

INVESTIGATION OF TURBULENT FLOW OVER A STEEP HILL USING LASER DOPPLER ANEMOMETRY

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Abstract.. *This paper investigates the effects that a steep topographical elevation exerts on the properties of a neutrally stratified turbulent boundary layer. The main concern of the present work is to experimentally investigate the influence of the separation region on the behaviour of the mean and turbulent velocity fields along the topography. Laser-Doppler anemometry was used to measure the longitudinal and vertical mean velocities and its fluctuation components, in a water-channel environment. Original and detailed measurements of the recirculation region downstream of the crest are presented. The results are compared with the predictions of the linear theory of Jackson and Hunt (1975). The behaviour of the fluctuations is analysed on the basis of the local equilibrium concept and the rapid distortion theory of Batchelor e Proudman (1954). The results for the turbulent stresses are compared with both experimental results and numerical predictions available in literature.*

Keywords: *Flow over hills, turbulence, laser-Doppler anemometry, experimental measurements.*

1. Introduction

A faithful modelling of atmospheric flows over topographic elevations has been the subject of innumerable studies since the second half of the past century. One of the pioneers in this attempt was Robert Long (1959), who conducted a series of water-channel experiments in the 1950's to study lee-wave generation. However, owing to theoretical and experimental difficulties, only from the 1970's on, consistent and systematic investigations on flow over complex terrain were developed.

The primary response of the boundary layer to a change in topography is the acceleration of the flow over the hill crest, known as the "speed-up" factor, and associated deceleration on the lee side. A handful of studies on flows over hills have focused on modeling the behaviour of the speed up factor subjected to different surface elevations and atmospheric conditions. An important step forward has been achieved with the linear theory proposed by Jackson and Hunt (1975) for a neutrally stratified boundary layer, which has been further extended in Hunt et al. (1988) to account for stability effects of the atmosphere. This theory has allowed for rapid calculations of the flow over moderate topography, providing a general picture of the perturbed velocity field.

However, the validity of this linear framework is severely diminished when considering flows over steep terrain. The main difficulty introduced by large elevations is the significant nonlinear effects originated from the presence of boundary layer separation downstream of the crest. Under such condition, linear theory is no longer useful for

predicting either the mean or turbulent fields. Indeed, up to date, no established theory is capable of accounting for the influence of steep topography on the flow field, especially when considering the behaviour of the turbulent structure.

In the present work, we are particularly interested on the flow over hills which are steep enough so that large separation regions are formed on its downstream side. Our main concern is to experimentally investigate the influence of the separation region on the behaviour of the mean and turbulent velocity fields along the topography. A neutrally stratified flow over a steep hill has been simulated in a water channel environment. Measurements of longitudinal and vertical components of mean velocity and its fluctuation components were conducted with the aid of laser Doppler anemometry. A flow visualization study is also presented. Mean velocity results are compared with predictions provided by the linear theory of Jackson and Hunt (1975). The turbulent flow field is analysed on the basis of local equilibrium and the rapid distortion theory, introduced by Batchelor and Proudman (1954) and described by Townsend (1976). The present results allow a thorough description of the inner region of the boundary layer, extending from the upstream region, across the separated zone to the downstream lee side. A flow visualization study was performed to furnish a qualitative location of the separation point and the extension of the recirculation region. This first information was further quantitatively evaluated by the LDA measurements.

Because many sources of air pollution are located in complex terrain, detailed and accurate information about the flow field are needed so that reasonable predictions of gas concentration can be made. However, despite the huge demand for a better understanding of the separation process downstream of steep slopes, only a small number of works have focused on this problem; and even fewer have accomplished a thorough experimental investigation of the recirculation region.

From this standpoint, the main contribution of the present paper is to provide detailed and refined experimental data on neutrally stratified flows over a steep hill, in particular inside the recirculation region. These data that may help to broaden our understanding of the complex phenomenon of flow separation from curved surfaces. Before extending this problem to more complex situations, it is crucial to have a well-established knowledge of the main mechanisms involved on the separation behind hill crests. Our aim was then to consider only the effects of hill shape and slope. Other contributions such as surface roughness or stratification effects are not in the scope of this work.

2. Some comments on previous works

Undoubtedly, one of the most significant contributions to the comprehension of flow over hills was introduced by Hunt and his co-workers. Based on the asymptotic structure of the boundary layer, Jackson and Hunt (1975) derived a set of linearised equations which can satisfactorily predict the flow over low to moderate surface elevations.

