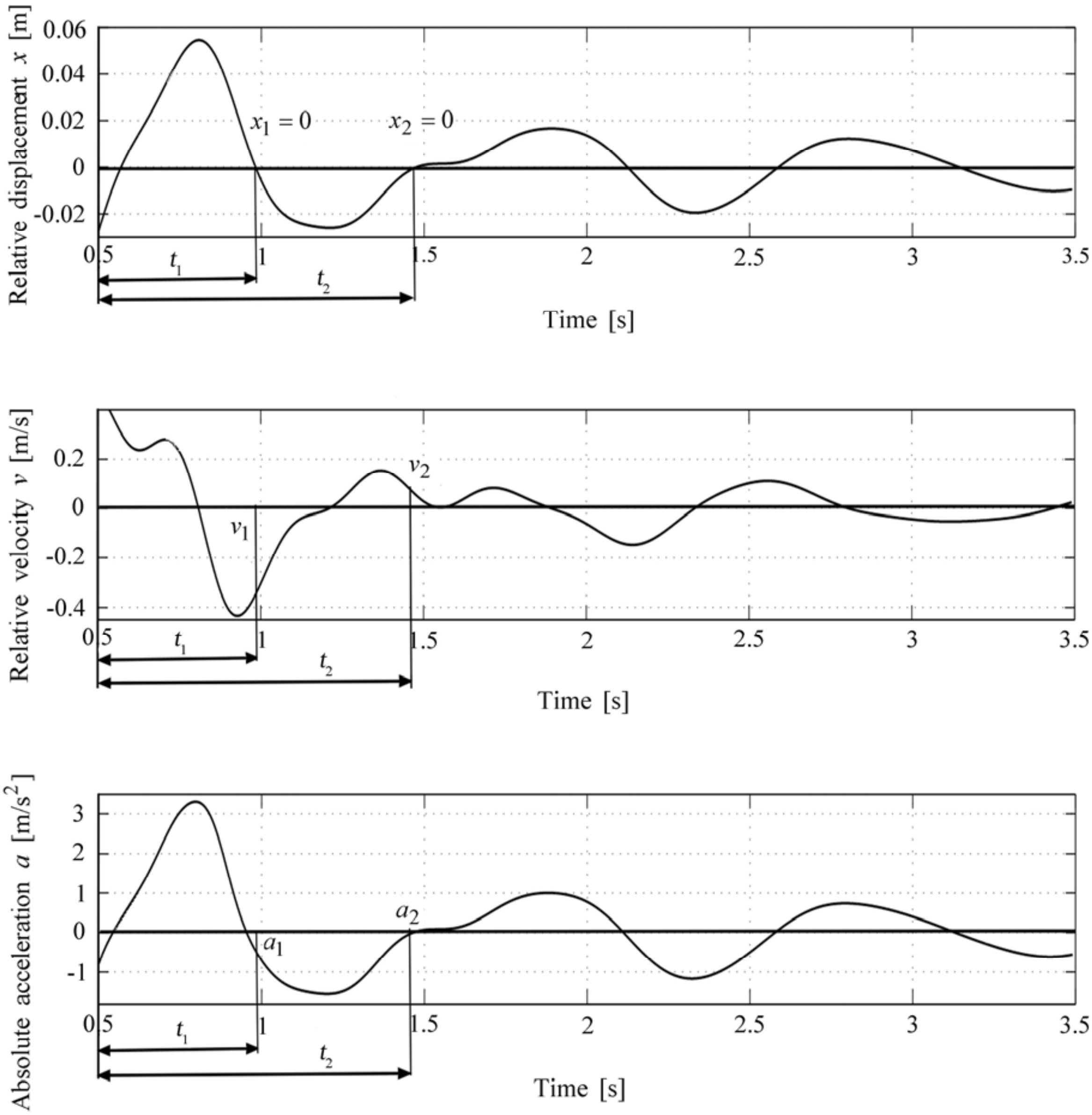

Figure 6. Example of the free vibrations of the mass *m* of the tested system.

