Dynamic method for measuring critical buckling load of sway frames

Boris Blostotsky1, Elia Efraim1
1Department of Civil Eng., Faculty of Engineering, Ariel University, 40700 Ariel, Israel
email: bx@ariel.ac.il, efraime@ariel.ac.il

ABSTRACT: The critical sway buckling load is the upper limit of the allowed vertical frame loading. This load is defined by the frame configuration, cross-sections of the columns and horizontal beams as well as configuration and stiffness of the connections between them. In known non-destructive methods of determining the side-sway frames critical load an implementation of vertical loading is technically difficult that is especially essential at full-size tests. The value of the applied load is limited also by significant deformation of the frame approaches the critical load value.

The paper deals with a new non-destructive method for experimental determining the critical sway buckling load of frames. For ensuring mobility of the frame in the sway mode the vertical loading is carried out by the flexible support between the frame beam and fixed base through the loading device. The critical vertical load of the “frame–traction element” combined system is higher than of single frame. This fact allows to load the system with loading that corresponds to critical load of a frame without loss of system stability. Procedure of the experiment includes: determining of free vibration natural frequency of the combined system “frame–traction element” for different tensions of traction element, determining the lateral stiffness of the system, calculation the corresponding stiffness of the frame and its critical load. The accuracy of the method is verified by comparison the experimental and theoretical results.

KEY WORDS: Dynamic test; Critical load; Sway mode; Buckling.

1 INTRODUCTION

Testing of portal frames in sway mode is performed in order to determine the maximal allowable load limited by plastic deformations of frames elements or their connections [1,2] and a critical load limited by frames stability [3,4]. Motivation to determine accurate value of critical force is caused by its using as follow:
- critical load is maximal allowable load for given configuration of frame and can be used as criterion of perfection or efficiency of the structure;
- it is used in calculation prescribed by standards for design of structural elements under combined action of compression and bending [5];
- it is used for verification of theoretical analysis of stability at various end conditions of frames columns [6].

In the dynamic method of non-destructive testing of frames for critical load [7,8,9] the relation of instability to structural stiffness is used. Many theoretical and experimental investigations confirm decreasing of free vibration frequency with increasing of compression forces [10]. The critical load determined based on frequency measurement results for various loads by extrapolation up to null frequency according to linear dependence of frequency squared on compression load. At frames testing in sway mode the vertical loading is performed by using of additional weights [9,11], directly via a lever system [12] or using special mechanism (gravity load simulator) [13].

In the present work a new non-destructive method for determination of frames critical buckling load in side-sway mode is proposed. The method allows performing measurements during the tests under loads that exceeds the frame critical loads without losing its stability. Technological advantage of this method is the ability to create loading correspond to critical loads of real full-size frames.

The aim of the present work is developing of more exact non-destructive experimental method for determination of critical load of frame in sway mode. Increasing in accuracy is achieved by conducting experiments in diapason of vertical loads includes the critical load as well as by using of loading system without friction.

2 THE IDEA AND DESCRIPTION OF THE METHOD

Figure 1 presents the setup scheme for testing of sway frames with columns (A) fully restrained at the base and with hinged or fixed attachment to girder (B). The traction element (C) (cable or rod) has a pinned connections to the base and the frame's beam and includes the device (D) for producing of tension force. Since the lateral vibrations of the frame cause vertical displacements of the girder, maintaining a stable tension is achieved by using hydraulic or pneumatic loading device or by including additional spring element in case of mechanical loading device (D).

The experiment consists in determining the natural frequency of the system vibrations at different values of tension force in traction element and accordingly compression forces in the frame columns. Adopting the lateral movement of the frame girder as degree of freedom, the generalized mass M' of the system can be presented in the following form:

\[ M' = M_f + 2M_e' + M_c' + M_a' \]  \hspace{1cm} (1)

where \( M_f \) - the mass of the frame girder;
**M'c** - the generalized column mass;

**M'e** - the generalized mass of traction element including tension device and flexible rod or cable;

**M'a** - the additional mass attached for increasing the generalization accuracy.

The main inaccuracy of mass generalization consists in determination of generalized column mass that depends on shape of vibration mode [14]. For columns with constant cross-section

\[ M'_c = 0.240 M_c \]

for rigid pinned end conditions, and

\[ M'_c = 0.365 M_c \]

for rigid connection at both ends. These values are obtained based on form of static deformation under uniformly distributed load. The influence of the error in calculated generalized column’s mass can be reduced by increasing translational moving mass by adding of mass **M'a** to a girder (Figure 1).

