TRANSITIONS IN SOME STAGNATION FLOWS OF VISCOELASTIC FLUIDS AT LOW REYNOLDS NUMBERS

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Flow Instabilities and Turbulence in Viscoelastic Fluids
Leiden, Holland
July 19-23, 2010
OUTLINE

• Objective
• Experimental and numerical results
  • Cross slot
  • Flow focusing
• Some analytical thoughts: Stagnation + “vortex” flow
• Closure
OBJECTIVE

• **Elastic instabilities** \((\text{Re}=0)\): enhanced mixing or upper limit in devices

• Transition from steady symmetric to steady asymmetric flow is our main interest

• When it occurs and what are the effects of solvent, inertia and extensional viscosity. Brief review in some simple flows

• Some findings about the asymmetric flow: decoupling into simpler flows

• Results: mostly numerical (some experiments) and analytical (work in
REVIEW

Viscoelastic instabilities in shear flows

Taylor-Couette flow  Larson et al., JFM 218 (1990) 573
Cone-plate flow  McKinley et al., JNNFM 40 (1991) 201
Lid driven cavity flows  Pakdel & McKinley, PRL 77 (1996) 2459

Underlying mechanism  McKinley et al, JNNFM 67 (1996) 19
                     Pakdel & McKinley, PRL 77 (1996) 2459

\[
\left( \frac{\lambda U}{R} \frac{\tau_{11}}{\tau_{12}} \right) \geq M_{\text{crit}}^2
\]

curved streamlines

Instability growth to elastic turbulence

Microfluidics & viscoelasticity
Squires & Quake, Rev. Mod. Phys. 77 (2005) 977

Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids
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NUMERICAL METHODS: SOLUTION OF THE GOVERNING EQUATIONS

- Finite-volume method (in-house code)
- Collocated block-structured mesh
- Non-orthogonal coordinates (Cartesian velocity and stress tensor)
- Diffusion: central differences (2nd order in uniform mesh)
- SIMPLEC algorithm
- Rhie-and-Chow to couple velocity and pressure
- Special scheme to couple velocity and extra stress


- Advection: CUBISTA high-resolution scheme (based on QUICK, 3rd order)


- Standard formulation and log-conformation formulation (allows higher De)


CROSS SLOT
Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids

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2D CROSS SLOT WITH UCM: EFFECT OF INERTIA

Poole et al., PRL 99 (2007) 164503

$De = \frac{\lambda U}{H}$

Inertia decreases degree of asymmetry and stabilizes the flow

Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids

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Increasing the solvent viscosity

Increases $D e_{\text{CR}}$

For $\beta > 3/9$ flow becomes asymmetric unsteady (as in flow focusing)
Increasing Re

Increases $D_{eCR}$
Decreases degree of asymmetry
For $Re > 2$ unsteady asymmetric flow

Poole et al., SoR 2007
2D CROSS SLOT: OLDROYD-B — STABILITY MAP

\[ \beta = \frac{1}{9} \]

Symmetric

Unsteady asymmetric

Steady asymmetric

Transitions in some stagnation viscoelastic flows at \( Re = 0 \)
Flow Instabilities and turbulence in viscoelastic fluids

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Increasing $\varepsilon$
Increases $D_{e_{CR}}$
Decreases degree of asymmetry ($\varepsilon<0.04$)
Increases degree of asymmetry and extension in $D_{e}$ ($\varepsilon>0.04$)
Asymmetric stable flow disappears for $\varepsilon>0.08$

Transitions in some stagnation viscoelastic flows at $Re=0$
Flow Instabilities and turbulence in viscoelastic fluids

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$\beta = 1/9; \, Re = 0$

Transitions in some stagnation viscoelastic flows at $Re=0$

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FLOW FOCUSING
(extensional flow “without” shear)
Cross-slot with 3 inlets and 1 outlet

Flow Focusing

Operational Variables

\[ Q_1, Q_2 \]
\[ Q_3 = 2 \times Q_2 + Q_1 \]