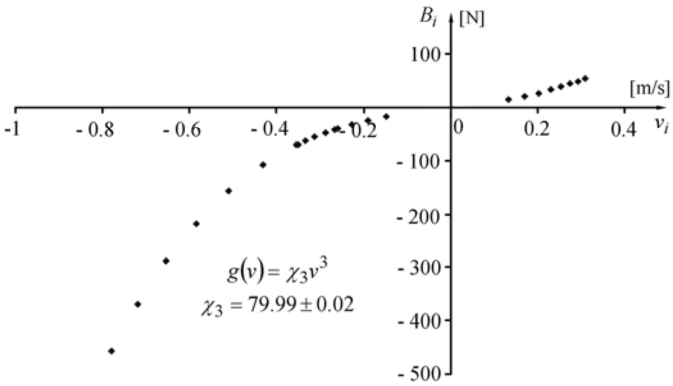

Figure 7. Dependence $B_i(v_i)$ obtained for the tested system.

For these values, assuming that the functions $g(v)$ and $f_s(x)$ are given, values of z_j have been calculated according to the formula (9). In the result, the dependence $z_j(x_j)$ has been obtained in the form of the points shown in the Figure 8.

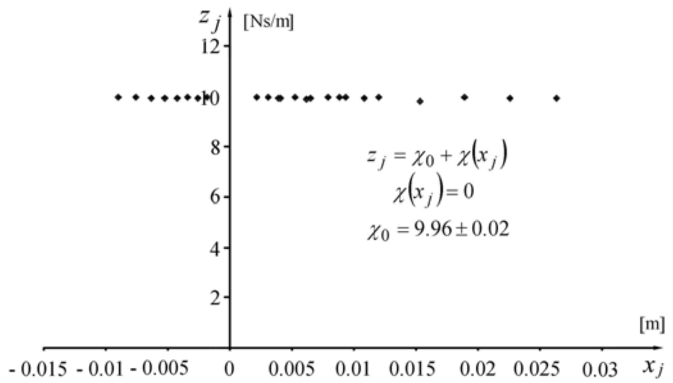

Figure 8. Dependence $z_j(x_j)$ obtained for the tested system.

Based on the graph of this dependency, it is easy to conclude that the function $\chi(x)$ of the tested system can be assumed to be equal to zero.

The further studies have been conducted, which took into account the above results, regarding the determination of the concrete values of the damping parameters. It has been assumed that while applying the harmonic force to the mass m_2, the vibrations of the mass m are described by the differential equation (18) and parametric identification applies to the constants χ_0 and χ_3.

The studies have been conducted in the frequency range from 1.1 Hz to 1.38 Hz. In this range, researchers have been looking for the resonant frequencies for the various levels of the constant amplitudes W of the pseudo-excitement force $w(t)$ of the form (16). This frequencies were determined as the extreme values of the dependences $X^2(\Omega)$, which are shown in the Figure 9. These relations have been determined by manipulating the values of the amplitudes P of the force applied to the mass m_2. This manipulation used a special method in which the acceleration a_A had a constant amplitude for each frequency. The values of these amplitudes were selected so that the amplitude of the corresponding resonant velocities were included in the range of up to 0.6 m/s. It corresponds to the non-linear part of the dissipative characteristic $g(v)$ (such as the Figure 7). In this speed, the acceleration was not purely sinusoidal (see Figure 10). Therefore, for acceleration amplitude (values a_{Ai}) adopted the product of mass m_2 velocity amplitude by frequency ω is: amplitude (a_{Ai}) = ω times amplitude (v_{Ai}). So the amplitude of the acceleration calculated a_A were determined on twelve different levels (i = 1, 2, …, 12) in the range of from about 0.1 m/s^2 to about 2.4 m/s^2.