Indeed, most of the subsequent works have addressed the study of flow and dispersion over low and moderate hill, but focused almost entirely on the behaviour of the speed up factor, e.g. Castro and Snyder (1982), Charruthers and Choularton (1982), Teunissen (1983), Bowen (1983), Arya and Gadiyaram (1986), Arya et al. (1987). Certainly, the interest raised by the demand in understanding the dynamics of pollutant's dispersion as well as optimizing the location of wind energy generators in hilly terrain have steered the course of those researches.

In an original work, Britter et al. (1981) studied the influence of surface roughness and separation on the characteristics of the "speed up" factor, i.e. the maximum increase in velocity at the hill top. Britter and his fellows conducted wind tunnel measurements along a two-dimensional hill, which profile followed a curve of Agnesi. The authors found that the perturbation velocity on the hill top could be estimated by assuming a linear superposition of the velocity changes produced by the changes in elevation and in surface roughness. In the lee side, however, this assumption was not valid since the separation region significantly altered the flow characteristics. As for the turbulent structure, Britter et al. were the first to correlate the fluctuation field to the effects of rapid distortion theory.

Bradley (1980) and Britter et al. (1981) can be considered some of the pioneers in trying to explain the behaviour of the turbulent field over hills. One of the first extensive field measurement campaigns was conducted Bradley (1980). Data were presented for the mean velocity field and turbulent components over Black Mountain, located in Canada. Calculations for the speed up factor were compared with the linear theory of Jackson and Hunt (1975), and an extensive analysis of the behaviour of the turbulent flow field over the elevation has been shown. The turbulent fluctuations were found to behave according to the rapid distortion theory, initially introduced by Batchelor and Proudman (1954). Bradley has observed some discrepancies between the measured data and the numerical predictions of Taylor (1977), which could not represent correctly the non-equilibrium region of the flow.

Zeman and Jensen (1987) have developed a new model to investigate the turbulent flow field over two-dimensional hills. They applied the von Mises transformation to the mean momentum equations, along with a second-order turbulence closure type. The model predictions were compared with some experimental results from the Askervein Hill project, showing good agreement. The hill perturbation pressure was calculated by means of potential flow theory. The authors focused their attention on the turbulence characteristics of the flow field over hills. They observed that in a region close to the ground the magnitudes of all non-zero stress components are increased relative to the upstream values, in accordance with the increase in mean velocity and shear. At higher levels the effects of rapid distortion of turbulence were found to decrease the magnitude of the longitudinal fluctuations while increasing the vertical fluctuation component. As Mason and King (1985), Zeman and Jensen (1987) have observed some discrepancies in the

behaviour of the fluctuation components between different works available in literature. The authors speculated that this fact is due to the upstream anisotropy of the flow, but no further research was conducted to prove this assertion.

A wide perspective of the state of the art of research of this topic can be obtained from Kaimal and Finnigan (1994), Belcher and Hunt (1998). These authors point out that the main limitation of field investigations on separated flows on the lee side of steep hills: the recirculation depth is usually of the order of hill height, which is just too high to be explored by the meteorological towers of field campaigns. From this perspective, wind tunnel studies still have much to contribute for the understanding of separation process in atmosphere.

3. Experimental apparatus

Several investigators have resorted to water experimentation in order to examine the flow over hills and other obstacles. The towing tank method has been extensively used in the past, as depicted by the works of Long (1959), Snyder (1985) and more recently by Gyüre and Jánosi (2003). Indeed, this method is quite appropriate when considering atmospheric flows dominated by blocking, lee waves, inversions and strong stratification effects. However, as explained by Meroney (1990), the towing tank method may distort surface layer predictions because the uniform approach profile simulated is not equivalent to shear flow found near the earth's surface. Water-channel experiments, on the other hand, do not suffer from this weakness. The mean and turbulent boundary layer profile can be accurately reproduced in this facility. The ratios of longitudinal and vertical fluctuation components of the simulated inner layer are comparable to atmospheric data, as presented in the results section.

All the experiments were conducted at the Hydraulics Laboratory of the Civil Engineering Department of the University of Oporto, Portugal. In the following sections the facilities, instrumentation and model used are described.

3.1 Water-channel

The Hydraulics Laboratory has two different water channels which work in a closed loop system. The one used in the present work is a 17 m long water channel, with cross sectional area of 0.60 m high per 0.40 m wide. The side walls were made of glass, so as to make it convenient to perform any visual inspection of the flow, as well as to permit an appropriate use of the laser-Doppler anemometer.

