ABSTRACT: The present work investigates the dynamic response of a fixed–bottom offshore wind turbine subjected to the combined wind-waves action employing different nonlinear wave kinematic models. Linear, 2nd-order and fully nonlinear models are implemented in the hydrodynamic module of a global hydro-aero-servo-elastic solver. All the wave models are based on the potential flow assumption. A first study of the structural response is performed in regular waves with increasing steepness considering the turbine both in parked condition and in power production. A more realistic simulation is then carried out with irregular waves and turbulent wind. Hydrodynamic loads associated to the three wave models are coupled with aerodynamic loads acting on the rotor of a 5-MW wind turbine. Hydro-aero-elastic calculations are performed using the NREL software FAST. The paper shows that from wave steepness $ka = 0.1$ on the 2nd-order model becomes inaccurate. It underestimates the structural loads as well as the resonant oscillations of the tower caused by the higher-order components.

KEYWORDS: Offshore Wind Turbines; Nonlinear waves; Structural dynamics; Springing.

1 INTRODUCTION

The development of more accurate simulation tools capable of capturing the effects of complex environmental conditions on large multi-megawatt wind turbines are strongly required within reliability–based design framework. A novel numerical package capable of predicting the nonlinear loads acting on offshore wind turbines (OWTs) exposed to nonlinear sea states has been recently proposed in [1, 2, 3]. The model reproduces the structural response by coupling a fully nonlinear wave kinematic solver, [4], [5], [6], [7], [8], [9] and [10], with a hydro-aero-elastic simulator of the entire system [11]. The numerical approach proposed in [1, 2, 3] proved to be computationally very efficient due to a domain decomposition strategy. The fully nonlinear numerical wave solver is invoked only on special sub-domains where nonlinear waves are expected, whereas on the remaining parts of the space-time domain the linear wave theory is assumed. Recently, in [12] it has been shown that 2nd-order wave contributions have significant effects on the loads assessment of OWT. However, in [13] we observed that, even for moderate sea-states, the 2nd-order wave theory may miss important effects on the structural response.

In the present paper, we integrate the linear, 2nd-order and fully nonlinear (without domain decomposition) wave models in a fully coupled hydro-aero-elastic solver and compare the structural response in order to have a clearer picture of the effects that higher-order contributions have as the wave steepness increases. It is performed first a study in regular waves with the turbine both in parked and operating conditions, then an extreme sea state is reproduced with irregular waves.

The paper is structured as follows: in Section 2 and Section 3 the Fully Nonlinear (FNL) and 2nd-order wave models are briefly recalled. Section 4 summarizes the global dynamic model and the main features of the baseline wind turbine used in this study are listed. In Section 5 the effects of the three wave models on the system response are presented and discussed. Finally, in Section 6 the main conclusions are drawn.

2 FULLY NONLINEAR IRREGULAR WAVES MODEL

2.1 Mathematical and numerical formulation

The 2D problem governing the nonlinear propagation of gravity waves is formulated within the potential-flow assumption. For an inviscid fluid in irrotational flow, the potential function $\Phi(t, p)$ describes the velocity field at time $t$ in each point $p \in \Omega(t)$. For an incompressible fluid, Laplace’s equation is valid in the whole domain

$$\nabla^2 \Phi = 0 \quad \forall p \in \Omega(t) \quad (1)$$

The 2D domain $\Omega(t)$ is bounded by four boundaries: inflow $\Gamma_{1i}(t)$, rigid bottom $\Gamma_b$, outflow $\Gamma_{1o}(t)$, and free-surface $\Gamma_f(t)$ (see Figure 1). On each of them, suitable boundary conditions have to be enforced. In particular, the impermeability condition $\nabla \Phi \cdot \vec{n} = 0$ is imposed on the bottom $\Gamma_b$, while the continuity with the linear wave kinematics has to be ensured on $\Gamma_{1i}(t)$ and $\Gamma_{1o}(t)$: finally, nonlinear kinematic and dynamic boundary conditions must be imposed on $\Gamma_f(t)$

$$\frac{D\Phi}{Dt} = \nabla \Phi, \quad \forall p \in \Gamma_f \quad (2)$$

$$\frac{D\Phi}{Dt} = -\frac{p_a}{\rho_w} - g\eta - \frac{1}{2} \nabla \Phi \cdot \nabla \Phi, \quad \forall p \in \Gamma_f \quad (3)$$

where $\vec{r}$ is the position vector of the water particle $p$ and $\eta$ the free surface elevation, $p_a$ and $\rho_w$ denote the atmospheric pressure and the water density, respectively.