Dimensionless Variables

\[ FR = \frac{Q_2}{Q_1} \]
\[ VR = \frac{U_2}{U_1} \quad (= FR) \]
\[ Re = \frac{\rho U_2 D}{\eta_0} \]
\[ El = \frac{De}{Re} \]
\[ De = \frac{\lambda U_2^2}{D} \]

All dimensions kept constant in experiments and calculations
FLOW FOCUSING: NEWTONIAN

Separation streamlines: nearly hyperbolic shape

\[ \varepsilon_H = \ln \left( \frac{D_1}{D_3} \right) = \ln \left[ \frac{3}{2} (1 + 2VR) \right] \]

\[ Q_1 = 0.01 \text{ ml/h} \]

\[ Q_2 = 0.3 \text{ ml/h} \]
\[ VR = 1, \text{ Re}_3 = 2.8 \]

\[ Q_2 = 0.9 \text{ ml/h} \]
\[ VR = 3, \text{ Re}_3 = 6.5 \]

\[ Q_2 = 15 \text{ ml/h} \]
\[ VR = 50, \text{ Re}_3 = 94.2 \]

\[ Q_2 = 18 \text{ ml/h} \]
\[ VR = 60, \text{ Re}_3 = 112.8 \]

Microfluidic flows of viscoelastic fluids
V BCR 2010

Oliveira et al. JNNFM 160 (2009) 31-39

Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP
Rio de Janeiro, Brazil, 14-16th July 2010
FLOW FOCUSING: PAA125

17

Oliveira et al. JNNFM 160 (2009) 31-39

$$Q_1 = 0.01 \text{ ml/h}$$

Increasing $$Q_2$$

Viscoelastic

$$Q_2 = 0.05 \text{ ml/h}, \ VR = 5$$  
$$Re = 0.23, \ De = 0.38$$  
Symmetric

$$Q_2 = 0.2 \text{ ml/h}, \ VR = 20$$  
$$Re = 0.87, \ De = 1.41$$  
Steady Asymmetric

$$Q_2 = 0.5 \text{ ml/h}, \ VR = 50$$  
$$Re = 2.15, \ De = 3.479$$  
Unsteady 3D

Microfluidic flows of viscoelastic fluids  
V BCR 2010

Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP  
Rio de Janeiro, Brazil, 14-16th July 2010
FLOW FOCUSING: PAA125

\[ Q_1 = 0.01 \text{ ml/h} \]

<table>
<thead>
<tr>
<th>( Q_2 )</th>
<th>VR</th>
<th>Re</th>
<th>De</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 ml/h</td>
<td>5</td>
<td>0.23</td>
<td>0.38</td>
</tr>
<tr>
<td>0.1 ml/h</td>
<td>10</td>
<td>0.45</td>
<td>0.723</td>
</tr>
<tr>
<td>0.2 ml/h</td>
<td>20</td>
<td>0.87</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Oliveira et al. JNNFM 160 (2009) 31-39

Microfluidic flows of viscoelastic fluids
V BCR 2010

Sousa, Afonso, Oliveira, Alves & Pinho - CEFT/FEUP
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FLOW FOCUSING: VISCOELASTIC

\[ \xi = \frac{1 - R}{1 + R} \]

\[ R = \frac{tr \tilde{V}^2}{tr D^2} \]

Astarita, JNNFM 6 (1979) 69
Thompson et al., JNNFM 86 (1999) 375
Mompean et al., JNNFM 111 (2003) 151

Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids

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FLOW FOCUSING: EFFECT OF VR

\[ F^* = \frac{F_W - F_E}{F_3} \]

Bistable flow
High VR:
constant \( D_{ec} \)
evolution independent of VR
supercritical pitchfork bifurcation

\[ F^* = 0.59 \sqrt{D_e} - 0.33 \]

Oliveira et al. JNNFM 160 (2009) 31-39
FLOWS FOCUSING: EFFECT OF $\beta$

$$\beta = \frac{\eta_s}{\eta_s + \eta_p}$$

Oldroyd-B

$\beta$ stabilizes the flow increases $\text{De}_c$
$\beta \geq 6/9$, no steady asymmetry