The stiffness of the system "frame–traction element" is calculated based on measured natural frequency as follows:

\[ K_f \approx \frac{\pi^2 M_f}{\omega^2} \]

The stiffness of the frame only **K** is calculated based on correlation with systems stiffness. The frames critical load determined based on frame's stiffness values.

### 3 DEPENDENCE OF THE FRAME STIFFNESS AND SYSTEM STIFFNESS ON VERTICAL LOAD

In figure Figure 2 the calculation scheme for stiffness and critical loads of the system "frame–traction element" is presented for case of frame with rigid girder and rigid connection of column with the base and pinned or rigid connection with the girder. The general load of the system includes a weight load **Q** of system elements that causes initial axial compression in the columns, and a tensile force **T** in the traction element. The axial load on each column is

\[ P_i = \frac{P}{2} = T/2 + Q/2 \]

where **P** is general load on the frame.

The lateral stiffness of system "frame–traction element" is determined by lateral displacement due to unit lateral force taking into account changing of direction of traction force **T** caused by lateral displacement of frame's girder. The lateral stiffness of the frame as the part of the loaded system is determined by lateral displacement of the girder due to unit lateral force with constant vertical direction of traction force.

\[ K'_f = K_f \frac{H^3}{EI} \]

where **H** - column height, **E** - the elasticity modulus, **I** - moment of inertia of columns cross-section. For rigid-rigid boundary conditions at columns ends the solution of elastic deformation equation is

\[ K'_f = 2\pi^2 \frac{L^4}{H^2} \left(1 - \frac{H}{L^2} \right) \]

The lateral stiffness of frame with rigid connection at both columns ends can be obtained from solution (7), when the condition of constant vertical direction of traction force assigned by \( L \rightarrow \infty \):

\[ K' = 2\pi^2 \frac{L^4}{2\tan(H/2)} \]

Figure 3 presents graphs of dependencies non-dimensional stiffness characteristics **K'_f** of the system "frame–traction element" and **K'** of the frame alone (when \( H/L=0 \)) on non-dimensional parameter \( \frac{H}{L} \) and **u'** ratio for rigid-rigid end conditions.
The values of \( K_s \approx K^* = 24 \) at \( u=0 \) are correspond to lateral stiffness of frames without axial forces in columns (case of \( Q = 0 \) and \( T = 0 \)).

The values of \( K_s \) and \( K^* \) at \( u=u_0 \) are correspond to lateral stiffness of frame under structural weight load only. The intersections of the graphs with \( u_0 \) axis determine the values \( u_{s,cr} \) and \( u_t \) that correspond to the critical loads of system and frame respectively.

The critical force \( P_{s,cr} \) in the columns and the tension force \( T_{s,cr} \) in traction element that correspond to the critical state of the system in sway mode are calculated from (5) and (3) when \( u=u_{s,cr} \). The critical force \( P_{cr} \) of frame only is determined by (5) when \( u=u_{s,cr} \).

An equation for determination \( u_{s,cr} \) for rigid-rigid end conditions followed from (7) satisfying conditions \( K^* = 0 \) [16] and has the following form:

\[
\frac{u_{s,cr}}{2} + \frac{H}{L} \left( 1 - \frac{u_0^2}{u_0^2} \right) \left( \frac{u_{s,cr}}{2} - \frac{u_{s,cr}}{2} \right) = 0
\]  

(9)

Solution of (8), that corresponds to null stiffness of frame, gives the critical value \( u_{s,cr} = \pi \) for frame with rigid-rigid connections (for frame with rigid-pinned connections \( u_{s,cr} = \pi/2 \)). The roots of equation (9) depend on two parameters: \( u_0 \) - non-dimensional parameter of initial weight load \( Q \), and \( H/L \) ratio - non-dimensional parameter of the system "frame–traction element". Taking the initial load on column as a part of its critical load as

\[
Q = q \cdot P_{cr}
\]  

(10)

we have for rigid-rigid end conditions

\[
u_0 = \sqrt{q \cdot \pi}
\]  

(11)

The values of \( u_{s,cr} \) for \( q=0, 0.5 \) are presented in table Table 1.