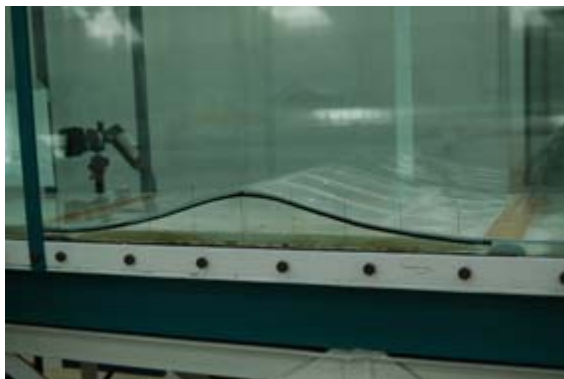
The water recirculation system consists of two underground tanks, four pumps with a maximum capacity of 150 l/s and one upper stabilizing tank. The working section was 3 m long, and was situated 7.3 m downstream of the channel entrance. The model of the hill was located 8 m from the channel entrance.

During a typical run, two pumps sufficed to be used to keep the system running in a steady state, with a maximum flow rate variation of $\pm 0.8\%$. The two pumps were used to supply to the stabilizing tank at 90 l/s. At the entrance of the channel, the water was made to pass through a series of screens and filters so as to stabilize, make uniform and suppress any excessive level of turbulence. The screens and filters were also used to control the grain-size of the particles in suspension in the water. To guarantee a flow rate control of 0.001 l/s, a magnetic flowmeter was installed in the supply line. The water depth along the channel was controlled by a vertical steel gate.

An illustration of the system is shown in Figure (1.a), and the hill is shown in Figure (1.b)



(a)



(b)

Figure 1: Illustration of the water-channel (a) and model hill (b) used to conduct the present experiments.

3.2 Model of the topographic elevation

For the present work, a smooth surface two-dimensional, axisymmetric hill has been built. Following the geometrical characteristics used by Loureiro et al. (2000, 2001), the shape of the hill is given by the “Witch of Agnesi” curve,

$$z = H \left[1 + \left(\frac{x}{L_H} \right)^2 \right]^{-1} \quad (1)$$

where H denotes the hill height, L_H is a horizontal length scale corresponding to half hill height. These characteristics lengths were chosen so as to generate a steep hill where separation would certainly occur.

The model was built using polished Plexiglass, which dimensions were $H = 60$ mm and $L_H = 150$ mm. These values give a maximum slope of $18,6^\circ$, a base length of 600 mm and an aspect ratio (L_H/H) of 5. This curve has been extensively used in literature, e. g. Britter and Hunt (1981) and Arya et al. (1987).

3.3 Instrumentation

A one component Dantec laser-Doppler anemometry system was used in the forward scatter mode to conduct the measurements of the mean and fluctuating velocity field. The laser source used to form the beams is a 2 W Ar-ion, operating in multi-mode. A Bragg cell unit was used to introduce a total optical-electronic shift of 0.6 MHz, allowing the resolution of the direction of the flow field and the correct measurement of near-zero mean velocities. The beams were made to pass through a series of conditioning optical elements, in order to achieve a small measurement volume and to improve the optical alignment. Front lenses of 310 mm focus length were mounted on the probe in order to accurately position the measurement volume on the centerline of the water channel. Before being collected by the photomultiplier, the scattered light was made to pass through an interference filter of 514.5 nm, so that only the green light is acquired. The signal from the photomultiplier was band-pass filtered and processed by a TSI 1990C Counter, operating in the single measurement per burst mode. A series of LDA biases were avoided by adjusting the strictest parameters on the data processor. For this set of experiments, a frequency validation setting of 1% was used with 2^5 cycles of comparison. For each point measured, a sample size of 10,000 values has been considered. Table (1) lists the main characteristics of the laser-Doppler system used.

Table 1. Main characteristics of the laser-Doppler anemometer in air at e^{-2} intensity.

Laser wavelength	514.5 nm
Measured half angle of the beams	3.415°
Fringe spacing	4.3183 mm
Frequency shift	0.60 MHz
Dimensions of the measurement volume:	
Major axis	2.53 mm
Minor axis	162.0 mm

This whole system was used to measure both the longitudinal and the vertical velocity components. This could be done simply by turning the probe around its axis, so that on both conditions, the fringe distribution was perpendicular to the measured velocity component. Typical uncertainties associated to the mean and fluctuating velocity data, U , W , S_u , S_w , S_{uw} , were estimated in $\pm 2.5\%$, $\pm 10\%$, $\pm 6.5\%$ and $\pm 12\%$ respectively.

4. Results

This section presents the mean velocity profiles over the hill and on its neighborhood. Firstly, the properties of the undisturbed boundary layer are shown and discussed. Then, the results for the perturbed flow field over the hill are presented. For clarity, and to identify which mechanisms are dominant in each region, the results will be particularly split into three blocks: data for the flow filed upstream of the separation point (first 3 stations), data for the recirculation region (next 7 stations) and data for the returning to equilibrium region (last 3 stations).

