UNIFORM FLOW OF VISCOELASTIC FLUIDS PAST A CONFINED CYLINDER

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Motivation

- **Relate rheology and negative wake**
- **Clarify discrepancies in literature (numerical works)**
- **Accurate numerical results**
- **Parametric investigation**

Experimental findings (Sphere) ↓

- **Boger fluids: no negative wake**
- **Shear-thinning: negative wake**

Hassager (1979), Bisgaard (1983); Arigo et al (1995); Arigo & McKinley (1998);Bush (1993,1994)

Numerical findings

- **Boger fluids: negative/ no negative wake**
- **Shear-thinning: negative wake**
- **Discrepancies: accuracy, mesh fineness (same model)**

Cylinder: Dou & PT (2003,2004);Kim et al (2005)

Sphere:Harlen (2002); Satrape and Crochet (1994)

Flow condition, fluids models and computational domain

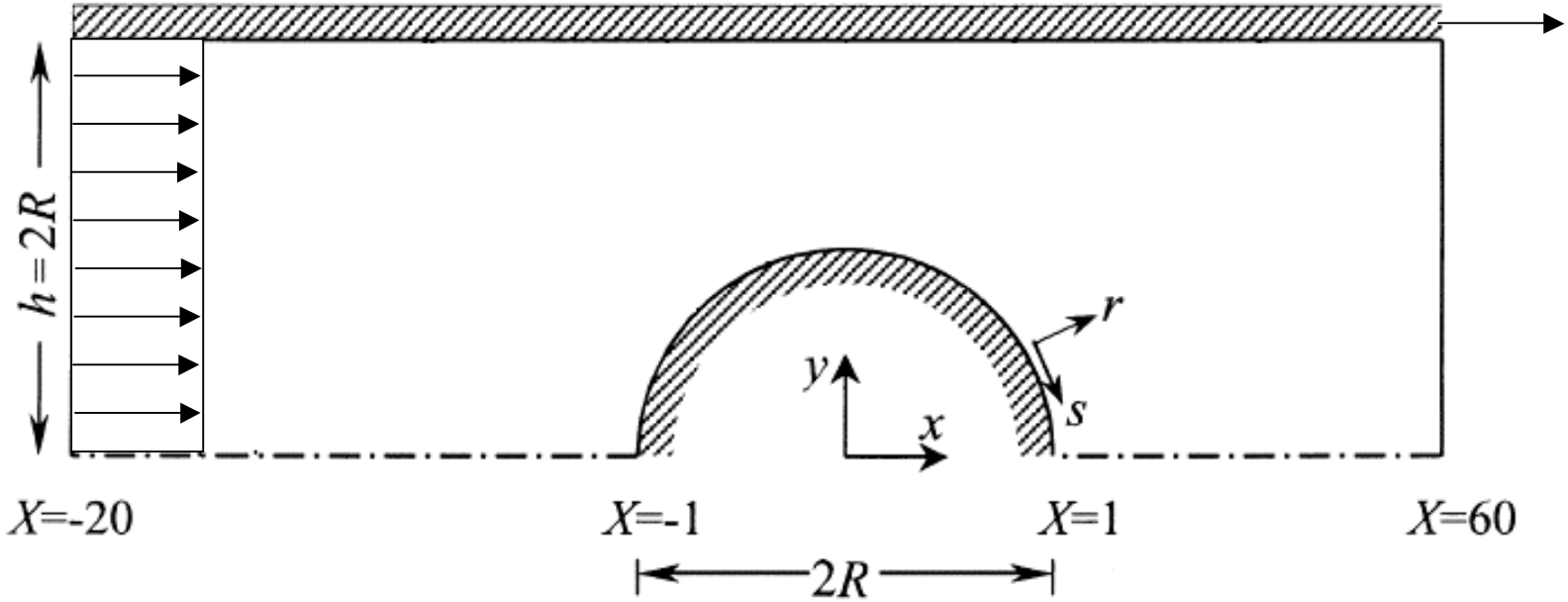
Flow conditions

- **uniform flow**
- **50% blockage**

Fluid Models

- **Boger fluids: UCM, Oldroyd-B, FENE-MCR**
- **Shear-thinning: PTT, (Giesekus)**

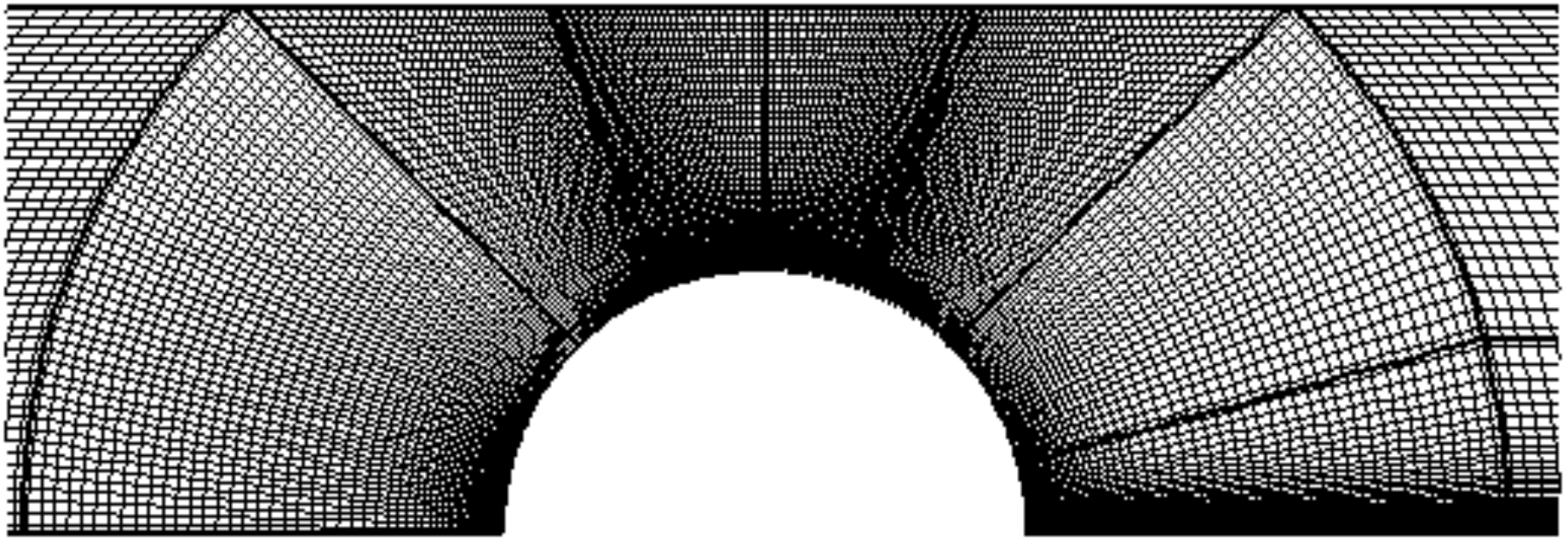
Computational domain



Meshes

22 560 total cells
135 360 DoF

Mesh M60WR



Mesh M120WR

90 240 total cells
541 440 DoF

Governing equations

1) Mass

2) Momentum

$$\frac{\partial}{\partial t}\left(J\rho u_i\right)+\frac{\partial}{\partial \xi_l}\left(\rho\beta_{lj}\tilde{u}_j\hat{u}_i\right)=-\beta_{li}\frac{\partial p}{\partial \xi_l}+\frac{\partial}{\partial \xi_l}\left(\beta_{lj}\tilde{\tau}_{ij,p}+\beta_{lj}\tau_{ij,s}\right)++J\rho g_i$$

3) Constitutive equation

$$\tau_{ij}=2\eta_s D_{ij}+\tau_{ij,p}$$

Newtonian solvent

PTT (linear stress coefficient) and Giesekus

$$\left(1+\frac{\lambda\varepsilon}{\eta_p}\tau_{kk,p}\right)\tau_{ij,p}+\lambda\left(\frac{\partial\tau_{ij,p}}{\partial t}+\frac{\partial\left(u_k\tau_{ij,p}\right)}{\partial x_k}\right)=\eta_p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\lambda\left(\tau_{jk,p}\frac{\partial u_i}{\partial x_k}+\tau_{ik,p}\frac{\partial u_j}{\partial x_k}\right)-$$

$$\lambda\frac{\xi}{2}\left(\tau_{jk,p}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)+\tau_{ik,p}\left(\frac{\partial u_j}{\partial x_k}+\frac{\partial u_k}{\partial x_j}\right)\right)-\frac{\alpha\lambda}{\eta_p}\tau_{ik}\tau_{kj}$$

Numerical method: brief description

1) Finite volume method (Oliveira et al,1998; Oliveira & Pinho, 1999)