According to determined values of parameters \( u_{s,cr} \) and \( u_0 \) and given value of weight load non-dimensional parameter \( u_0 \), the domain of dependency (8) for \( K^* \) is \( u \geq 0 \) and domain for \( K_s \) from (7) is \( u_{s,cr} \geq u \geq u_0 \).

The case of \( H/L=0 \) corresponds to permanent vertical direction of the tensile force \( T \), so there is no dependence of the roots values \( \pi \) and \( \pi/2 \) on initial weight load \( q \) and they equal to the critical load parameters of the frame alone.

### Table 1. The values of non-dimensional parameter \( u_{s,cr} \) of system "frame–traction element" for different values of \( H/L \) ratio and two states of initial weight load.

<table>
<thead>
<tr>
<th>Column ends conditions</th>
<th>H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>rigid-rigid</td>
<td>q=0</td>
</tr>
<tr>
<td></td>
<td>q=0.5</td>
</tr>
<tr>
<td>rigid-pinned</td>
<td>q=0</td>
</tr>
<tr>
<td></td>
<td>q=0.5</td>
</tr>
</tbody>
</table>

When \( u_{s,cr} < u < u_{s,cr} \) the load acting on the system larger then frame's critical load. During the vibrations the forces acting on girder from columns coincide with direction of girder's displacement [16], the frame stiffness is negative [15]. The system's stability is provided by lateral component of inclined force in traction element that larger than columns' reaction. At \( u=u_{s,cr} \) the frame's reactions are equal and opposite to the lateral component of the tensile force and this state is limit of system stability. For example, for rigid-pinned or rigid-rigid columns connections according to roots values presented in Table 1 for \( H/L=1 \) and \( q=0.5 \) \( P_{s,cr}/P_{cr}=2.78 \) and for \( H/L=2 \) and \( q=0.5 \) \( P_{s,cr}/P_{cr}=7.29 \).

Stability of the system at loads which exceed frame's critical value allows testing the frames under load that closer or exceeds its critical buckling load. Limitations of this possibility are absence of plastic deformations in frame elements or in connections between them and buckling of columns in non-sway mode.

### 4 DETERMINATION OF CRITICAL LOAD BASED ON TESTING RESULTS

As testing results the values of lateral stiffness of the system "frame–traction element" are determined for different values of tensile force in traction element and as a consequence vertical load on the columns. Based on these values determining of the frame critical load is required. If the testing results include measurements at loads exceeding the frame's critical load, the last one is determined by interpolation of the results. Otherwise – by extrapolation. In both cases this value is defined as load value corresponding to null stiffness [15] [16]. For developing the mathematical model the expression for \( K_s \) from (7) taking into account (3) with (8) can be presented as follow:

\[
K_s = K + \frac{T}{L}
\]  

(12)

The physical meaning of (12) is that lateral stiffness of the system is the sum of stiffness of the frame and the flexible traction element. According to (12) for each experiment based on measured values of \( K_s \) and \( P \) the frame lateral stiffness is obtained as

\[
K = K_s - \frac{T}{L}
\]  

(13)

Figure 4 presents the dependence graph of lateral stiffness ratio \( K/K_s \) on non-dimensional load parameter \( P/P_{cr}=(u/l)^2 \).
The graph illustrates the possibility of approximation by straight line that cross over the point (0,1) on stiffness axis and point (1,0) on load axis that is described by following equation:

\[
\frac{K}{K_0} = 1 + \frac{P}{P_{cr}}
\]

The relative deviation of linearly approximated values \(K/K_0\) from the exact theoretical values for all values \(K/K_0\) in loads range \((0,1)\), not exceed 1.4%, and for loads range \((0,1.2)\) not exceed 1.8%.