Hereafter, all the data are shown in reference to a Cartesian coordinate system located at the symmetrical axis of the hill. All the profiles were measured at the centerline of the water-channel, at a flow rate of 4.0 l/s and water height of 236 mm. Measurements were taken at 13 stations along the test section, and Figure (2) illustrates its spatial distribution.

4.1 Undisturbed Boundary Layer

The standard approach to the study of flow over hills is based on the concept of perturbation techniques. Therefore, the flow over the hill is treated as the incoming undisturbed flow, (U_d) plus a perturbation ΔU , introduced by the presence of the hill, i.e. $U = U_d + \Delta U$. The next two pictures characterizes the non-perturbed boundary layer profile measured at the station $x = -750$ mm.

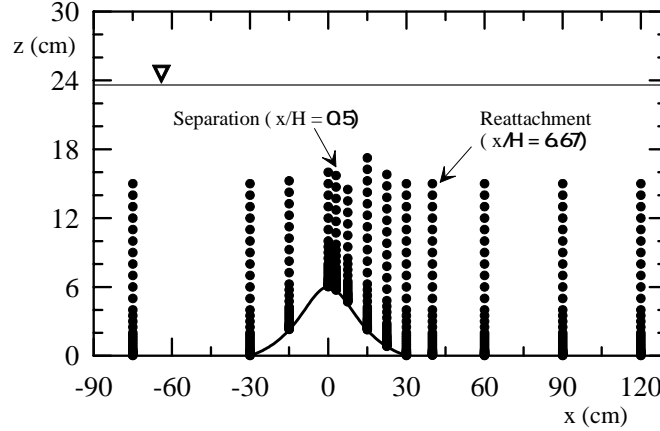


Figure 2: Description of the spatial distribution of the measured profiles.

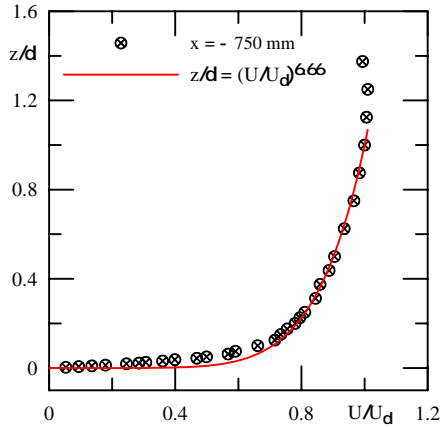


Figure 3: Mean longitudinal velocity profile in physical coordinates.

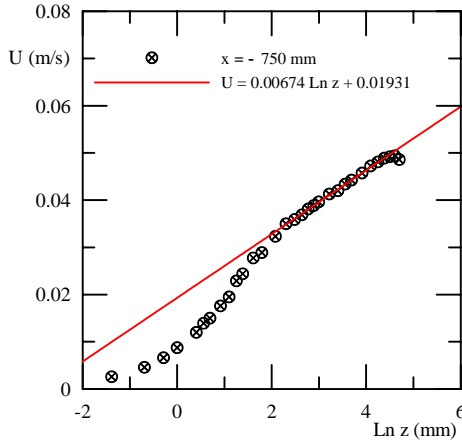


Figure 4: Mean velocity profile in inner coordinates.

Figure (3) shows the longitudinal mean velocity profile in physical coordinates. The vertical and horizontal coordinates are normalized by the boundary layer thickness, d , and by the free stream velocity of this profile, U_d , respectively. Figure (4) presents a log-linear plot where the data is adjusted by the logarithmic law shown in Eq. (2):

$$U = \frac{u_\tau}{\kappa} \ln \left(\frac{z}{z_0} \right), \quad (2)$$

where k is the von Karman constant. The values for the friction velocity, u_τ , and for the roughness parameter, z_0 , were obtained through the curve-fitting of the Eq. (2) to the logarithmic portion of the boundary layer.

From the best-fit of the logarithmic law presented in Figure (4), the value of the friction velocity was calculated, giving $u_\tau = 0.072 U_d$. This value is in 5% agreement with the data of Britter et al. (1981), $u_\tau = 0.0685 U_d$, and Athanassiadou and Castro (2001), $u_\tau = 0.06 U_d$. In fact, all the characteristic parameters of the undisturbed boundary layer are in quite good agreement with other authors, thus certifying that the simulated flow is representative of atmospheric flows. A summary of the comparison with other authors' data is presented in Table (2). The global and local properties of the undisturbed boundary layer are presented in Table (3).

4.2 Mean Velocity Field

Table 2. Comparison of the characteristic parameters of the undisturbed boundary layer with data from other authors.

