On Measuring the Supply Chain Flexibility

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ABSTRACT

This paper presents some lines of thinking related with the establishment of a concrete mathematical basis to measure the ability of enterprises in a supply chain to maintain equilibrium under unexpected variations. It will be argued that flexibility will be the most appropriate term to classify the concept behind such idea and that agility can be understood as a special case of flexibility. Then, a simple theory of flexibility directed to demand variations in the supply chain is deduced and commented, as well as some results achieved by dynamic supply chain simulation presented and discussed.

1. Introduction

Flexibility and agility are two expressions already well established on the field of Business, even if neither of them had been quantified in the form of a metric that could be applied without misleading by any manager in any part of the world. There is still a diffused and cloudy idea about what in fact each of those terms mean regarding business and management, due to the fact they are multi-variable concepts and normally used to express the same ideas in day-by-day language. Apparently, the handling of such designations depends on the school of thinking as well as on the area of work it is related to. For instance, flexibility is more used in manufacturing, where it means something like the ability to answer fast and reliable to needs of changing in the product mixing, while agility is most of the times associated with supply chain and thought to be something like the ability of fast response to changes in the overall enterprise (Goldman et al., 1995). Probably, one also can recognise in the usage of such terms a certain pattern of the kind which let Wendy Currie* (2000) say “the historical analysis of these change management panaceas shows that eventually their popularity and applicability declines and they are replaced by new panaceas which, although labelled differently, are in many ways similar to the predecessors”, while referring to concepts like TQM, JIT, BPR and PI. The fact is that it continues to be extremely difficult to find a single and wide accepted definition for flexibility and for agility, and even more difficult to find their metrics.

For that reason we decided to start not using such words. Instead we will start using the concept of elasticity, which is already deeply treated on Physics by the Theory of Elasticity, thus meaning it will be understood by all the scientific community.

Then, the supply chain will be considered a mathematical system with the form of a network with nodes and arcs where materials and information flow. In such a system, each node will exhibit a certain ability to return to its equilibrium concerning the flow traversing it (we will only consider the flow of materials), with such flow being highly dependent on the material sink and source policies governing each node. Obviously, the complexity of the problem will exponentially increase with the increase of the number of nodes in the chain and their own complexity. Based on those aspects will try to establish a practical measure of flexibility.

In this paper we first make some comments regarding nomenclature and then we will derive the theoretical basis behind our perspective of measuring flexibility. This has naturally resulted from the need of trying to quantify flexibility while developing a Dynamic Supply Chain Simulator (Feliz, Brito, 2003)2 capable of using such metrics. Finally, we present some practical results obtained by simulating a didactic supply chain case.

2. Some Theoretical considerations

Although the Theory of Elasticity is somehow a field of knowledge demanding the manipulation of advanced mathematics, mainly differential equations, the approach we present here will be less complex than that and somehow a mixture between signal processing and elasticity. Our expectation is that this will turn these ideas wider applicable in practice and will lead to a simple method of measuring the ability of a system to change state while maintaining the equilibrium.

The theoretical basis of elasticity comes from the problem known in literature as the Simple Harmonic Oscillator, which since long time ago is completely described in Physics literature. An excellent and clear description can be found in the book from Murray R. Spiegel, “Schaum’s Outline of Theory and Problems of Theoretical Mechanics with an Introduction to Lagrange’s Equations and Hamiltonian Theory”, McGraw-Hill, 1967.

From reading its 4th chapter one can realize that the elastic forces are in fact forces directed to the state of equilibrium, but also recognize that those forces are more related with the ability to oscillate around such a state than with the ability of returning to it. Elasticity, as a quality derived from the elastic constants of the materials, must then be understood as an ability to oscillate and not precisely as an ability to return to the equilibrium. Figure 1 represents a simple system oscillating around an equilibrium state (Qo) with a certain period (To) of oscillation, period that is dependent on the elastic constant. The higher the value of this constant (or the elasticity) the higher the rate of oscillation and

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Thus the lower the period $T_o$.

\[ \text{Fig. 1 A pure elastic behaviour} \]

But this of course does not serve our intents, and then we simply refuse to use the term elasticity to express the idea of maintaining the equilibrium when a change of state in the supply chain occurs.