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FLOW FOCUSING: EFFECT OF $\varepsilon$

$\varepsilon$ stabilizes the flow
increases $De_c$
decreases degree of asymmetry
$\varepsilon \geq 0.04$ steady asymmetry disappears
(Transition directly to unsteady flow)

Similar levels of normal stresses achieved near critical conditions
Extensional properties decisive for onset of flow asymmetry

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FLOW FOCUSING: NUMERICAL VERSUS EXPERIMENTS (PAA 125)

Experimental
PAA 125 + NaCl

Numerical
UCM, 2D, Re=0

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STAGNATION FLOW
Symmetry & asymmetry
Some observations from numerics on cross flow
DIFFERENCE OF TWO SYMMETRIC FLOWS FAR FROM TRANSITION

\[ De = 0.20 - De = 0.19 \]
DIFFERENCE OF TWO SYMMETRIC FLOWS CLOSE TO TRANSITION

\[ De = 0.308 - De = 0.307 \]
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STREAMLINES FOR SYMMETRIC FLOW

$De = 0.309$

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STREAMLINES FOR CRITICAL FLOW

\[ De = 0.310 \]

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SMALL DIFFERENCE BETWEEN TWO ASYMMETRIC FLOWS

\[ De = 0.312 - De = 0.311 \]

\[ De = 0.315 - De = 0.314 \]

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LARGER DIFFERENCES

Asymmetric- critical
\[ De = 0.32 - De = 0.31 \]

Asymmetric- asymmetric
\[ De = 0.34 - De = 0.32 \]

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STAGNATION + VORTEX FLOW
An analytical solution
PROBLEM FORMULATION: UCM

Stagnation flow
\[ u_{sta} = ax \]
\[ v_{sta} = -ay \]

“Vortex” flow
\[ u_{vor} = b_u y \]
\[ v_{vor} = b_v x \]

Stagnation + “vortex” flow
\[ u = ax + b_u y \]
\[ v = -ay + b_v x \]

Transitions in some stagnation viscoelastic flows at Re=0

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GENERAL SOLUTION & CONSTANT SOLUTION

\[
\begin{align*}
\tau_{xx} + De \left[ (b_u y + ax) \frac{\partial \tau_{xx}}{\partial x} + (b_v x - ay) \frac{\partial \tau_{xx}}{\partial y} - 2 \left( a \tau_{xx} + b_v \tau_{xy} \right) \right] &= 2a \\
\tau_{xy} + De \left[ (b_v x - ay) \frac{\partial \tau_{xy}}{\partial x} + (b_u y + ax) \frac{\partial \tau_{xy}}{\partial y} - \left( b_u \tau_{xx} + b_v \tau_{yy} \right) \right] &= b_u + b_v \\
\tau_{yy} + De \left[ (b_v x - ay) \frac{\partial \tau_{yy}}{\partial x} + (b_u y + ax) \frac{\partial \tau_{yy}}{\partial y} - 2 \left( b_u \tau_{xy} - a \tau_{yy} \right) \right] &= -2a
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \tau_{ij}}{\partial x_k} &= 0 \Rightarrow \\
\tau_{xx} &= -\frac{2 \left[ a + 2a^2 De + b_v \left( b_u + b_v \right) De \right]}{-1 + 4 \left( a^2 + b_u b_v \right) De^2} \\
\tau_{xy} &= -\frac{b_u + b_v + 2a \left( b_u - b_v \right) De}{-1 + 4 \left( a^2 + b_u b_v \right) De^2} \\
\tau_{yy} &= \frac{2 \left[ a - 2a^2 De - b_u \left( b_u + b_v \right) De \right]}{-1 + 4 \left( a^2 + b_u b_v \right) De^2}
\end{align*}
\]