2) Structured, colocated and non-orthogonal meshes

3) Momentum (u_i)

$$\frac{\partial}{\partial t}\left(J\rho u_i\right)+\frac{\partial}{\partial \xi_l}\left(\rho\beta_{lj}\tilde{u}_j\hat{u}_i\right)-\frac{\partial}{\partial \xi_l}\left(\frac{\eta_p}{J}\beta_{lj}\beta_{lj}\frac{\partial u_i}{\partial \xi_l}\right)-\frac{\partial\left(\beta_{lj}\tau_{ij,s}\right)}{\partial \xi_l}=-\beta_{li}\frac{\partial p}{\partial \xi_l}+\frac{\partial}{\partial \xi_l}\left(\beta_{lj}\tilde{\tau}_{ij}\right)+J\rho g_i-\frac{\partial}{\partial \xi_l}\left(\frac{\eta_p}{J}\beta_{lj}\beta_{lj}\frac{\partial u_i}{\partial \xi_l}\right)$$

solvent polymer

4) Discretization (formally 2nd order)

Diffusive terms: central differences (**CDS**)

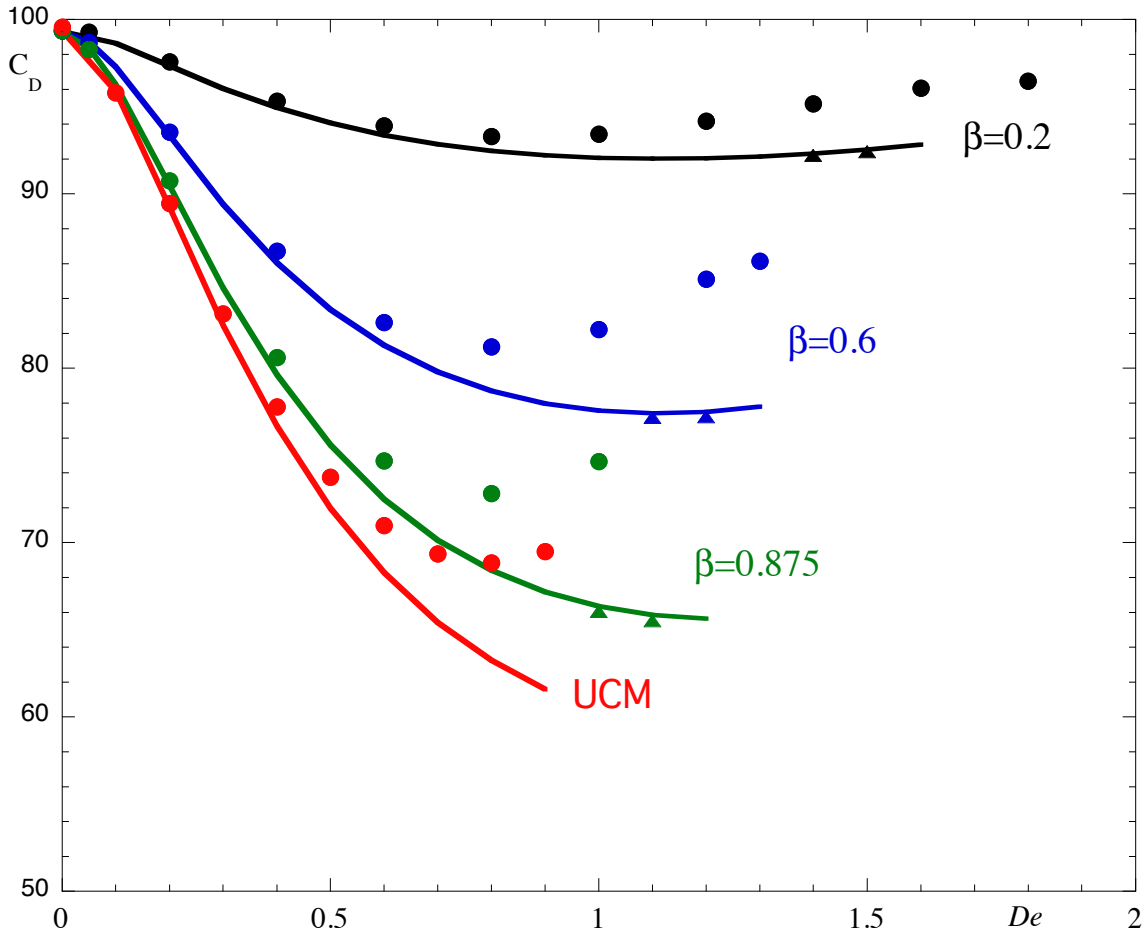
Advective terms: **CUBISTA (deferred correction)**

(Alves et al, 2000, 2003)

5) Special formulations for cell-face velocities and stresses

Results: Oldroyd-B e UCM: C_d

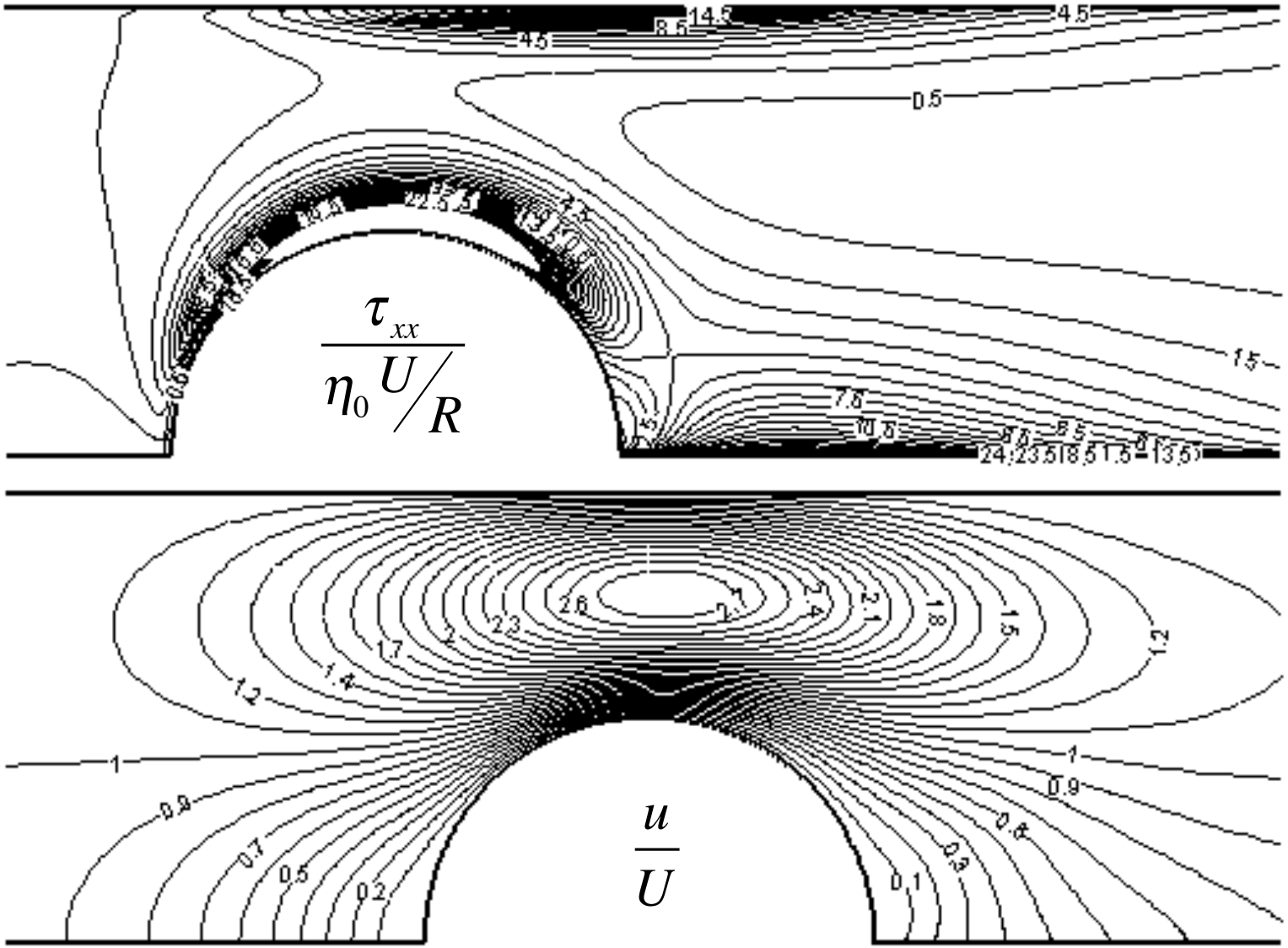
Deborah number $De = \dfrac{U\lambda_p}{R}$ $\qquad Re = 0$ $\qquad \beta \equiv \dfrac{\eta_p}{\eta_p + \eta_s}$