However, if we now introduce a certain force of dumping in the Simple Harmonic Oscillator, such a system will in fact start to exhibit a tendency to return to its state of equilibrium, thus behaving much more similar to the effects we are interested to measure concerning the supply chain responsivity. In figure 2 it is shown the three different modes how this new system can approach the new state. They are known in literature as the over-damped mode, the critical-damped mode, and under-damped mode.

\[ \text{Fig. 2 A dumped elastic behaviour} \]

Once in the over-damped mode the system naturally returns to $Q_o$ after the initial perturbation is removed, but this time is longer than the optimal response time and thus it will turn the system low. On the other extreme, when under-damped, it will approach $Q_o$ swinging and thus exhibiting great instability. Anyhow, once critical-damped, the system not only will reach the equilibrium faster but also spending the minimal energy. Systems in such conditions are said to be well-damped or tuned to their characteristic frequency of oscillation $f_o = 1/T_o$.

In our point of view, this dumped behaviour already satisfies a certain idea of flexibility, for it shows how the system will return to the equilibrium after an instantaneous perturbation. Therefore we are tempted to conclude that a system will be more flexible when its Time to Return to Equilibrium ($T_q$) is reduced. At the same time, we conclude that a measure of such a time interval would in fact give us an idea of the flexibility.

Another result of this approach is that agility could be interpreted as a specific behaviour of flexibility, as it points towards the elegancy of movements and thus to the ability of achieving the goals with less energy spending. Thus, agility would be understood as the critical-damped condition of flexibility. These would lead us to the following statements:

The system is agile when it faster recovers the equilibrium without going into oscillations, implying the minimal energy consumption.

or, saying in another way:

The system is agile when operating in a regime or well-dumped flexibility.

3. The metrics for flexibility

In the previous section we have suggested that flexibility would be inversely proportional to the Time to Return to Equilibrium ($T_q$), thus, a simple way to express the flexibility is inverting the things and consider:

\[ \text{FLEXIBILITY} \sim \frac{1}{T_q} \]  

This expression is equivalent to the Rate of Returning to Equilibrium, or, in other words, to the Ability to Absorb Chocks.

This is of course a general definition for flexibility, and so it must hold for any systems as a basic concept, not only for supply chains. Anyhow, the challenge now is to find a way by which flexibility can be measured in practice. For that purpose we will use a simple representation of a state of equilibrium along the time. In fact, if we believe flexibility to represent the ability of the system to maintain its equilibrium, then a way to turn “visible” such ability can be plotting over time or the state itself or the function that represents the equilibrium of that state.

With such intents, let us focus the attention on just a node of the supply chain and on the state of its inventory. Obviously, the node will be in equilibrium (in terms of materials flow) when the rates of input and output of materials will be the same. That is, when:

\[ R_{\text{out}} = R_{\text{in}} \]  

Being $R$ a quantity of materials per unit of time, a way of calculating these quantities in practice will be to consider the amounts of materials ($Q_o$) entering the node during the time interval ($dT_o$) of two consecutive arrivals from the supplier, and the amount of materials ($Q_{\text{out}}$) leaving the node during the time interval ($dT_{\text{out}}$) between consecutive services to the clients. Equation (2) can then be written as:

\[ \left( \frac{Q_{\text{out}}}{Q_{\text{in}}} \right) \left( \frac{dQ_{\text{out}}}{dT_{\text{out}}} \right) = \frac{dT_{\text{out}}}{dT_{\text{in}}} \]  

or, arranged in another way:

\[ \left( \frac{Q_{\text{out}}}{Q_{\text{in}}} \right) \times \left( \frac{dT_{\text{out}}}{dT_{\text{in}}} \right) = 1 = \text{EQUILIBRIUM} \]  

Anyhow, if we look at the node only at the moments the material arrives from the supplier ($dT_{\text{in}} = T_{\text{in}}$), we can say the node will be at perfect equilibrium when the quantity of materials going out equals the quantity of material coming in. In any other circumstances, the node will be out of equilibrium.

Thus, a simple and practical form to represent this equilibrium function is to compute it every time the material arrives, and this will be achieved by counting $Q_{\text{in}}$ and $Q_{\text{out}}$ during each time interval $dT_{\text{in}}$ followed by an operation of division.

It is also important to notice that with this method the function EQUILIBRIUM(t) will be a discrete function “sampled” by the frequency of arrival, as shown in figure 3 (such frequency is given by $f_\text{in} = 1 / T_{\text{in}}$, as it also can be deduced from the same figure). This important aspect shows that the node is not “during all the time connected” to its suppliers, and for that reason it will face “certain” difficulties on following “certain” demand changes, resulting obvious that systems with longer $dT_{\text{in}}$ will have more
difficulties to recover from demand “chocks”. We let to the Supply Chain Managers the investigation on how to reduce $dT_{in}$.