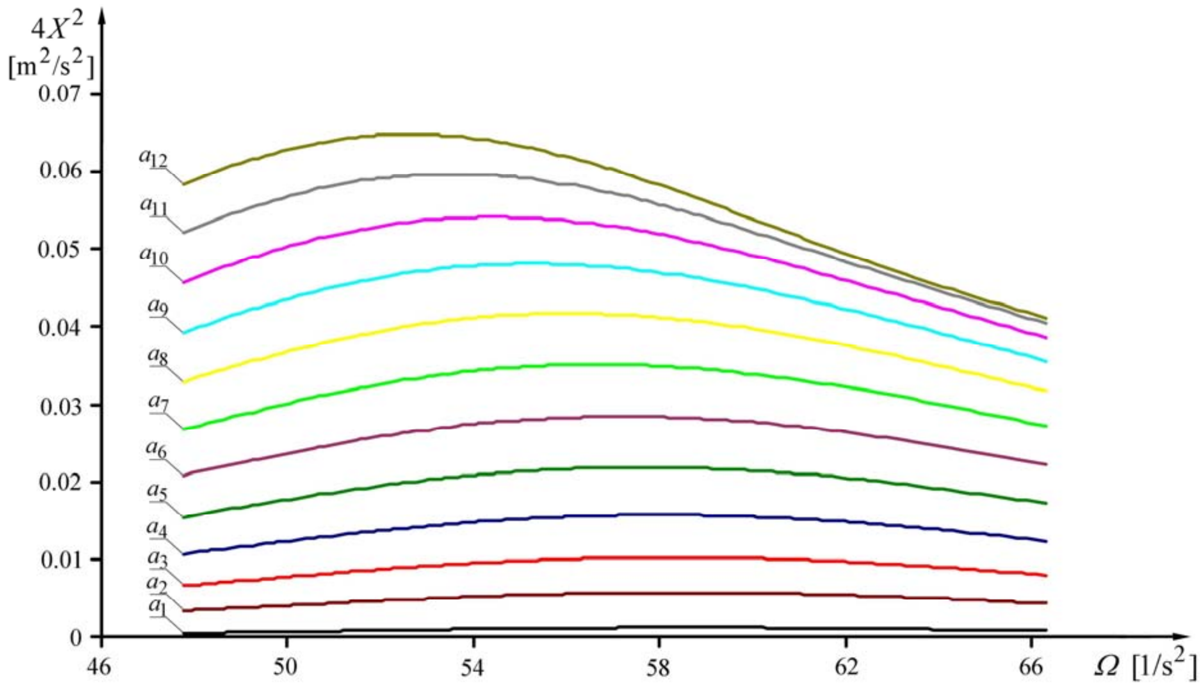

Figure 9. Special frequency characteristics of the relative displacements for the mass m.

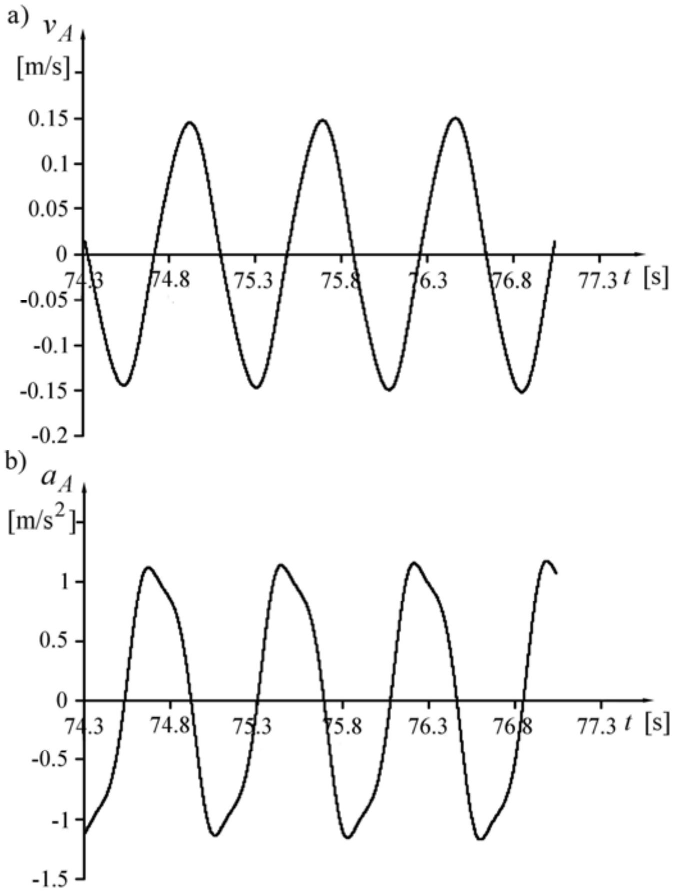

Figure 10. An example the time charts mass m_2:
a) – absolute velocity, b) – absolute acceleration.

Obtained resonance values X_r and ω_r were used to prepare the graph of the relation between the square of the resonance frequency Ω_r and the fourth power V_r^4 of the amplitude of the resonance velocity V_r using the relation $V = \omega X$. This relationship has been shown in the Figure 11.