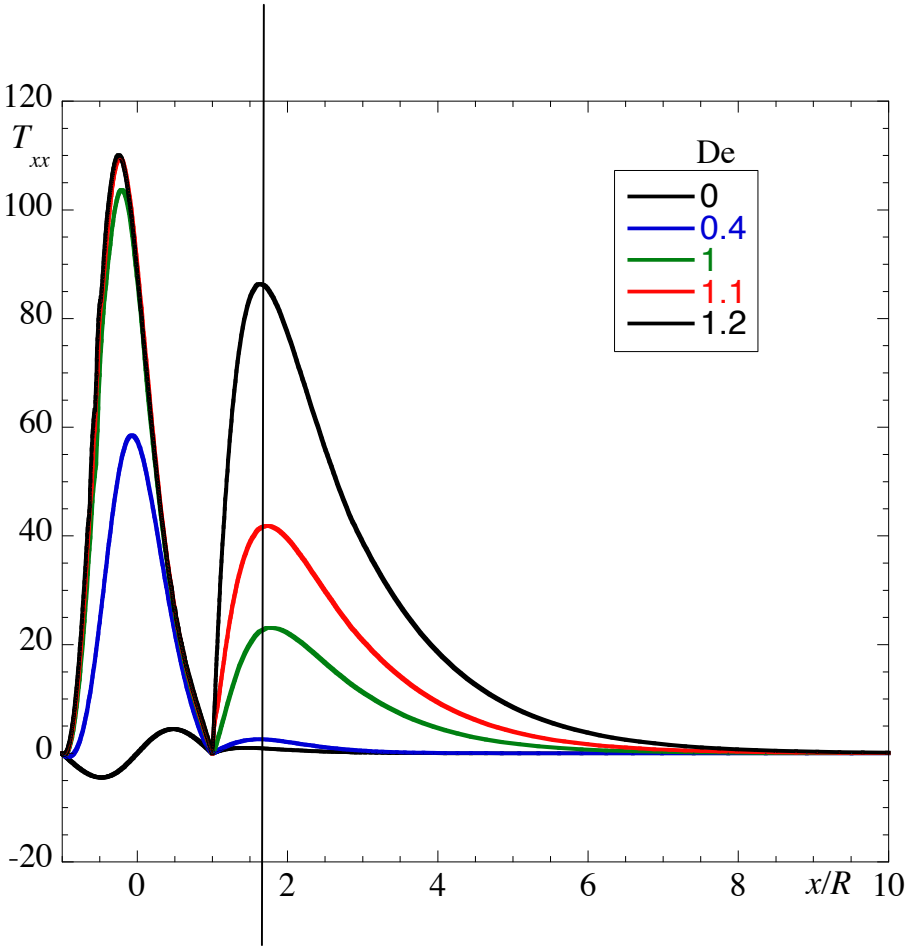
- Dou&PT (2003)

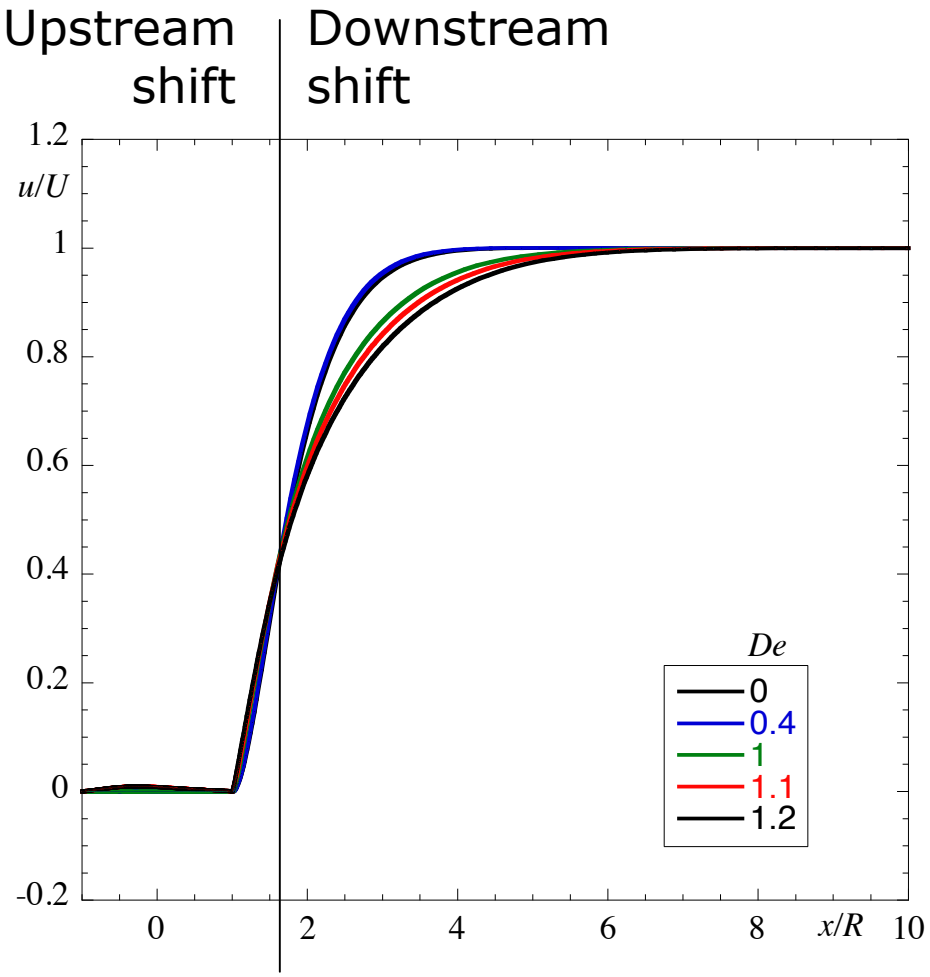
Results: Oldroyd-B, β= 0.6 (1)