![Figure 1: Two–dimensional domain of the time-depending Laplace equation.](image-url)
(MEL) approach, which splits the problem into a kinetic and time evolution sub-parts.

In the Eulerian step, at a given time instant, the solution of the Boundary Value Problem (BVP) for the Laplace equation with mixed Dirichlet and Neumann boundary conditions is reformulated in an integral representation of the velocity potential (using Green’s second identity). The boundary integral equation is discretized in space by means of quadratic isoparametric boundary elements [14]. We refer to [1, 15] for additional details.

The Lagrangian step (time evolution) consists in the time integration of Eqs. (2) and (3) where the potential and the free-surface profile at the new time step $t + dt$ are provided. The fourth-order Runge-Kutta (RK4) algorithm is used for the time integration scheme.

3 SECOND ORDER NONLINEAR IRREGULAR WAVES

The 2nd-order wave theory is implemented starting from the second order velocity potential. To solve Laplace’s equation the 2nd-order wave theory is implemented starting from the Boundary Value Problem (BVP) for the Laplace equation with mixed Dirichlet and Neumann boundary conditions is reformulated in an integral representation of the velocity potential (using Green’s second identity). The boundary integral equation is discretized in space by means of quadratic isoparametric boundary elements [14]. We refer to [1, 15] for additional details.

The numerical implementation of the 2nd–order theory (as well as the third order) is performed using the Inverse Fast Fourier Transform (IFFT). First–order terms, that involve single summations, require the definition of the first–order coefficients, see [12]. The horizontal particle velocity and the horizontal particle acceleration may be obtained from the sum of the first–order wave components used to discretize the wave spectrum:

\[ \eta_1(t) = \sum_{m=1}^{N} A_m \cos(\omega_m t - \phi_m) \]  

The 2nd-order term is obtained as a sum of differences and sums of frequencies as follows:

\[ \eta_2(t) = \eta_2^-(t) + \eta_2^+(t) \]  

\[ \eta_2^\pm(t) = \sum_{m=1}^{N} \sum_{n=1}^{N} \left[ A_m A_n \left( B_{mn}^\pm \cos((\Psi_m pmn + \Psi_n pmn) - \phi_m - \phi_n) \right) \right] \]  

where $\Psi_m pmn = \omega_m t - \phi_m$ and $\Psi_n pmn = \omega_n t - \phi_n$, $A_m$, $A_n$, $\omega_m$ and $\omega_n$ are, respectively, the amplitude, the frequency, and the circular frequency of the m–th and n–th wave components. $B_{mn}^+$ and $B_{mn}^−$ are the second–order transfer functions [16]. Similarly, the second–order velocity potential is obtained as a sum of a first order and a second order term $\Phi(t) = \Phi_1(t) + \Phi_2(t)$. $\Phi_2(t)$ is defined by differences and sums of frequencies: $\Phi_2 = \Phi_2^+ + \Phi_2^−$, where

\[ \Phi_2^\pm = \frac{1}{2 \pi} \sum_{m=1}^{N} \sum_{n=1}^{N} \left[ A_m A_n g^2 \cosh(k_{mn}^\pm (h + z)) \right] \]  

\[ D_{\pm} \sinh(\phi_{\pm} \pm \psi_{\pm}) \]  

In the above equation $k_{mn}^\pm = |k_m \pm k_n|$ and $D_{\pm}$ are transfer functions (not reported here, see [12]). The horizontal particle velocity and the horizontal particle acceleration may be obtained with the second order velocity potential by taking the gradient of $\Phi$.