	S_u / u_t	S_w / u_t
Athanassiadou and Castro (2001)	2.19	1.12
Britter et al. (1981)	2.12	-
Gong and Ibbetson (1986)	2.20	1.00
Khurshudyan (1981)	2.50	1.20
Zeman and Jensen (1983)	2.43	1.23
Present work	2.12	0.69

Table 3. Global and local properties of the undisturbed boundary layer.

α (mm)	U_d (m/s)	u_t (m/s)	z_0	l (mm)	h_m (mm)
100	0.0482	0.0035	0.0571	6.0	60.1

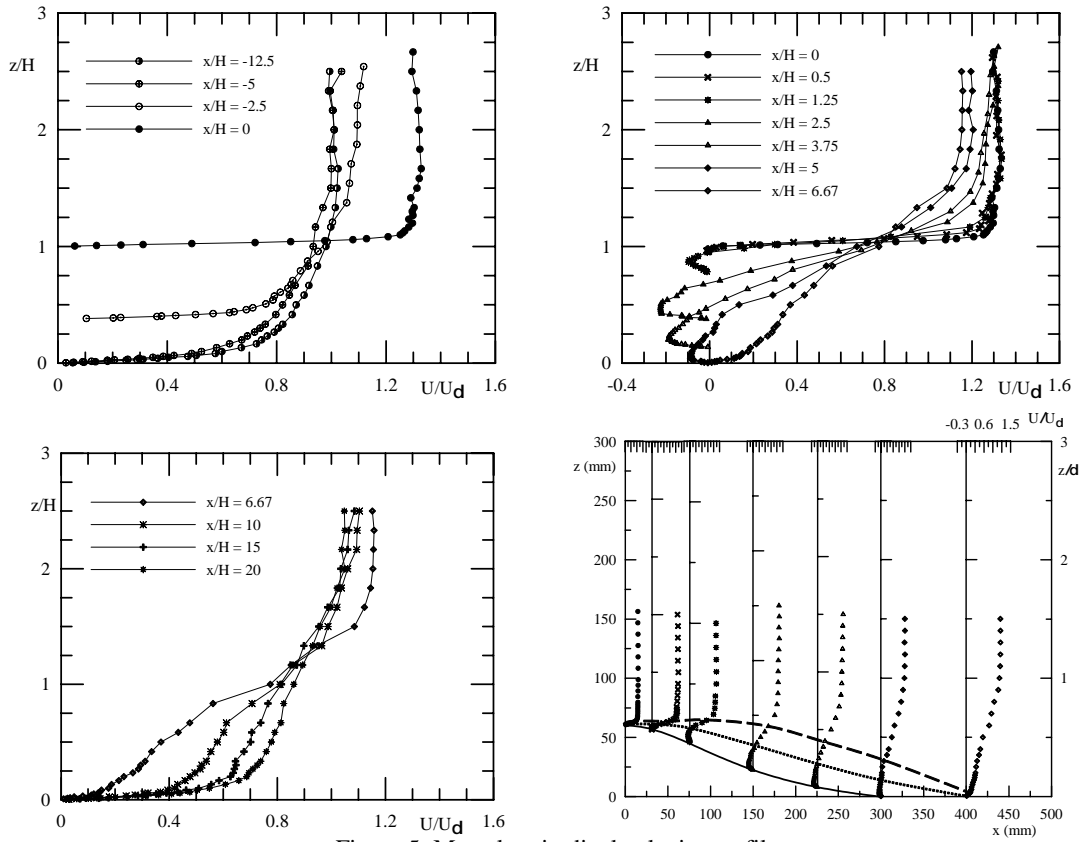


Figure 5: Mean longitudinal velocity profiles.

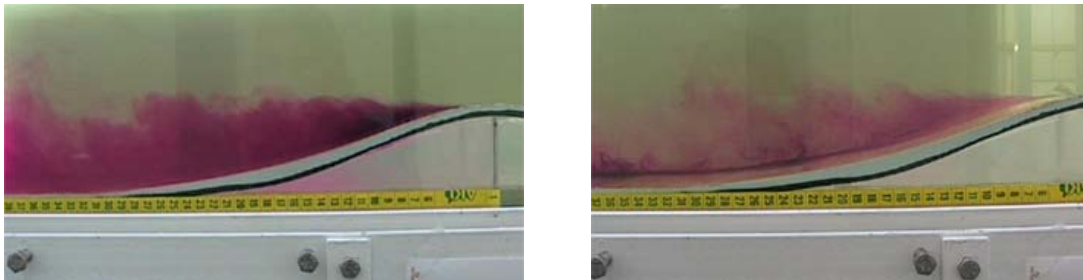


Figure 6: Flow visualization.

Longitudinal mean velocity profiles are presented in Figure (5). Plots on the top of Figure (5) show the upstream profiles, on the left, where a pronounced acceleration can be noticed; and the velocity profiles measured on the downwind slope of the hill, on the right.