$$De = 1.2$$

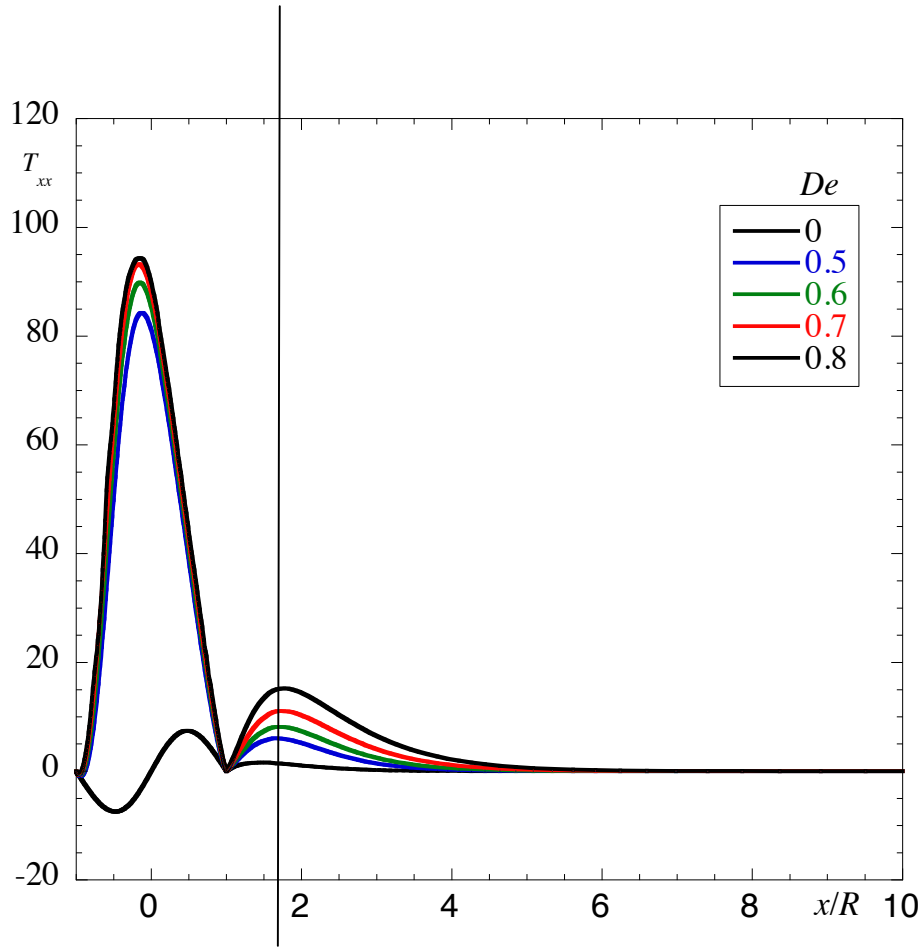


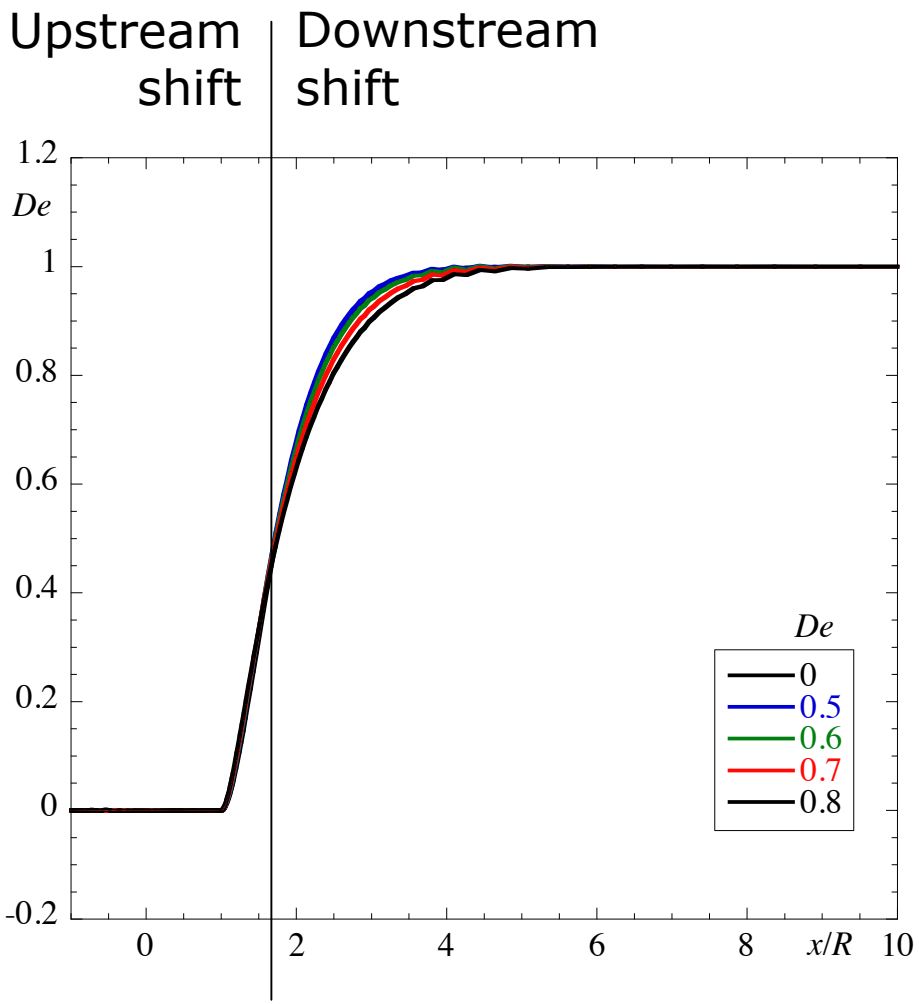
Results: Oldroyd-B, β= 0.6 (2)





Results: UCM (1)

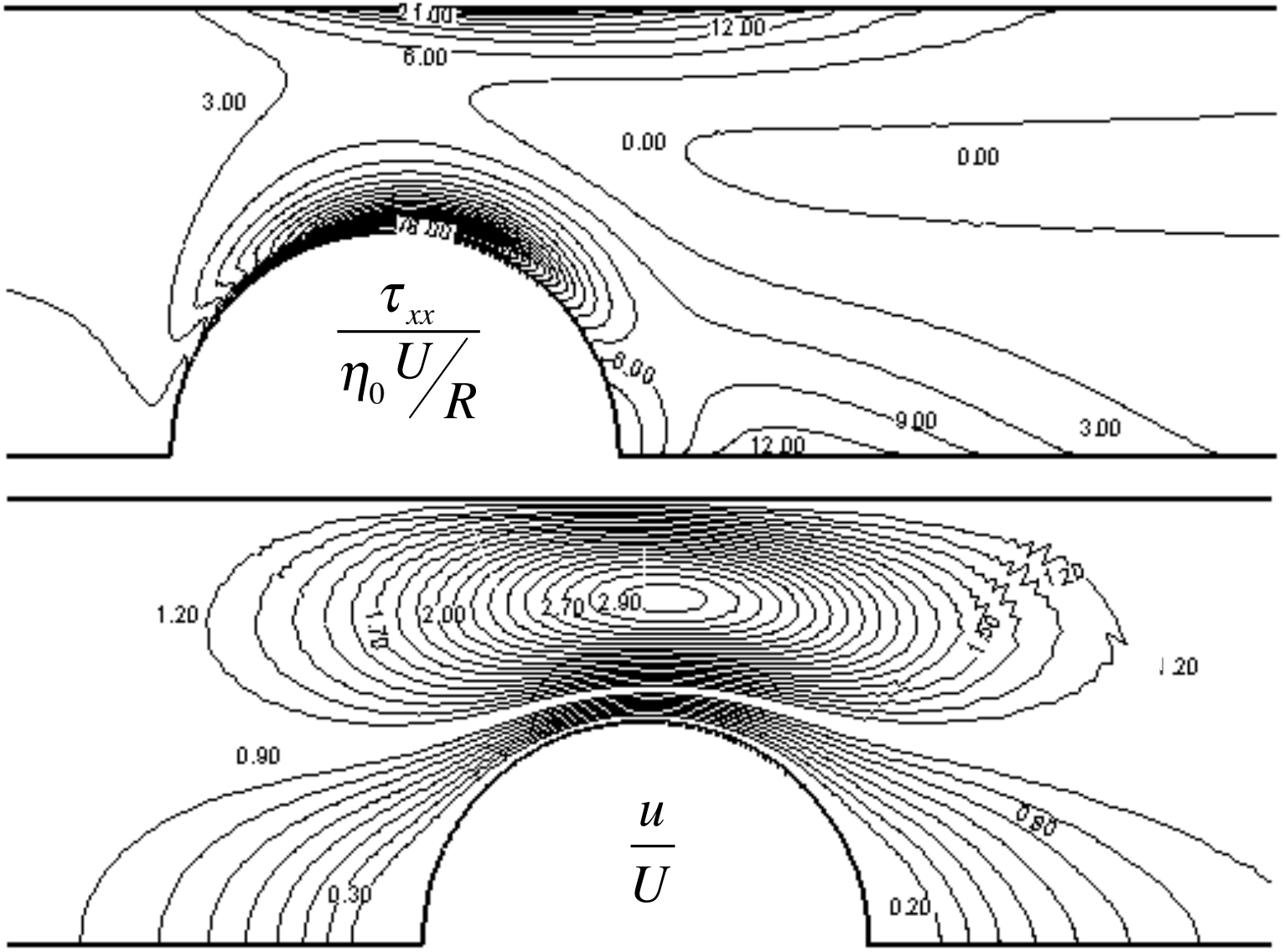




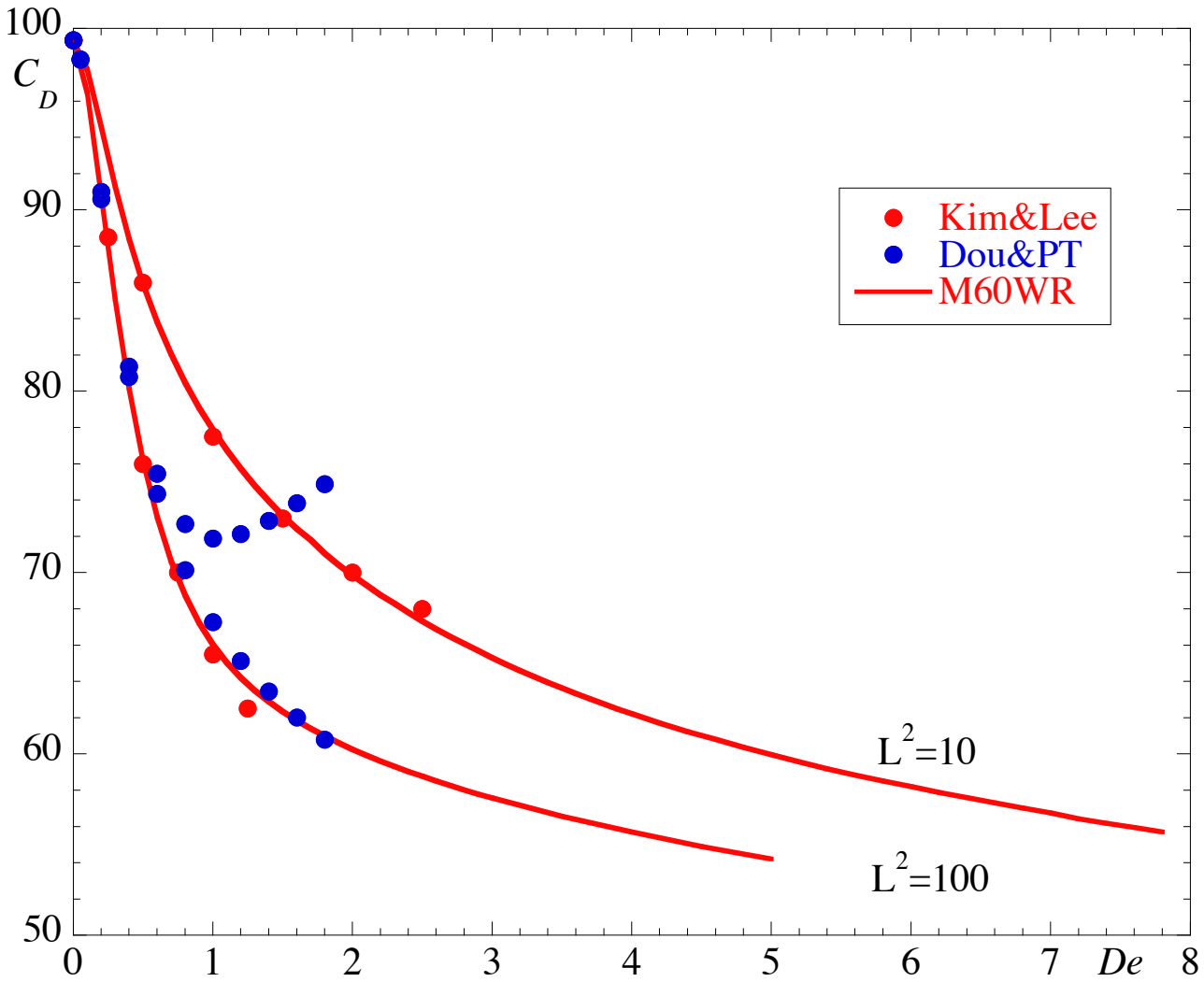
- Upstream shift size decreases when β increases (UCM) $\rightarrow 2^{nd}\tau_{xx} \downarrow$
- No negative wake