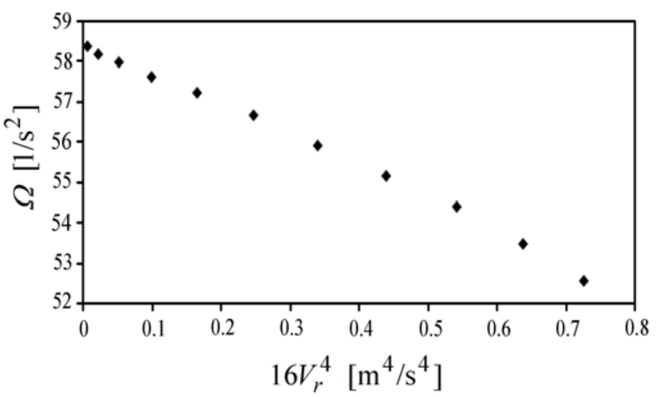

Figure 11. Dependency $\Omega_r(V_r^4)$ of the tested system.

Approximation of this dependency by the linear function of the form (28) gave the results:

$$l_0 = 58.48 \pm 0.054[1/\mathrm{s}^2] \quad l_1 = 124.8 \pm 2.35[1/\mathrm{s}^2] \qquad (31)$$

Then, based on the relations (29) the following values have been obtained:

$$\chi_0 = 25.98 \pm 0.47[\mathrm{Ns/m}] \quad \chi_3 = 79.02 \pm 0.75[\mathrm{Ns/m}] \quad (32)$$

7 REMARKS AND CONCLUSIONS

This paper presents a proposal for a procedure to determine the mathematical form of the function that describes the dissipative properties of the materials. It has been assumed that these properties can be determined by an analysis of the vibration damping of the concentrated mass *m* that is attached to the complex mechanical vibrating system using the tested material. The proposed procedure is based on the two independent methods of identification. The first of these methods belongs to the group of so-called nonparametric methods [25, 26]. It allows to define the shape of the function that describes the damping force as a function of velocity and to check if this force does not depend on the displacement. The second method is based on the frequency characteristics – on the dependency of the amplitude of velocity and the frequency in the specially defined, by equation (25), resonance of the relative displacement. This is a typically parameter method in which it is assumed that the shape of the function describing the effect of the dissipative forces is already known. Both of the presented methods are complementary and the nonparametric method should be considered as the initial method.

Both methods refer to the identification of the relative vibration's damping of the simple single-mass system attached to the complex system with many degrees of freedom and an unspecified construction (see Figure 2). It should be stressed that the type and the structure of this system is not relevant in the case of the methods being used. It is however important that the point *A* should perform an oscillating movements under the influence of the excitations applied to the complex system. Additionally, both methods can also be used in the absence of the complex system. In this case, the point *A* is motionless, the tested system is a system with one degree of freedom and the exciting force should be applied directly to the mass *m*. Example of such non-parametric method has been already presented by the authors in the separate article [24].

Experimental verification tests described in the section 6 of this paper were conducted for the case in which the complex oscillating system consists of two masses only and is linear. Conducting similar studies for more complex systems seems to be important and is already planned by the authors. However, at this stage of the research it can be already seen that the resonance method gives less accurate results than the non-parametric method. This applies especially to the parameter κ_0 describing the linear part of the damping function. The numerical value obtained by this method ($\chi_0 = 25.98$) is significantly different than the reference value ($\chi_0 = 10$). This is mainly because of the shape of the dependence $l_0(\chi_0)$ of the form (29). It can be easily calculated that the error in estimating the value of l_0 causes about ten times higher error in the value of χ_0. May be noted that the exact value of the constant l_0, for selected numerical values: c = 900 N/m, m = 15 kg, χ_0 = 10 Nm/s should be based on formula (29), have a value of l_0 = 59.78. This value in comparison with the value of l_0 = 58.98 is not a big error (it is Δl_0 = 1.3 which is approximately 2.2 % of the value of l_0). Meanwhile, the results of the first method (see Figure 8) allow to determine the true value of χ_0 equal to 10 Nm/s.

Another thing which is worth emphasizing is that the frequency characteristics shown in the Figure 9 are similar to the frequency characteristics of the Duffing system of the soft characteristics of elasticity (resonant frequency decreases with the increase of the excitement amplitude) [27, 28, 29]. Observation of these characteristics can lead to the conclusion of non-linear characteristics of elasticity of the tested system. It would lead to the false conclusions about its behavior under different dynamic loads (e.g. transition to chaos [30]). This implies that the first method is important from the point of view of modeling and identification of real mechanical systems.

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