This solution absorbs the constants on the rhs of constitutive equation.
HOMOGENEOUS SOLUTION (1)

\[
\begin{align*}
\tau_{xx} + De \left[ (b_u y + ax) \frac{\partial \tau_{xx}}{\partial x} + (b_v x - ay) \frac{\partial \tau_{xx}}{\partial y} - 2(a \tau_{xx} + b_v \tau_{xy}) \right] &= 0 \\
\tau_{xy} + De \left[ (b_v x - ay) \frac{\partial \tau_{xy}}{\partial x} + (b_u y + ax) \frac{\partial \tau_{xy}}{\partial y} - (b_u \tau_{xx} + b_v \tau_{yy}) \right] &= 0 \\
\tau_{yy} + De \left[ (b_v x - ay) \frac{\partial \tau_{yy}}{\partial x} + (b_u y + ax) \frac{\partial \tau_{yy}}{\partial y} - 2(b_u \tau_{xy} - a \tau_{yy}) \right] &= 0
\end{align*}
\]

Solution hypothesis (1): \( \tau_{ij}(x, y) = \tau_{ij}(\phi) \) with \( \phi = kx + Ty \)

\[
\begin{align*}
m\phi De \sqrt{a^2 + b_u b_v} \frac{d\tau_{xx}}{d\phi} &= (-1 + 2a De) \tau_{xx} + 2b_v De \tau_{xy} \\
m\phi De \sqrt{a^2 + b_u b_v} \frac{d\tau_{xy}}{d\phi} &= b_u De \tau_{xx} + b_v De \tau_{yy} \\
m\phi De \sqrt{a^2 + b_u b_v} \frac{d\tau_{yy}}{d\phi} &= -(1 + 2a De) \tau_{yy} + 2b_u De \tau_{xy}
\end{align*}
\]

\[
\begin{align*}
k &= \frac{Tb_v}{-a \pm \sqrt{a^2 + b_u b_v}} \\
m &= \pm 1
\end{align*}
\]
HOMOGENEOUS SOLUTION (2)

Solution hypothesis (2): \( \tau_{ij}(\phi) = \alpha_{ij}\phi^q \)

as in stagnation flow \(^1,^2\)

\(^1\) Renardy JNNFM 138 (2006) 204-205
\(^2\) Becherer, Morozov, van Saarloos JNNFM 153 (2008) 183-190

\[
\begin{align*}
\left[ -1 + 2aDe - mqDe\sqrt{a^2 + b_u b_v} \right] \alpha_{xx} + 2b_v De\alpha_{xy} \right] \phi^q &= 0 \\
\left[ b_u De\alpha_{xx} + b_v De\alpha_{yy} - \left( 1 + mqDe\sqrt{a^2 + b_u b_v} \right) \alpha_{xy} \right] \phi^q &= 0 \\
\left[ 2b_u De\alpha_{xy} - \left( 1 + 2aDe + mqDe\sqrt{a^2 + b_u b_v} \right) \alpha_{yy} \right] \phi^q &= 0
\end{align*}
\]

\[
\begin{align*}
\alpha_{xx} &= \frac{2b_v De\alpha_{xy}}{-1 + 2aDe - mqDe\sqrt{a^2 + b_u b_v}} \\
\alpha_{yy} &= \frac{2b_u De\alpha_{xy}}{-1 + 2aDe + mqDe\sqrt{a^2 + b_u b_v}}
\end{align*}
\]
HOMOGENEOUS SOLUTION (3)

Back-substituting, three possible values of $q$ and three possible stress fields

\[
q = \frac{2}{m} - \frac{1}{m \text{De} \sqrt{a^2 + b_u b_v}}
\]

\[
\alpha_{xx} = \frac{b_v \alpha_{xy}}{-a + \sqrt{a^2 + b_u b_v}}
\]

\[
\alpha_{yy} = \frac{b_u \alpha_{xy}}{a + \sqrt{a^2 + b_u b_v}}
\]

\[
q = -\frac{1}{m \text{De} \sqrt{a^2 + b_u b_v}}
\]

\[
\alpha_{xx} = \frac{-b_v \alpha_{xy}}{a}
\]

\[
\alpha_{yy} = \frac{b_u \alpha_{xy}}{a}
\]

\[
q = -\frac{2}{m} - \frac{1}{m \text{De} \sqrt{a^2 + b_u b_v}}
\]

\[
\alpha_{xx} = \frac{-b_v \alpha_{xy}}{a + \sqrt{a^2 + b_u b_v}}
\]

\[
\alpha_{yy} = \frac{b_u \alpha_{xy}}{a - \sqrt{a^2 + b_u b_v}}
\]