3.1 Numerical implementation

The numerical implementation of the 2nd–order theory (as well as the first order theory) may be efficiently performed using the Inverse Fast Fourier Transform (IFFT). First–order terms, that involve single summations, require the definition of the first–order coefficients, see [12].

Using the IFFT, in order to get a result compatible with the period of the simulation desired, a correct discretization of the wave spectrum is needed. Let $T_{sim}$ and $N$ be the simulation time and the number of samples, respectively. The time increment is $\Delta t = T_{sim}/(N - 1)$, while the time vector is defined as $t_p = p\Delta t$ with $p = 0, 1, 2, ..., N - 1$. The increment of the wave circular frequency used to discretize the wave spectrum is $\Delta \omega = 2\pi/(\Delta t N)$ and the frequency vector is defined as $\omega_m = m\Delta \omega$ with $m = 0, 1, 2, ..., N - 1$. In this way the IFFT returns a correct result compatible with the simulation time $T_{sim}$ and with the number of samples $N$.

Note that before using the IFFT to compute the 2nd–order terms, Eq. (5) is rewritten as

\[ \eta_2^\pm(t_p) = \frac{1}{2 \pi} \sum_{m=1}^{N} \sum_{n=1}^{N} \left[ A_m A_n B_{mn}^\pm \exp(-i(\omega_m pmn + \omega_n pmn) t_p) \right] \]  

which

\[ X_{mn}^\pm = A_m A_n B_{mn}^\pm \exp(-i(\phi_m pmn + \phi_n pmn)) \]  

The second order wave kinematics components may be evaluated using the same procedure and changing only the expression of the Fourier coefficients. For a complete description of the model and for the details of the numerical implementation we refer to [12].

4 GLOBAL SOLVER

Aero-hydro-servo-elastic computations are carried with FAST [11]. FAST is based on a combined modal and multibody dynamics formulation and can be used to model the rigid and flexible bodies of a wind turbine. Aerodynamic loads acting on the blades are calculated by means of the Blade-Element Momentum theory using the software AeroDyn [17], [18]. Hydrodynamic loads are computed within the FAST by means of the Morison equation [19]. These loads are made of two contributions: the viscous and the inertial terms. In addition to the nonlinear kinematics, we point out that the square of the water velocity in the viscous term also introduces higher-order components: whereas, the inertial term is purely affected by the nonlinear wave kinematics. Turbulent wind is generated with TurbSim [20].

The turbine model used in this study is the 5-MW Reference Wind Turbine for Offshore System Development [21], whose main characteristics are listed in Table 1. The diameter and the wall thickness vary linearly with the tower height. The base diameter of 6 m is equal to the diameter of the monopole. A water depth of 20 m is considered in all the simulations presented in the next section.

Table 1: Key properties of the NREL 5–MW Baseline Wind Turbine

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating Power</td>
<td>5 MW</td>
</tr>
<tr>
<td>Rotor Orientation, Configuration</td>
<td>Upwind, 3 Blades</td>
</tr>
<tr>
<td>Rotor, Hub Diameter</td>
<td>126 m, 3 m</td>
</tr>
<tr>
<td>Hub Height</td>
<td>90 m</td>
</tr>
<tr>
<td>Cut-In, Rated, Cut-Out Wind Speed</td>
<td>3 m/s, 11.4 m/s, 25 m/s</td>
</tr>
<tr>
<td>Cut-In, Rated Rotor Speed</td>
<td>6.9 rpm, 12.1 rpm</td>
</tr>
<tr>
<td>Rated Tip Speed</td>
<td>80 m/s</td>
</tr>
<tr>
<td>Rotor Mass</td>
<td>110 t</td>
</tr>
<tr>
<td>Nacelle Mass</td>
<td>240 t</td>
</tr>
<tr>
<td>Tower Mass</td>
<td>347 460 t</td>
</tr>
<tr>
<td>Pile length, diameter</td>
<td>30 m, 6 m</td>
</tr>
<tr>
<td>Tower top diameter, wall thickness</td>
<td>3.87 m, 0.019 m</td>
</tr>
<tr>
<td>Pile wall thickness, total weight</td>
<td>0.060 m, 187.90 t</td>
</tr>
</tbody>
</table>