Results: UCM (2)

$De = 0.8$

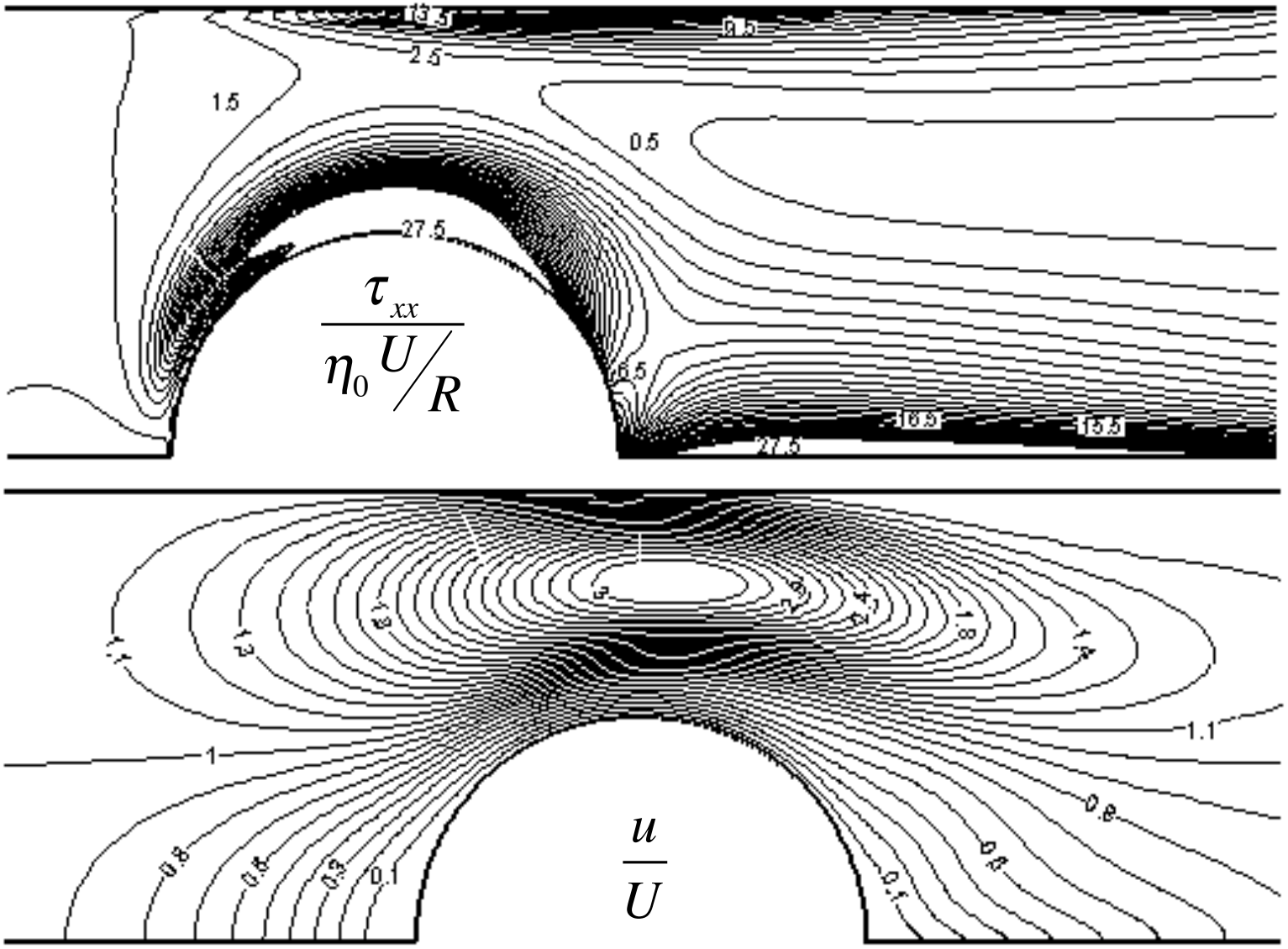


Results: FENE-(M)CR

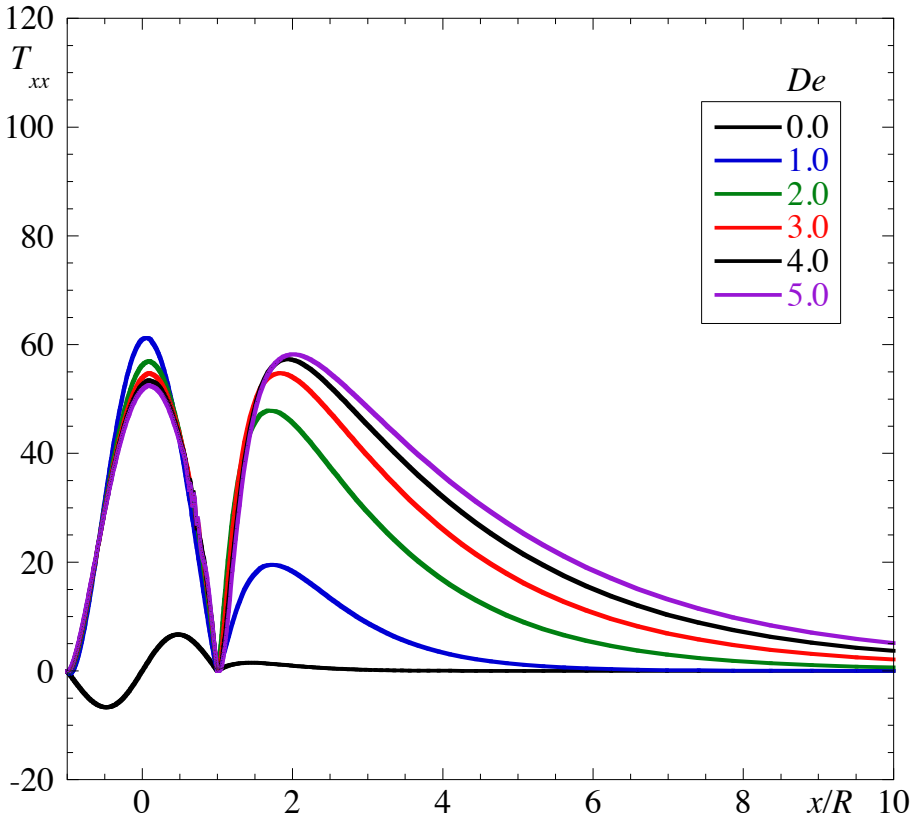


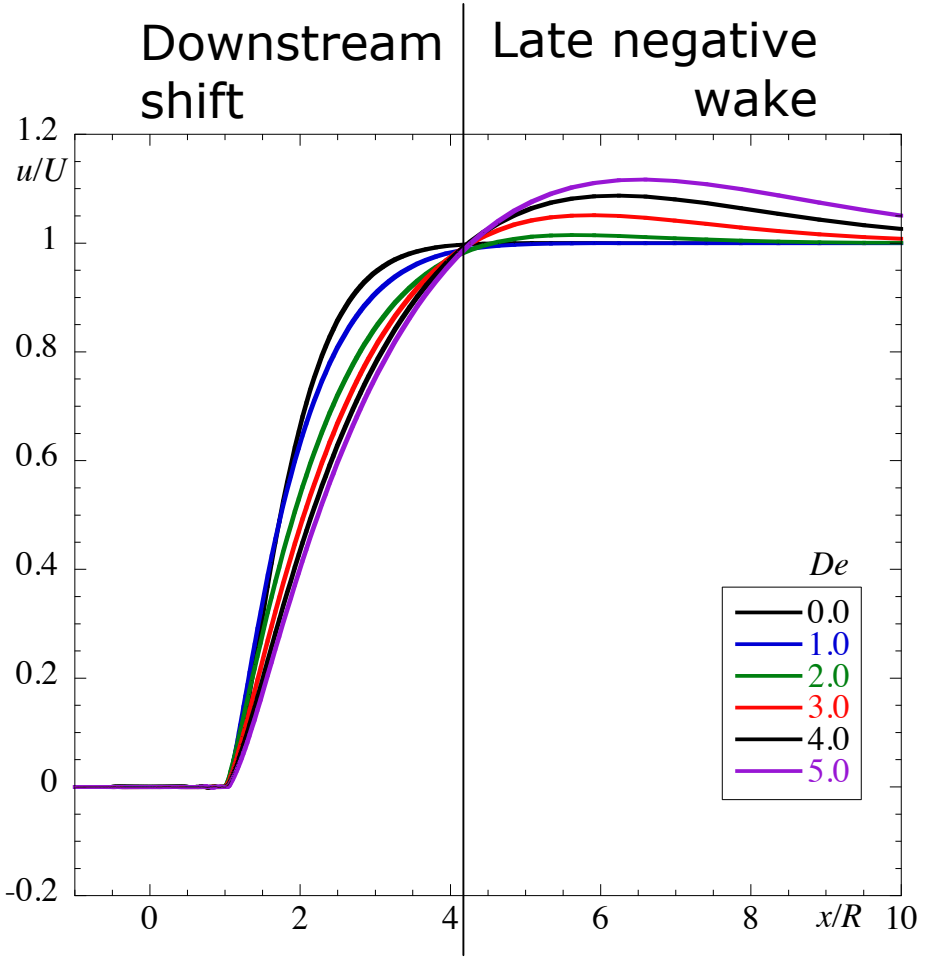
Results: FENE-(M)CR- L^2=100 (1)