Homogeneous solution is sum of all

Momentum not yet enforced

No boundary conditions imposed
MOMENTUM EQUATION (1)

\[ \frac{\partial}{\partial y} \left( -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) - \frac{\partial}{\partial x} \left( -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) = 0 \]

**Case 1**

\[ b_u = \frac{a^2}{b_v} \]

\[ b_u = b_v \]

\[ b_u = \frac{1 - 9a^2 De^2}{9b_v De^2} \]

\[ b_u = \frac{1 - 4a^2 De^2}{4b_v De^2} \]

\[ m = -1 \] avoids singularity at \( x=0, y=0 \) when \( De \ll 1 \)

singularities at all \( De \)

\[ b_u = b_v \]

possible forms to obey simultaneously momentum & UCM

Stagnation + “vortex” flow

\[ u = ax + b_u y \]

\[ v = -ay + b_v x \]

Note change of signs at \( De= 1/(3a) \) and \( 1/(2a) \)

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Transitions in some stagnation viscoelastic flows at \( Re=0 \)
Flow Instabilities and turbulence in viscoelastic fluids

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MOMENTUM EQUATION (2)

Case 2

\[ b_u = -\frac{a^2}{b_v} \]  
\[ b_u = \frac{1 - a^2 De}{b_v De^2} \]  
\[ b_u = -2ia - b_v \]  
\[ b_u = 2ia - b_v \]

\[ \text{singularities} \]

ok, but needs to be compatible with case (1)  
(restrict values of \(a\) and \(De\))

We will consider no contributions from case 2  
to the solution (k=0 and \(\alpha_{xy}=0\))

Case 3

After substitution of stresses all terms in equation are multiplied by \((1+m)\). 
Since \(m=-1\), momentum is automatically satisfied

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\[ \tau_{xx} = \frac{b_v \alpha_{xy1}}{-a + \sqrt{a^2 + b_u b_v}} \frac{2}{m} \frac{1}{mDe \sqrt{a^2 + b_u b_v}} \phi - \frac{b_v \alpha_{xy3}}{a + \sqrt{a^2 + b_u b_v}} \frac{2}{m} \frac{1}{mDe \sqrt{a^2 + b_u b_v}} \phi - \frac{2(a + 2Dea^2 + b_u b_v De + b_v^2 De)}{4a^2 De^2 - 1 + 4b_u b_v De^2} \]

\[ \tau_{xy} = \alpha_{xy1} \phi + \alpha_{xy3} \phi - \frac{b_u + b_v + 2aDe(b_u - b_v)}{4a^2 De^2 - 1 + 4b_u b_v De^2} \]

\[ \tau_{yy} = \frac{b_u \alpha_{xy1}}{a + \sqrt{a^2 + b_u b_v}} \frac{2}{m} \frac{1}{mDe \sqrt{a^2 + b_u b_v}} \phi + \frac{b_u \alpha_{xy3}}{a - \sqrt{a^2 + b_u b_v}} \frac{2}{m} \frac{1}{mDe \sqrt{a^2 + b_u b_v}} \phi - \frac{2(-a + 2Dea^2 + b_u b_v De + b_u^2 De)}{4a^2 De^2 - 1 + 4b_u b_v De^2} \]

with \( \alpha_{xy1} = \alpha_{xy1}(a, b_u, b_v), \alpha_{xy3} = \alpha_{xy3}(a, b_u, b_v) \) such as

\[ \alpha_{xy1} = \frac{\alpha_1(-a + \sqrt{a^2 + b_u b_v})}{b_v} \]
\[ \alpha_{xy3} = \frac{\alpha_3(a + \sqrt{a^2 + b_u b_v})}{b_v} \]