5 EFFECTS OF WAVES MODELS ON THE TURBINE RESPONSE

5.1 Regular waves

We first address the case of regular waves. Regular waves are generated with circular frequency $\omega = 0.6283 \text{ rad/s}\ (T = 10 s)$ and increasing steepness $ka$ from 0.05 to 0.3 with increment of 0.05. Simulations have been performed considering two configurations: i) parked condition, that is the rotor idles with blades pitched to feather (at a pitch angle of 90°) and no wind is
blowing; ii) power production, that is rotor speed of 12.1 rpm with blades pitched to 17.5° and a hub-height constant wind of 20 m/s is generated. The second configuration is meant to reproduce, at least in terms of global damping, a realistic case when the turbine is operating [1]. A total simulation time of 30 T is considered for each condition.

Figures 2 shows the time series of the wave elevation (WaveElev), tower-base fore-aft bending moment (TwrBsMyt) and tower-top fore-aft deflection (TTDspFA) corresponding to the linear, 2nd-order and FNL wave kinematics when the turbine is parked and in power production. The wave steepness is $ka = 0.20$.

The maximum wave elevations obtained with the linear, 2nd-order and FNL models are 3.86 m, 4.80 m and 5.7 m, respectively. In the parked case, linear and 2nd-order wave kinematics provide similar results for the tower-base shear force (not shown in figure) and bending moment. In contrast, the FNL produces an increase of about 20% on TwrBsFxt and 50% on TwrBsMyt. The maximum tower motion increases dramatically from approximately 2 cm (linear and 2nd-order) to 10 cm (FNL). A clear low-frequency component of 0.02 Hz is observed. This contribution is much more relevant in the FNL case because it is associated with the difference frequency $3f - f_n$, where $f = 0.1$ Hz is the wave frequency and $f_n = 0.28$ Hz is the first natural bending frequency of the tower. Conversely, linear and 2nd-order models give a very small contribution at the frequency $3f$, which is mainly related to the viscous drag contribution in the Morison equation.

When the blades rotate under a constant wind speed, TwrBsFxt (not reported here) and TwrBsMyt maximum values increase both of about 15%. Due to the rotor thrust caused by the wind action, the tower top oscillates around a mean value of 22.7 cm. The FNL minimum peak is 15.5 cm while the linear and 2nd-order minimum peaks are 19 cm. We observe that the low-frequency oscillation, characterizing the tower motion in parked condition, almost disappears in the present case.

5.1.1 Power Spectral Densities

The power spectral densities (PSDs) of the wave elevation, tower-base fore-aft bending moment and tower-top fore-aft deflection corresponding to the linear, 2nd-order and FNL wave kinematics when the turbine is parked (upper group) and in power production (lower group) are shown in Figure 3. The wave steepness increases from 0.05 to 0.30. For $ka = 0.05$ no differences exist in the PSDs of the structural loads by using the three different models for the wave kinematic as well as by changing the operating conditions.

In the PSD of the sea surface elevation process, the dominant peak occurs at 0.1 Hz (since $T = 10$ s). As the steepness increases, peaks gradually appear at $2f$ and $3f$, that is at 0.2 Hz and 0.3 Hz.

The system response has a main peak at 0.1 Hz. At $ka = 0.05$ (Figure 3(a)), the nonlinear load contributions are negligible as a consequence of the almost linear forcing. Increasing the $ka$, the nonlinearities in the structural loads become relevant. An evident peak at the natural frequency of the structure, i.e. at $f_n = 0.28$ Hz, appears in the parked case, while it vanishes in the power production. This behavior, studied in [1], is related to the aeroelastic damping induced by the rotation of the blades. However, we anticipate that the peak in the TTDspFA PSD at $f_n$ (see Figure 3(a) parked case) is order of magnitudes smaller than the one that would be caused by the nonlinear wave kinematics (see Section 5.2).