$\beta = 0.9$ $De = 3$



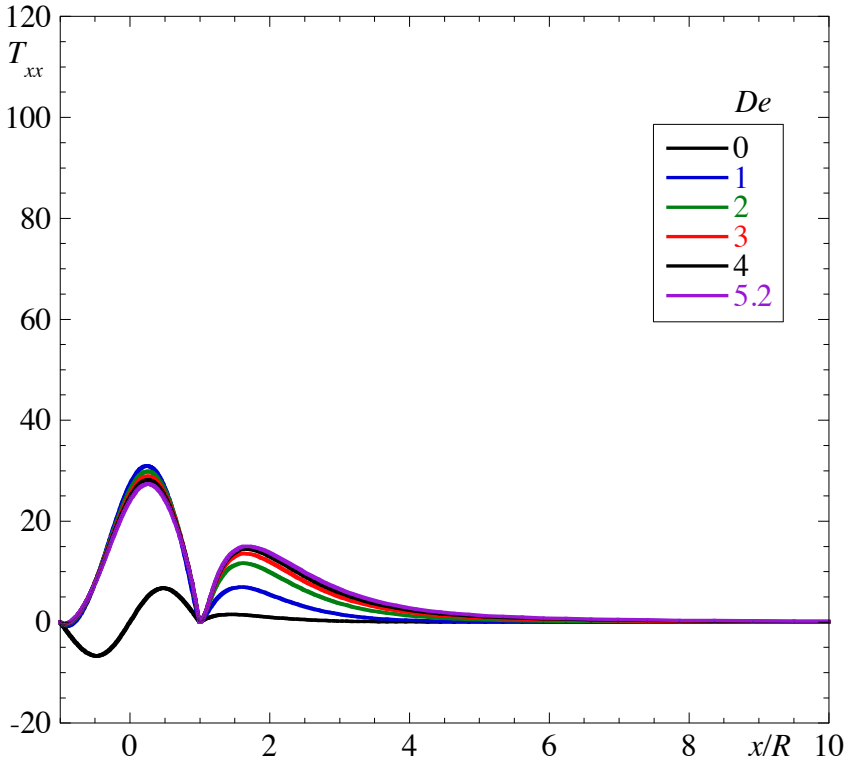
Results: FENE-(M)CR- L^2=100 (2)



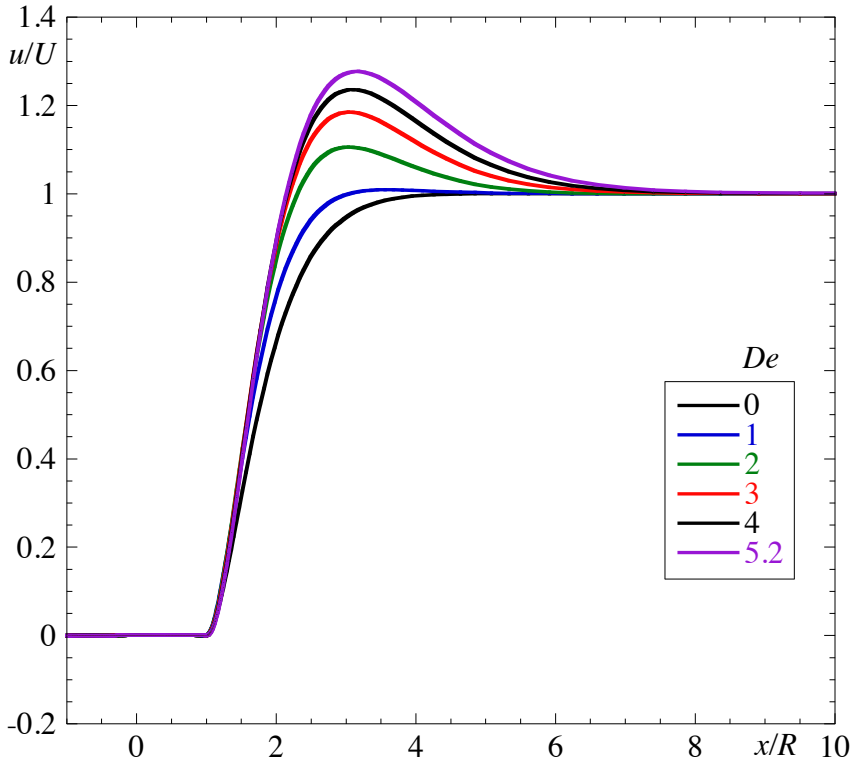


Agreement with Kim et al (2005)