The separation point was observed to occur approximately at $x = 30$ mm, ($x/H = 0.5$), and reattachment was observed at around $x = 400$ mm, ($x/H = 6.67$). The bottom left picture presents the velocity profiles downstream of the wake, where the boundary layer is returning to its equilibrium conditions. An overview of the recirculation is shown in the bottom right plot of Figure (5). The dashed line indicate the border of the recirculation region, estimated by the point of maximum shear stress of each profile. The dotted line links the points where the longitudinal mean velocity is zero.

Figure (6) illustrates the flow visualization procedure used to qualitatively verify the location of the separation point and the extension of the recirculation region. The picture on the right allows the inspection of a pronounced mixing layer region downstream of the crest, while on the right the reattachment can be estimated by the direction of vortices' motion. An illustration of this process is shown in Figure (7).

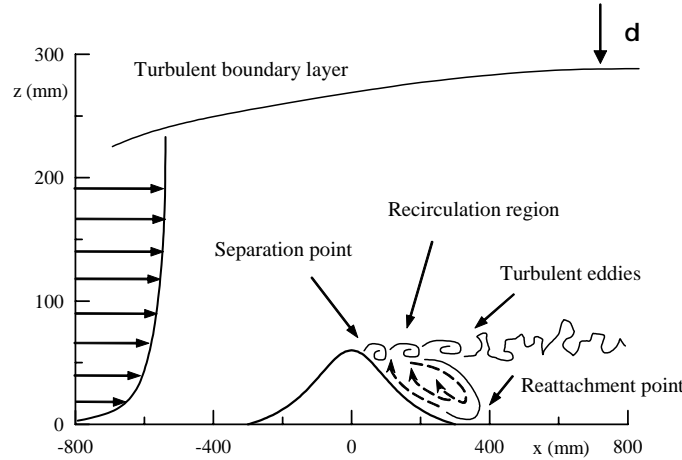


Figure 7: Schematic view of the separated region.

Following the model proposed by Jackson e Hunt (1975), the inner layer was estimated to be $l = 6$ mm, and the middle layer, $h_m = 60.1$ mm. The position of the maximum speed-up ($z = 2$ mm) is exactly predicted by the linearised theory. However, the value of \mathbf{Ds}_{max} , 2.5, is underestimated by 30%.

The changes in Reynolds stresses are shown in Figures (8) to (10). The measurements indicate a clear decrease in the longitudinal normal stresses, \mathbf{S}_u , on the top of the hill, followed by a large increase on the lee side. For \mathbf{S}_w , on the other hand, an increase of about 50% is observed on the hill top. This result is in accordance to the rapid distortion theory, e.g. Batchelor e Proudman, 1954.

In fact, under equilibrium region, i.e. in the upstream undisturbed profile and in the near-wall region of the flow, the turbulent intensity components increase proportionally to the changes in surface stresses. Britter et al. (1981) remarked that, based on estimates of the linearised theory, the following relations hold

$$\frac{\Delta \sigma^2}{\sigma^2} = \frac{\Delta \sigma_u^2}{\sigma_u^2} = \frac{\Delta \sigma_w^2}{\sigma_w^2} \cong 4 \frac{H}{L}, \quad (3)$$

where \mathbf{Ds} are the changes in the components of turbulent stresses from their upstream values. For the present data, a value of 3.5 has been found.

In the outer region of the flow where rapid distortion theory applies, Britter et al. have further remarked that the following relationship is valid

$$\frac{\sigma_u^2(x, z)}{\sigma_u^2(z_s)} = 1 - \frac{4}{5} \frac{\Delta u(x, z)}{U_0(z_s)}, \quad \frac{\sigma_w^2(x, z)}{\sigma_w^2(z_s)} = 1 + \frac{4}{5} \frac{\Delta u(x, z)}{U_0(z_s)} \quad (4)$$

The changes in longitudinal turbulence intensity as predicted by rapid distortion theory are presented in Figure (11).