Still in parked case, as the steepness increases, more and more energy is provided at $3f$, therefore, as confirmed in Figures 3(b)-3(d) (see PSDs of TTDspFA) the peak moves gradually from $f_n$ to $3f$, that is from 0.28 Hz to 0.3 Hz.

When the turbine is in power production (see Figure 3 lower panels) the transient behavior dissipates quite soon and the system responds at the loading frequencies. We only observe an augmentation of the energy at the frequencies $2f$ and $3f$ as the steepness increases. Note that to facilitate the comparisons among the figures, the minimum frequency is 0.05 Hz, so that the PSD at 0.02 Hz is not shown in these plots.

5.1.2 Amplitudes vs. wave steepness

The response amplitudes at given frequencies are plotted against the increasing steepnesses in Figure 4. The figure shows how the amplitudes of TwrBsFxt, TwrBsMyt and TTDspFA at 0.1, 0.2 and 0.3 Hz are influenced by the steepness of the incident wave system. No substantial differences between the parked (not shown here) and the power production configurations are observed.

At 0.1 Hz the amplitudes are overestimated by the linear and weakly nonlinear wave models (see Figure 4(a)). At 0.2 Hz the linear and 2nd-order models give the same results, while the FNL model captures a significant increase of the amplitudes for all the three response channels (Figure 4(b)).

The estimated amplitudes of TwrBsFxt, TwrBsMyt and TTDspFA at 0.2 Hz are the same when the linear or 2nd-order models are used. This happens because at $2f$ wave kinematics nonlinearities are negligible with respect to the 2nd-order components introduced by the square of the velocity in the viscous term of Morison’s equation. Thus, both linear and nonlinear wave models undergo the same growth as the steepness increases. On the contrary, the FNL model shows a remarkable increase due to the interaction of the 3rd-order components ($3f - f$), see Figure 4(b).

At 0.3 Hz, see Figure 4(c), the 2nd-order model differs from the linear one due to the interaction $2f + f$, however it underestimates the actual increase of the amplitudes due to the wave steepening. We point out that after $ka = 0.2$ the growth rate of the FNL curve becomes smaller. This may be justified by the frequent use of smoothing and regridding in the FNL numerical solver to avoid wave breakings. Above $ka = 0.2$, the solution may result not as much accurate as below 0.2.

5.2 Irregular waves

We consider an extreme sea state characterized by $H_s = 9$ m, $T_p = 11.8$ s and a mean hub-height wind speed $U = 33$ m/s [22]. Given the wind speed above the cut-out level, the turbine is set in parked condition. A total simulation time of 300 s is reproduced.

Figure 5 shows time series of the hub-height longitudinal wind velocity (WindVXi) (top panel), the wave elevations (WaveElev) (second panel from the top) and the system response: tower-base shear (TwrBsFxt) and bending moment (TwrBsMyt) and, in the bottom panel, the tower-top fore-aft displacement (TTDspFA).

Figure 6 shows the estimated power spectral density functions of WaveElev, TwrBsFxt, TwrBsMyt, and TTDspFA. In the surface elevation PSD the dominant peak occurs at 0.085 Hz (since $T_p = 11.8$ s). At this frequency, the PSD of the tower-base fore-aft shear force (TwrBsFxt) is overestimated by the linear and 2nd-order wave models. Since the turbine is parked, the dominant loads are hydrodynamic; thus, the tower-base shear force mainly reflects the effect of these hydrodynamic forces.