\( a = 1, b_u = 0, b_v = 0 \Rightarrow \text{Becherer et al. JNNFM 153 (2008) 183} \)
STREAMLINES AND STRESSES (1)

Stream function \[ \psi = axy + b_u \frac{y^2}{2} - b_v \frac{x^2}{2} \]

Stream function of vortex \[ \psi_1 = \psi_{total} - \psi_{stagnation} = b_u \frac{y^2}{2} - b_v \frac{x^2}{2} \]

\[ b_u = b_v \]
\[ b_u = \frac{1 - 9a^2 De}{9b_v De^2} \]
\[ b_u = \frac{1 - 4a^2 De}{4b_v De^2} \]

(1) \[ De < \frac{1}{\sqrt{9a^2}} \]

\[ b_u = b_v = 2.85; a = -1; \]
\[ De = 0.2 \]
Transitions in some stagnation viscoelastic flows at Re=0

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STRESSES (2)

(1a) $De < \frac{1}{\sqrt{9a^2}}$  
$De = 0.2; a = -1$

$b_v = b_u = 2.5$

$b_v = b_u = 2.3$

$b_v = b_u = 1.5$

$b_v = b_u = 1.2$

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(2) $De < \frac{1}{\sqrt{9a^2}}$

$$b_u = \frac{1 - 9a^2De^2}{9b_vDe^2}; a = -1; b_v = 5; De = \frac{1}{3\sqrt{a^2}} - 0.001$$
STREAMLINES AND STRESSES (4)

$\tau_{xx}$

$b_v = 5$

$\tau_{yy}$

$\tau_{yy}$

$b_v = 0.5$

$y = 0.000025$

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STREAMLINES AND STRESSES (5)

(3) $De > \frac{1}{\sqrt{9a^2}}$

$$b_u = \frac{1 - 9a^2De^2}{9b_vDe^2}; a = -1; b_v = 5; De = \frac{1}{3\sqrt{a^2}} + 0.001$$

$b_u, b_v$

Opposite signs

Vortex

$De < De_c$  Vortex enclosing stagnation point is not possible.

$De > De_c$  Vortex enclosing stagnation point is possible.

Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids

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STREAMLINES AND STRESSES (6)

\[ b_v = 5 \]
\[ \tau_{xx} \]

\[ \tau_{yy} \]

\[ b_v = 0.5 \]
\[ \tau_{xx} \]
\[ \tau_{yy} \]

\[ y = 0.000025 \]

Transitions in some stagnation viscoelastic flows at Re=0
Flow Instabilities and turbulence in viscoelastic fluids

Afonso, Cruz, Alves & Pinho - CEFT/FEUP
Leiden, Holland, 19-23\textsuperscript{th} July 2010
(4) $De > \frac{1}{\sqrt{9a^2}}$ - Circular vortex

$$b_u = \frac{1 - 9a^2De^2}{9b_vDe^2}; a = -1; b_v = 0.23999; De = \frac{1}{3\sqrt{a^2}} + 0.01$$
Transitions in some stagnation viscoelastic flows at Re=0

Flow Instabilities and turbulence in viscoelastic fluids

y = 0.000025
CLOSURE

• Steady symmetric to steady asymmetric is a purely elastic instability. Inertia and solvent delays and eliminates this transition.

• This transition exists with bounded extensional viscosity, but is weakened with $\varepsilon$

• Steady asymmetric flow is a combination of a planar stagnation and a vortex

• Analytical solution obtained enforcing UCM constitutive equation and momentum. It shows closed vortex cannot exist below $De<1/(3a)$

• Behavior of the solution currently under investigation: need to impose BC

• Need for stability analysis on the analytical solution.
ACKNOWLEDGEMENTS


• CNpQ 200120/2009-3