Results: FENE-(M)CR- L^2=10

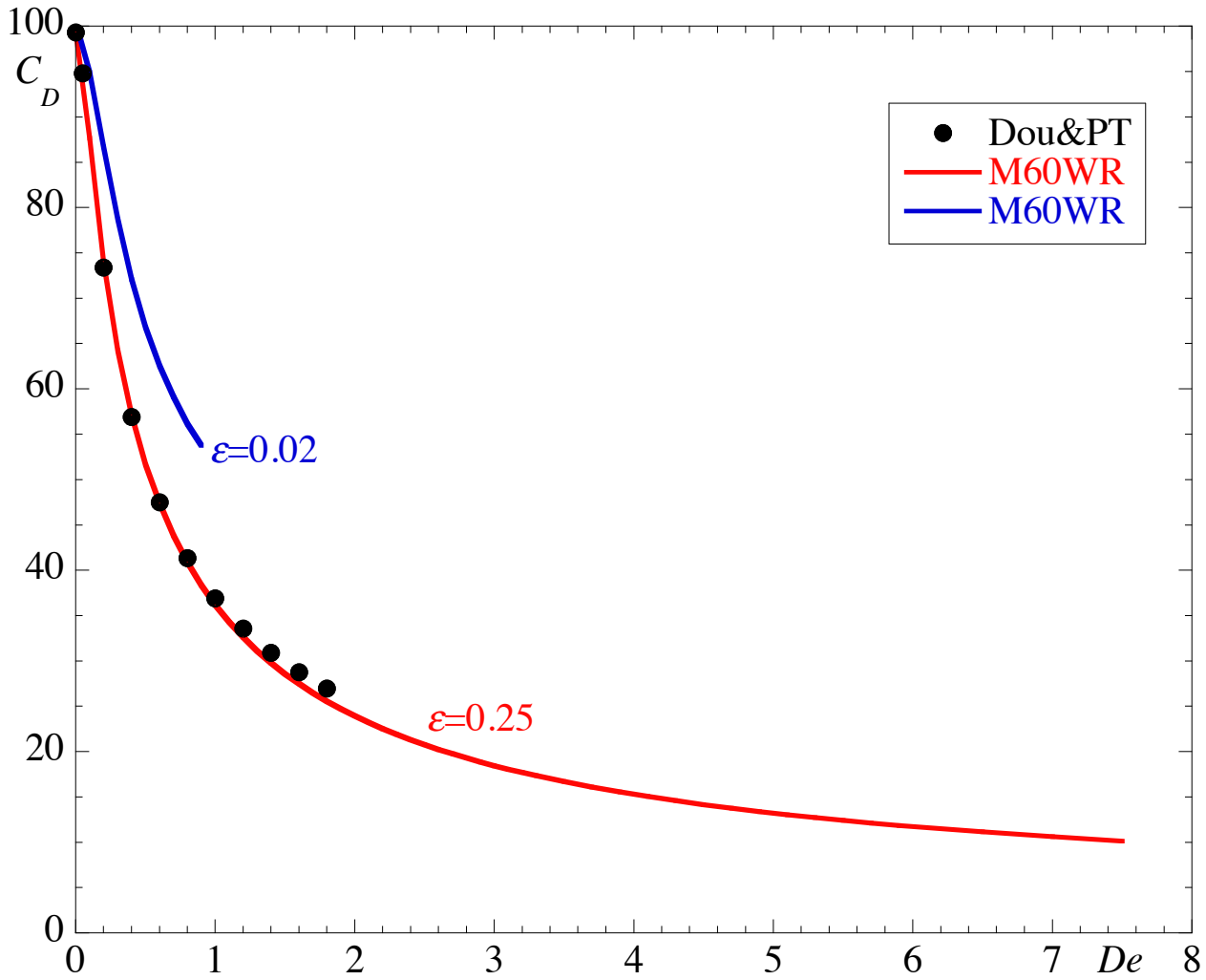


Upstream shift and
strong negative wake



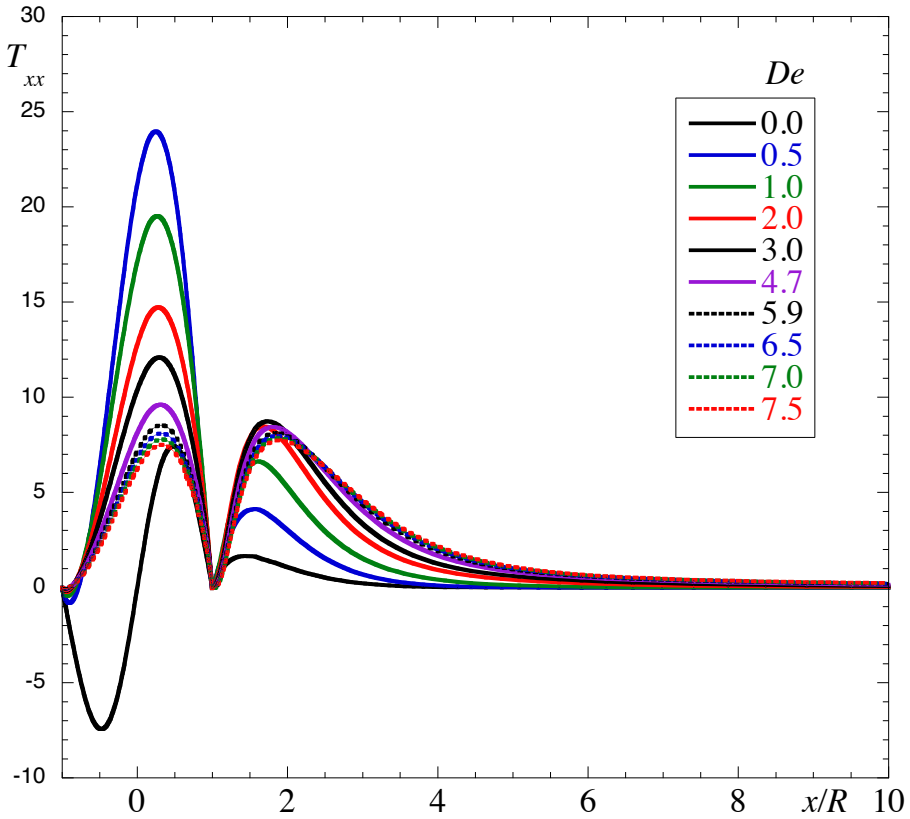
Results: PTT (ξ=0)

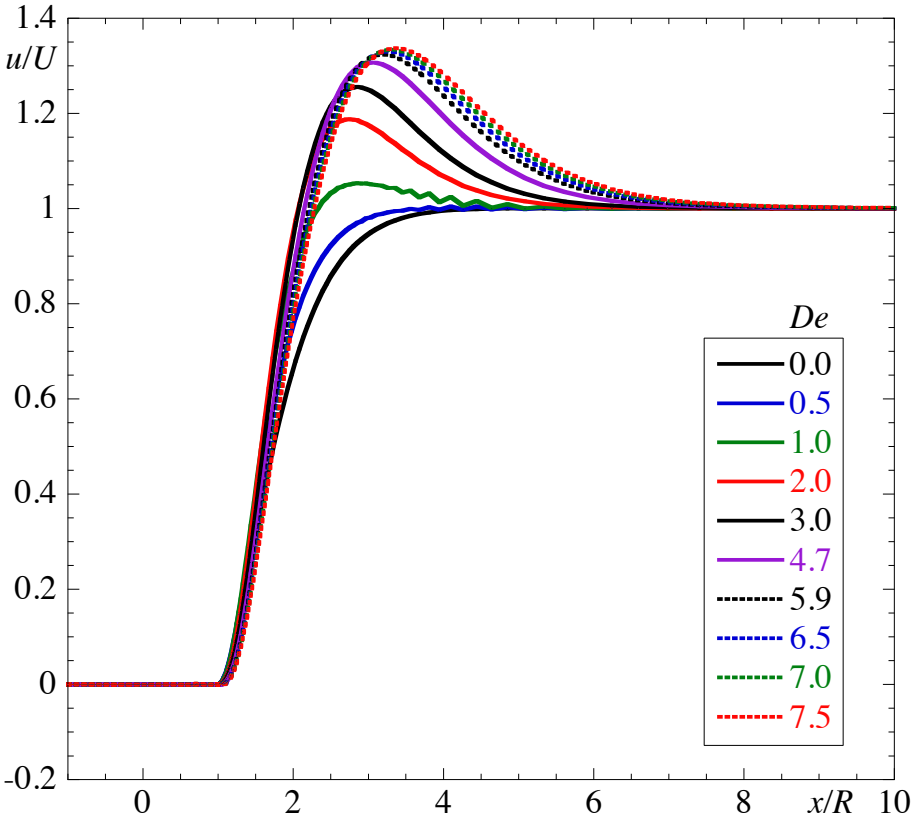
$$\beta = 1$$



Results: PTT (ξ=0) ε=0.25 (1)