Moreover, due to the large lumped mass at the top of the tower (with an associated large moment arm), the tower-base bending moment PSD follows the tower-top motion, with a smaller amount of energy associated with hydrodynamic loading at frequencies around 0.85 and 0.17 Hz. Peaks in the PSDs of the tower-base bending mode (TwrBsMyt) and tower-top fore-aft deflection (TTDspFA) exhibits significant amplifications at the tower fundamental frequency when the FNL model is used. These amplifications are justified by the springing-like vibrations clearly visible also in the time series of Figure [?]. In this case the structure is excited by a spectrum of wave components causing the excitation of its natural frequency. In the previous case of regular waves, we observed a peak at 0.3 Hz (see for instance Figure 3(d)) that here is not present. Higher-order wave components (captured by the FNL model) cause a very strong
amplification of the response at 0.28 Hz. The structure experiences resonant vibrations such that the PSD at 0.28 Hz is one order of magnitude larger than in the case of regular waves. For this reason the peaks at the sea fundamental frequencies are very much attenuated in the PSD of TwrBsMyt and no longer visible in the TTDspFA PSD, see Figure 6.

6 CONCLUSIONS

A comparison of the effects of linear, weakly and fully nonlinear regular and irregular wave models on the dynamic response of a fixed-bottom offshore wind turbine has been presented. In regular waves, for very small wave steepnesses, the structure responds with the same amplitudes at the main wave frequency regardless of the wave theory used. As the steepness increases, the structure responds also at the higher-order loading frequencies. In this case, differences between linear, 2nd-order and FNL models become dramatically relevant. From $ka = 0.01$ on, the linear and 2nd-order theories become inaccurate. The structure responds more and more at the loading frequencies of $f, 2f$ and $3f$ so that the PSDs between the parked and operating condition assume the same shape.

The structural response under a severe sea state with irregular nonlinear waves shows that when the turbine is parked, it becomes extremely sensitive to resonant vibrations. The nonlinear kinematics excites the first natural frequency of the tower causing a very high peak in the tower motion PSD. This peak, associated with springing-like phenomena, is remarkably underestimated (more than 50% less) by both the linear and 2nd-order wave models.

A more comprehensive study aimed at establishing the effects on the long-term design loads as well as at assessing the accumulated fatigue damage (considering other sea states) when linear, 2nd-order and FNL models are employed will follow. The large differences between the FNL and 2nd-order models is expected to have great importance on these structural aspects.

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REFERENCES


Figure 2: Time series of the wave elevation (first panel from top), tower-base fore-aft bending moment (TwrBsMyt) and tower-top fore-aft deflection (TTDspFA) for the parked (second and third panels from top) and operation (forth and fifth panels from top), corresponding to the linear (blu dotted line), 2nd-order (green dashed line), FNL with (red solid line) wave kinematic models with $\kappa\alpha$ 0.20.
Figure 3: Spectra of the wave elevation (first row), tower base fore-aft bending moment and tower-top fore-aft deflection for the parked (second and third rows) and operation (forth and fifth rows) conditions corresponding to the linear (blue dashed dotted), second-order (green dashed), FNL with (red solid) wave kinematic models with increasing $ka$.

(a) $ka = 0.05$.  
(b) $ka = 0.10$.  
(c) $ka = 0.20$.  
(d) $ka = 0.30$.  

Parked case.  

Power production.
Figure 4: Amplitudes vs. wave steepness of tower-base shear force and bending moment (TwrBsFxt, TwrBsMyt) and tower-top fore-aft deflection (TTDispFA) corresponding to the linear (blue with stars), 2nd-order (green with circles) and FNL with (red with crosses) wave kinematic models at different frequencies with the blades rotating under a steady hub-height wind speed (constant pitch).
Figure 5: Time series of the hub-height longitudinal wind velocity (WindVxi), wave elevation (WaveElev), tower base fore-aft shear force (TwrBsFxt), tower base fore-aft bending moment (TwrBsMyt) and tower-top fore-aft deflection (TTDspFA), corresponding to the linear (blu dotted), 2nd-order (green dashed), FNL (red solid) wave kinematic models.

Figure 6: PSDs of the wave elevation (WaveElev), tower base fore-aft shear force (TwrBsFxt), tower base fore-aft bending moment (TwrBsMyt) and tower-top fore-aft deflection (TTDspFA), corresponding to the linear (blu dotted), 2nd-order (green dashed line), FNL (red solid line) wave kinematic models.