4.3 Turbulent structure

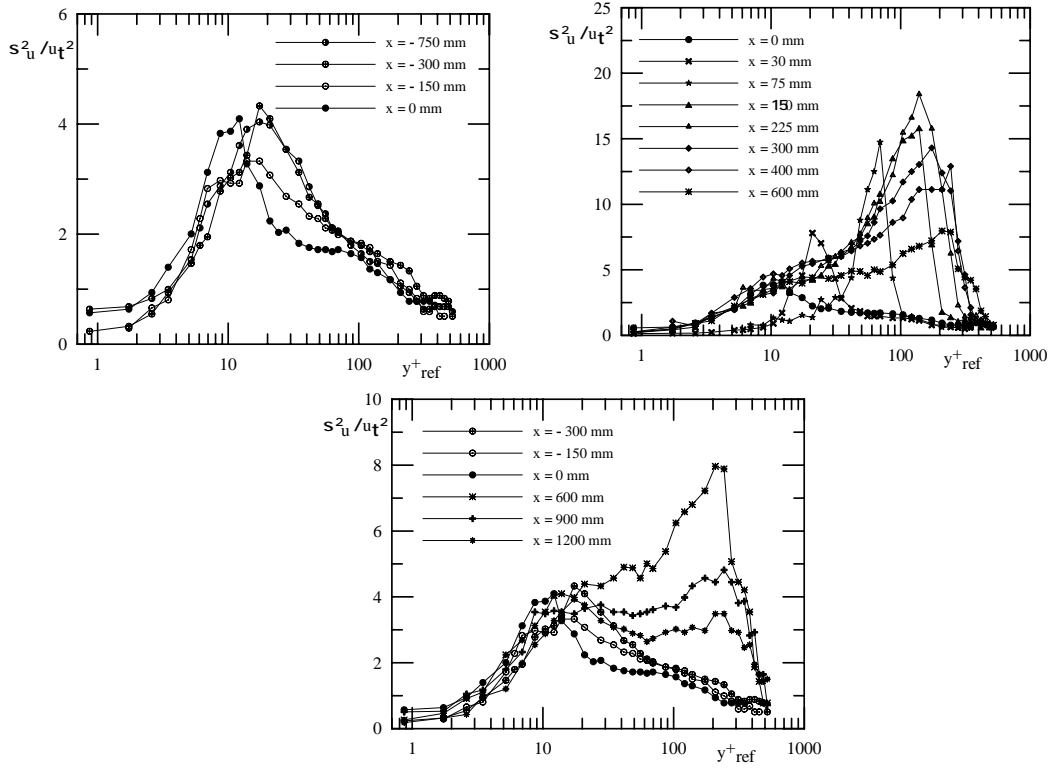


Figure 8: Longitudinal turbulent stresses.

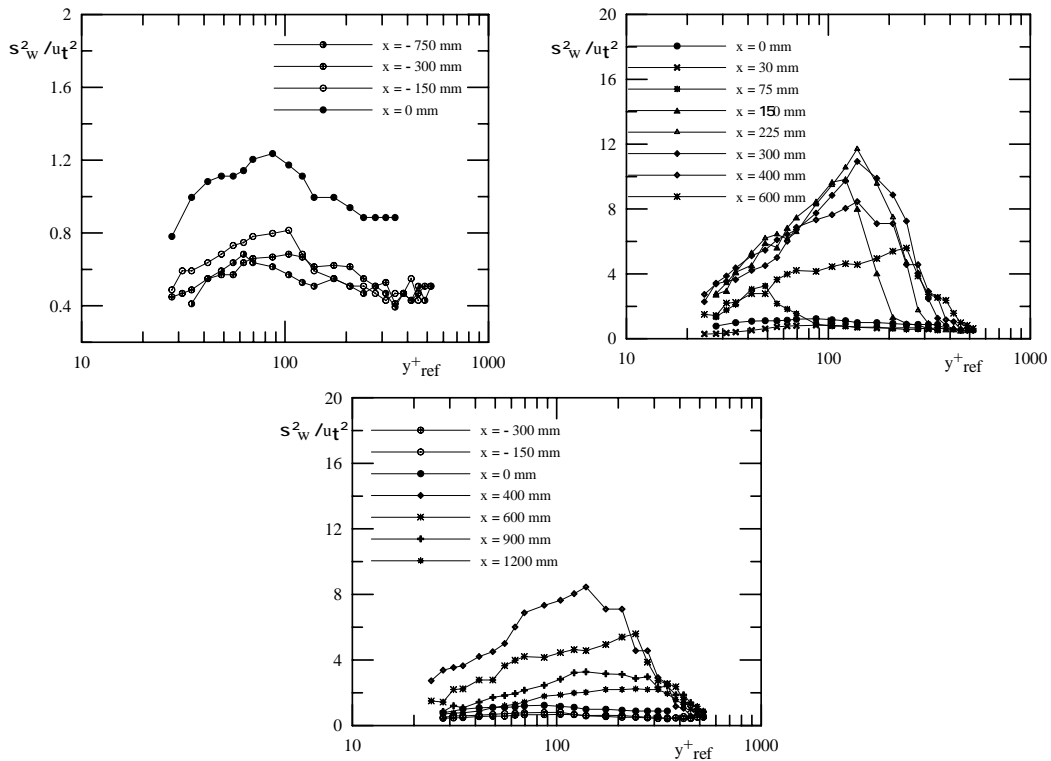


Figure 9: Vertical turbulent stresses.