At the moment, and if for simplicity we consider the frequency of arrival to be fixed, the important is to realise that any measure of the Time to Return to Equilibrium ($Tq$) by means of this equilibrium function will lead to values multiples of $dT_{in}$ that is:

$$Tq = n \ dT_{in}$$

(5)

Where $n$ is an integer assuming values 0, 1, 2, 3, etc., representing the number of arrivals from the supplier. Finally, comparing equation (5) with equation (1) we will deduce:

$$FLEXIBILITY \sim \frac{1}{n} \ \frac{1}{dT_{in}} = \frac{1}{n} \ f_{in}$$

(6)

That is, the flexibility will be directly proportional to the frequency the material is arriving from the supplier ($f_{in}$), as well as inversely proportional to the number of such arrivals ($n$). And this is an interesting result, first because it comes close to a range for this metric, saying the flexibility belongs to the interval $[0, f_{in}]$ with an exception of $\infty$ when the perturbation does not affect at all the equilibrium ($n = 0$), and also because it is in accordance with the “empiric” principle of reducing the reorder quantity in warehouses and the lot quantity in manufacturing systems. Actually, these results fit perfect with the common idea that a system will get more flexible as its frequency of replacing material rises. It also results from these considerations that to achieve the optimal flexibility (or the state of agility) in practice one must adjust (by trials?) $f_{in}$ and the reorder quantities in order to minimize $Tq$ without letting the system entering in oscillation. In principle, we believe this can be achieved using the scheme suggested in figure 4 with steps of demand to induce perturbations in the system, by means of simulation.

Although this seems a reasonable proposal to measure the flexibility, it is important to recognize it comes out from a one dimensional analysis of the problem. In fact, it just considers the variable $time (t)$ and completely ignores the variable material quantity ($Q(t)$), and this would mean a system would be the same flexible whether going to equilibrium by moving lots of material or by moving only little quantities of it. Of course, it seems reasonable to consider the second case would be more flexible, despite the same $Tq$. And this reasoning leads us to a final representation of flexibility based on the “energy spent on imbalance”.

To do so, lets notice that equation (3) can also be written in the following form:

$$(Q_{out} / dT_{out}) \cdot (Q_{in} / dT_{in}) = 0$$

(7)

what, considering again $dT_{out} = dT_{in}$, leads to:

$$(Q_{out} \cdot Q_{in} ) / dT_{in} = 0$$

(8)

which is another way of representing the EQUILIBRIUM function for the inventory and can be though as an “Imbalance Ratio”, measured on inventory units (SKU) per unit of time. This already corresponds to a certain “energy” spent per unit of time, and thus gives us an idea of the “Power Spent on Imbalance” or rigidity, which in fact can be seen as the inverse of flexibility. By this definition, the system is more flexible when its rigidity gets low, and a simple way to measure it will be integrating the EQUILIBRIUM function over time and divide its value by the total time of imbalance. Thus, if for a certain period of time $dT_{in}[j]$ it holds that there was $Q_{in}[j]$ units of material going into the node and $Q_{out}[j]$ units of material going out, the rigidity can be calculated as:

$$RIGIDITY = \frac{\sum_j Q_{in}[j] \cdot Q_{out}[j]}{\sum_j dT_{in}[j]}$$

(9)

And then,

$$FLEXIBILITY = \frac{1}{RIGIDITY}$$

(10)

It is based on this measure the matrix of rigidity of a simulated case will be presented later in this paper.

4. Multi-function flexibility of a single-node system

Even considering a single node system, it is important to remember we are suggesting only a way to measure the inventory flexibility in respect to demand variations. Anyhow, in any general single-node system one have to consider a set of output functions $OUT[i](t)$ being driven by a set of input functions $IN[i](t)$, all of them running along a certain independent variable $t$, as depicted in figure 5. That is, we must start with the assumption that any of these functions can be graphically represented in time by some
kind of graph.

Observing this figure, it can be easily understood that in general one talks about flexibility of the \( OUT(t) \) functions in respect to variations of the \( IN(t) \) functions. If for instance a generic output function \( OUT(t) \) is dependent on the generic input functions \( INa(t), INb(t) \) and \( INc(t) \), then one will be automatically able to talk of more than one flexibility, as at least the following three obvious types could be considered:

a) flexibility of \( OUT(t) \) to variations of \( INa(t) \).

b) flexibility of \( OUT(t) \) to variations of \( INb(t) \).

c) flexibility of \( OUT(t) \) to variations of \( INc(t) \).