5 TESTING OF THE "FRAME – TRACTION ELEMENT" SYSTEM

The objectives of this investigation:

- verification of frames lateral stiffness dependency on load with constant vertical direction \(0<P<P_{cr}\);
- verification of lateral stiffness dependency of frame and system under loading by flexible traction element;
- precision assessment of determining frame critical load;
- accuracy assessment of determining frame critical load

The experimental setup (Figure 5) consist of two column made of aluminium sheet with thickness 20 mm, thickness 2 mm and height 550 mm, rigidly connected at the base. At the upper ends the columns are rigidly connected to the girder with mass 152 gr, the bending stiffness of which significantly greater than columns stiffness. The flexible traction element pin-jointed to the girder and system's mass, including masses of all movable system elements, generalized to the degree of freedom of the girder's lateral movement is 235 gr. The generalized mass of the frame alone is 195 gr.

Investigation of stiffness dependence on variable load with constant vertical direction is conducted by loading the girder by variable weight [9]. Linear extrapolation based on results of measured stiffness using least squares method up to value of \(K=0\) give 51.9 N. The deviation of results from regression line not exceeds 3%.

The graph illustrates the possibility of approximation by straight line that cross over the point (0,1) on stiffness axis and point (1,0) on load axis that is described by following equation:

\[
\frac{K}{K_0} = 1 + \frac{P}{P_{cr}}
\]

The relative deviation of linearly approximated values \(K/K_0\) from the exact theoretical values for all values \(K/K_0\) in loads range \((0,1)\), not exceed 1.4%, and for loads range \((0,1.2)\) not exceed 1.8%.

In case of \(H/L=0.75\) \((L=940\, \text{mm})\) with increasing of loading the system natural frequency and stiffness are decrease. The system is stable under maximal load that exceed the critical load of frame alone. The dependence \(K(P)\) coincide linear with aforementioned accuracy of linear approximation including the range of negative stiffness. In case of \(H/L=1.96\) \((L=280\, \text{mm})\) with increasing of loading the system's natural frequency and stiffness are increase.

Assessment of accuracy and reliability of critical load determination is conducted by comparison of experimental values obtained as results of three types of loadings and calculated theoretical values. The elasticity module that determined by measuring of stiffness of console rod is 65 GPa. The comparison of results is presented in table Table 2.

From the comparison of experimental values follows that estimation of critical load value \(P_{cr} = 52.25\, \text{N}\) is reliable and sufficiently accurate. Satisfactory value of critical load at constantly vertical loading has been achieved through careful alignment of columns before loading, that allowed loading the frame with load up to \(0.8P_{cr}\). Considerable deviation of theoretical values towards increase is explained by non-conformity of columns end conditions to theoretical perfectly
rigid-rigid due to elasticity of column's connections with the base and the girder.

The constant vertical load produced by the weight of the frame elements and variable load produced by traction element are considered. Analysis of physical states of the system "frame–traction element" depend on the frame loading and ratio of columns heights to traction element length is performed. It is established that depend on this ratio the lateral stiffness of the system can increase or decrease, but it always higher that the frames lateral stiffness.

The system is stable under the loads that exceed the critical one for the frame only. The testing load is limited by buckling of the system, buckling the columns in non-sway mode, plastic deformations of frame elements and their connections. Assessment of accuracy of the linear approximation model of frame stiffness dependence on vertical load has been performed by comparison with its exact solution. Inaccuracy of the linear model of frame stiffness dependence on vertical load not exceed 1.4% in the load range (0-1.0)P_{cr} and 1.8% in the range (0-1.2)P_{cr}.

The proposed method allows improving reliability and increase the determining accuracy of sway frame critical load. Frame loading method using flexible traction element allows produce loads corresponds to critical loads of real full-size frames without special devices for decreasing of friction influence.

**REFERENCES**