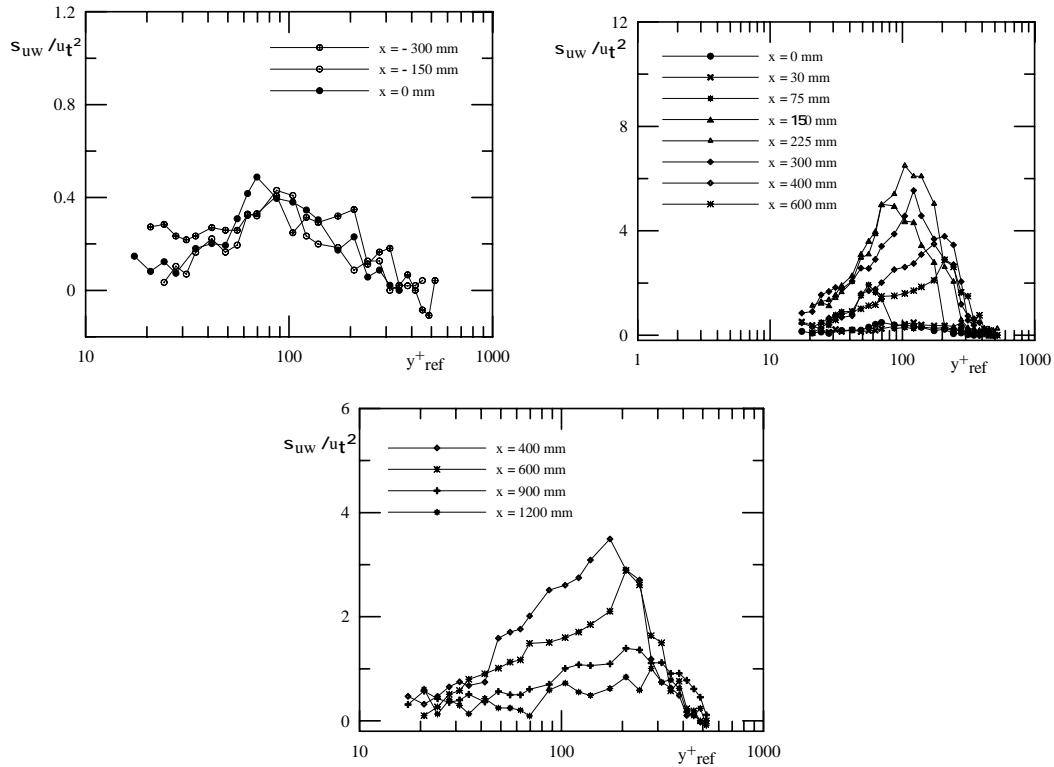


Figure 10: Shear turbulent stresses.

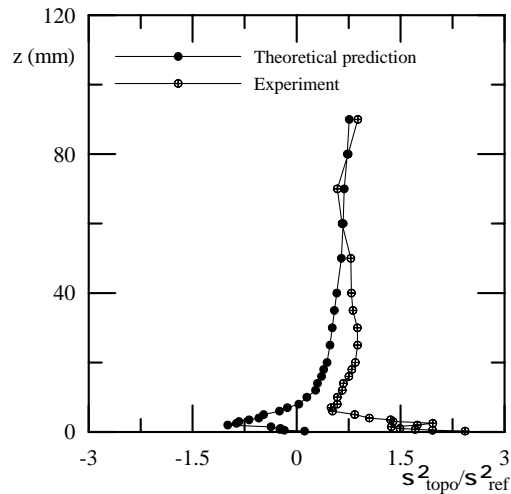


Figure 11: Rapid distortion effects.

5. Final remarks

This work has experimentally investigated the influence of the separation region on the behaviour of the mean and turbulent velocity fields along the topography. A neutrally stratified flow over a steep hill has been simulated in a water channel environment. Measurements of longitudinal and vertical components of mean velocity and its fluctuation components were conducted with the aid of laser Doppler anemometry. A flow visualization study was also presented.

At the present stage, the research has striven in characterising the flow in four distinct regions: in the undisturbed region upstream of the hill, on the top of the hill, inside the and in the undisturbed region downstream of the hill.

Mean velocity results are compared the predictions of the linear theory of Jackson and Hunt (1975). The turbulent flow field is analysed on the basis of the local equilibrium concept and the rapid distortion theory, introduced by

Batchelor and Proudman (1954) and described by Townsend (1976). The present results allow a thorough description of the inner region of the boundary layer, showing good agreement with the predictions of the rapid distortion theory.

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