This means in general one will talk about flexibility of a property in respect to variations of a certain factor. If that property depends on many factors, then many flexibilities can be considered for that property. To turn the things even more complex, one could also think on the input crossed variations...

So, even being the flexibility a simple concept, it usually turns complex if accuracy is important and if the output functions are dependent on more than a single input. This explains why till now many authors try to classify flexibility following the different levels strategy proposed by Leslie K. Duclos (2000), where he obviously assumes flexibility is a multi-dimensional construct.

At the same time, from the point of view presented here, the general flexibility of a node will no longer be a scalar but a matrix connecting the equilibrium functions of the properties (outputs) to the factors (inputs). The flexibility of a general single node system must then be represented by:

\[
\text{FLEXIBILITY} = \begin{bmatrix}
    f_{11} & f_{12} & \cdots & f_{1M} \\
    f_{21} & f_{22} & \cdots & f_{2M} \\
    \cdots & \cdots & \cdots & \cdots \\
    f_{N1} & f_{N2} & \cdots & f_{NM}
\end{bmatrix}
\]  

(11)

Where \( f_{ij} \) is the individual flexibility of the \( OUT(i(t)) \) property in respect to variations of the \( IN(j(t)) \) factor. Each of the \( f_{ij} \) values can be quantified as long as for each \( OUT(i(t)) \) one will establish a way to compute the respective EQUILIBRIUM function and a way of exciting it with the appropriate input step. In each case the big challenge is to define the EQUILIBRIUM function. What is this function for the costs, for example? Is it easy to describe? And for many other terms of the Supply Chain?

5. **Nodes flexibility (the supply chain flexibility)**

But practical systems are most of the times more complex than a single node system, and when talking of Supply Chains the complexity usually rises. We have shown in the previous section that in general the flexibility of a node would be represented by a matrix, from what follows that the flexibility of a group of nodes would be a matrix of matrices, and thus something too complex to easily be handled in practice. However, if we are only interested on a particular component of its output vector, that is, on just one output function (the inventory, for example) and consider it only depends on a variable (the demand, for example), the node flexibility already can be reduced to an easier measurable scalar quantity. For simplicity, we will assume each node is represented by such a simplistic model.

In that case, the Supply Chain is then reduced to a collection of inventory nodes, each one with a certain flexibility dependent on its previous element (the supplier) as well as on the reorder quantities. Therefore, the flexibility of the overall Supply Chain (in respect to a certain demand pattern) is again a matrix where the individual flexibilities will be plotted.

As an example to compute some practical results we will use the Supply Chain structure drawn in figure 6, which represents the “Cranfield Blocks Game”", a management game developed by Richard Saw (2002).

![Figure 6](image)

This is a seven node Supply Chain (from Depot1 to the Factory) where the direct demand (inputs) from customers (\( Cj \)) is injected at the depot level, and the Supplier is considered an infinite source of materials. For simplicity, only one product is being considered.

From what concerns our interests this can be thought of a system with four (4) inputs driving certain output functions at each of its seven (7) nodes. Thus, by means of injecting steps of demand at each input, one will be in principle able to measure how each node functions (like inventory, costs, etc.) will respond to such “shocks”, and through such measure represent the correspondent overall flexibility matrix.

In general, a simple way to write down this matrix will be plotting the individual flexibilities in columns ordered by increasing level of the facilities on the chain, as next figure suggests, which fast let us to have an idea on how the chain is susceptible to input “chocks”, as it will be shown in a simple example presented soon.

![Figure 7](image)

Following this representation, the flexibility of the chain will be the flexibility of its “arms” connecting the last customers to the primer suppliers. Thus, it is somehow understandable that a
particular Supply Chain would show high flexibility on serving certain customers and at the same time exhibit disastrous flexibility on serving others. A nice result from this approach, however, is that “the flexibility of an arm will be extremely dependent on the flexibility of its slower node”, since the speed on a chain is the speed of its slowest element, what means it can sometimes be very important to ensure a good cooperation between partners in the Supply Chain.

5.1 The Inventory-flexibility-to-demand

Following these ideas, we now present the determination of the Inventory-rigidity-to-demand matrix for the “Cranfield Blocks Game”, achieved by simulation. For convenience, we will use the rigidity representation, as it always deals with finite quantities.

In this case, all the facilities on the game have been configured to use the \((r, Q)\) stock reorder policy and the demand at each Depot have been initially kept constant at the average value the manual version of the game states. Anyhow, at a certain moment of the simulation process, the demand of a certain Depot received a sudden increment on its value, and the new situation was maintained till the end of the simulation. This procedure was done in each of the Depots separately. In each case, it was measured at each node of the Supply Chain the rigidity, applying the equation (9), and then the matrix represented. Also to simplify the read of the results the null values have been substituted by dashed lines in the matrix. For the simple propose of this paper, each simulation have been executed with only 3 replications, but the averages and irrespective uncertainties have been computed for 95% confidence as if more than 20 samples would have been used.

At the figure 8 is represented how the stock level of Depot1 behaved when subjected to a demand step of 2.5 times its usual average value. Notice that even if the system is answering good to such requirements there was introduced on it a certain instability that did not exist before.

5.2 Interpreting the matrix

It is now clear from this matrix that a step on demand of amplitude 2.5x on the Depot3 will lead us observe a “Power Spent on Imbalance” of 4.6 SKU/day in this facility, as well as 1.6 SKU/day at the Warehouse1, 5.0 SKU/day at the Warehouse2 and 4.6 SKU/day at the Factory. For instance, it can also be expected that the same kind of step at the Depot3 will lead to a much higher rigidity at that facility as well as to a higher influence on the Warehouse2, which will exhibit 7.4 SKU/day of imbalance.

Anyhow, such a step on Depot3 will never influence Warehouse1 behavior. And so on…
Another interesting aspect one can infer from this matrix is that probably the imbalance of 5.0 SKU/day shown by Warehouse2 is not motivated by the steps, as it maintains almost constant on the cases C1 and C2. This is probably a residual imbalance due to the natural uncertainty on the Warehouse2 demand due to unsynchronized Depot3 and Depot4 reordering moments.

A last interesting aspect coming out from this data is that it was impossible to obtain a reliable value on the C4 case, as the uncertainties are of the same order of the averages. And in fact, as the next figure shows, it was observed such a high stockout ratio on Depot4 that hardly it could lead to reliable values for the rigidity.

This also have been confirmed by the “Satisfaction ratio” observed at the customer C4, presented in the figure 12, which have dramatically rolled down after the “chock” of demand.

Based on this case, a deeper study concerning the flexibility dependency on reorder policies, reorder levels and step amplitudes will be published in a paper to come.

6. CONCLUSIONS

This results, achieved with the help of simulation, let us conclude that the proposed methodology to measure rigidity in a Supply Chain is feasible and the results can be somehow easily interpreted, therefore conducting to the possibility of applying a reasoning and, trough it, to support decisions.

At the same time, we also conclude that flexibility depends more on the suppliers than on the customers, which automatically remind us the idea of good cooperation with suppliers. It seems also natural flexibility results form an interrelation from where concepts as cooperation, visibility and trust automatically flow. At the same time, it have also been shown that flexibility can depend on the resources used and on how they are used, like the number of vehicles dedicated to delivery, for example, or the model used for reordering the material.

Of course that it is obvious the flexibility can be potentially made higher if working with more than one supplier, using redundancy, the usual way to think on flexibility, but that does not give the manager any quantitative values for reasoning, as it is no more than a holistic proposal, forgetting that more than a supplier can imply serious difficulties in relationship, due to concurrence.

Finally, one must remember that the present results have been obtained considering only “positive” steps of demand, that is, incrementing it, and nothing have been said for the opposite case, when there is a sudden decrement on demand. Anyhow, as the usual Supply Chain is mainly a flow of products in only one way, it results a natural true the best approach on theses cases it is to maintain the minimum stock possible, which in the limit would be a single unit of material (JIT).

Future research on this matter could include studies about other kind of Supply Chain structures, not only didactic but also representing real systems, and, using the kind of analysis we presented here, enrich the knowledge about the idea of flexibility with the final intent to turn it more scientific that what in fact seems at the moment. Tests could be made about flexibility on many aspects of the Supply Chain, as for instance with different reorder policies, or demand amplitudes, as well as with diverse resources and policies related with transportation, or even other. This seems, in fact, an open field for tests using simulation.

References:


4 This game is also described at J. Manuel Feliz-Teixeira, António E. S. Carvalho Brito, “Distributed Application for Supply Chain Management Training”, GEIN - Faculdade de Engenharia da Universidade do Porto, Portugal, 2003.

5 Richard Saw, “Cranfield Blocks Game”, private communication, Centre for Logistics and Supply Chain Management (CSCM), University of Cranfield, 2002.