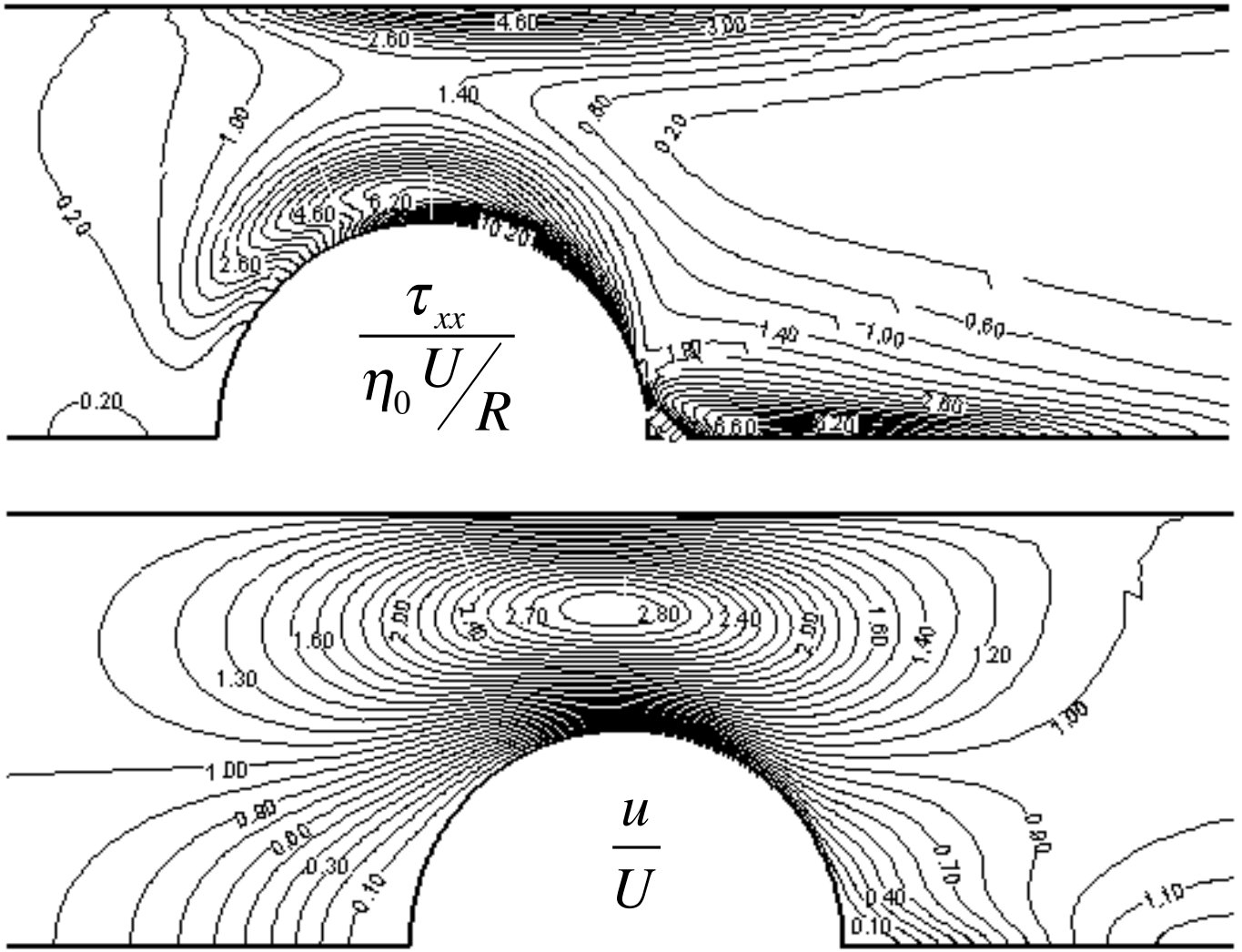
Upstream shift and
strong negative wake



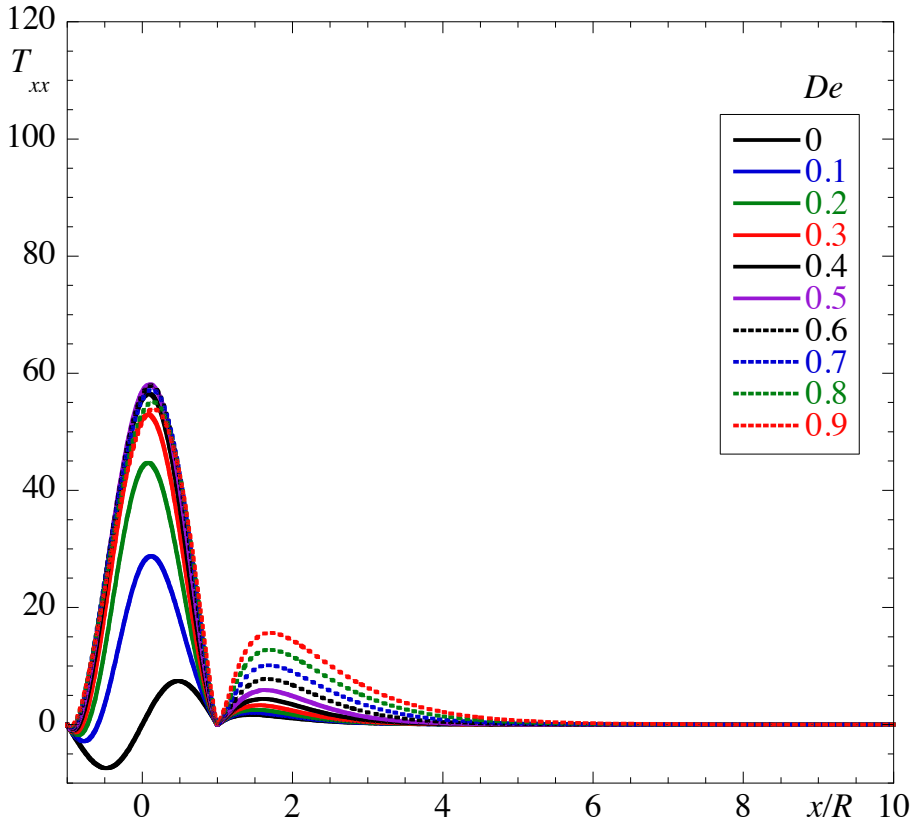


Results: PTT (ξ=0) ε=0.25 (2)

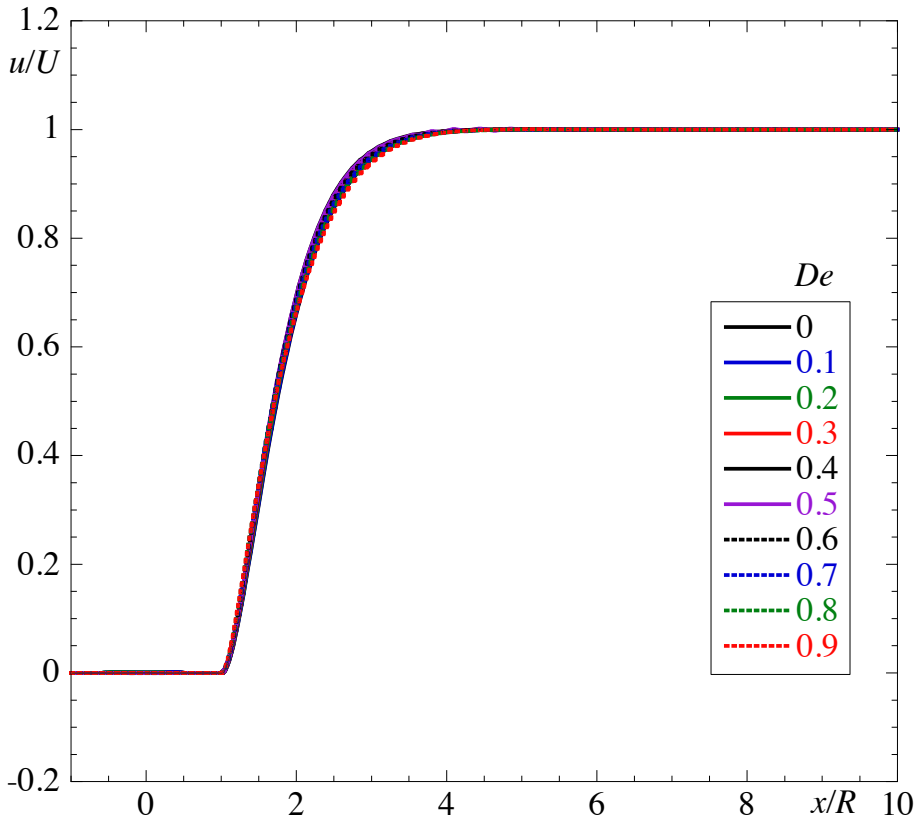
$$De = 3$$



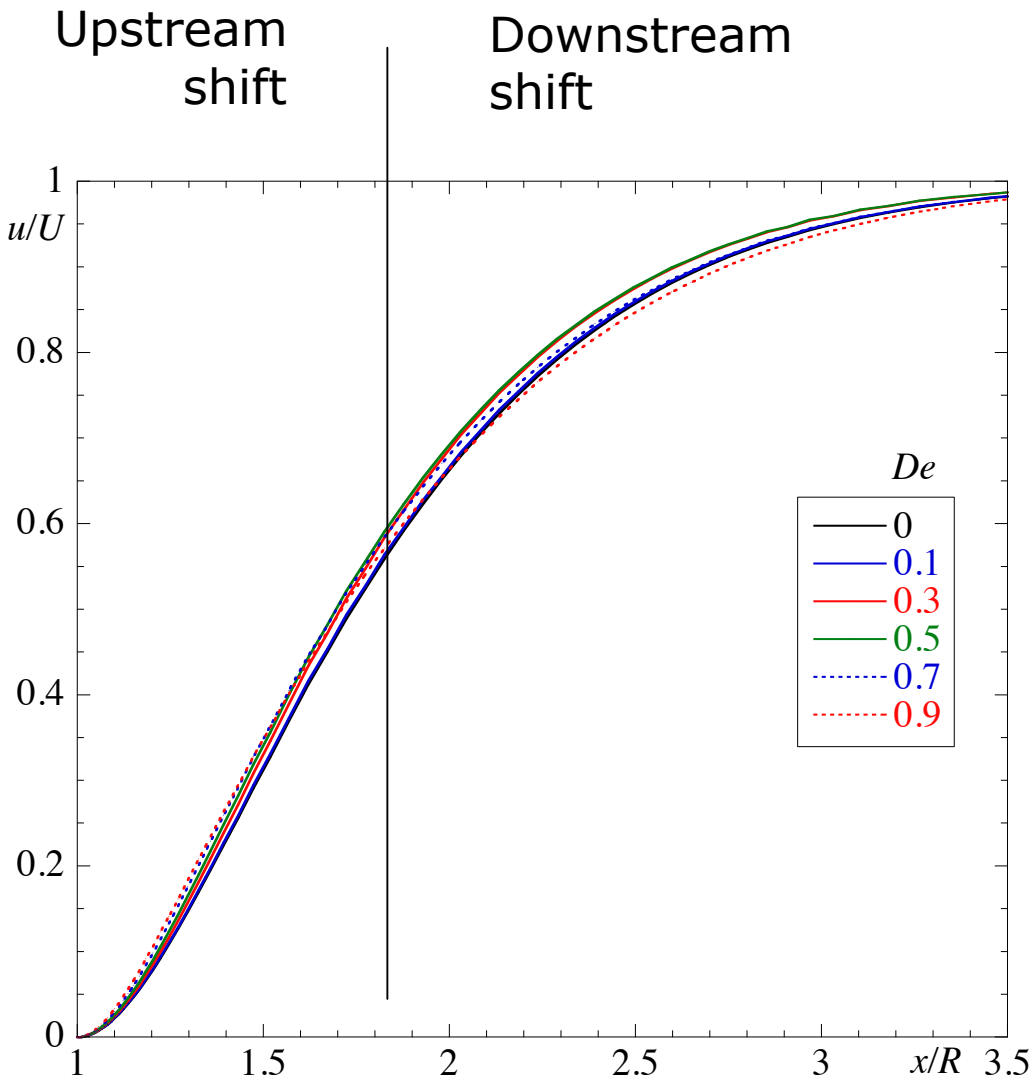
Results: PTT (ξ=0) ε=0.02 (1)



No negative wake

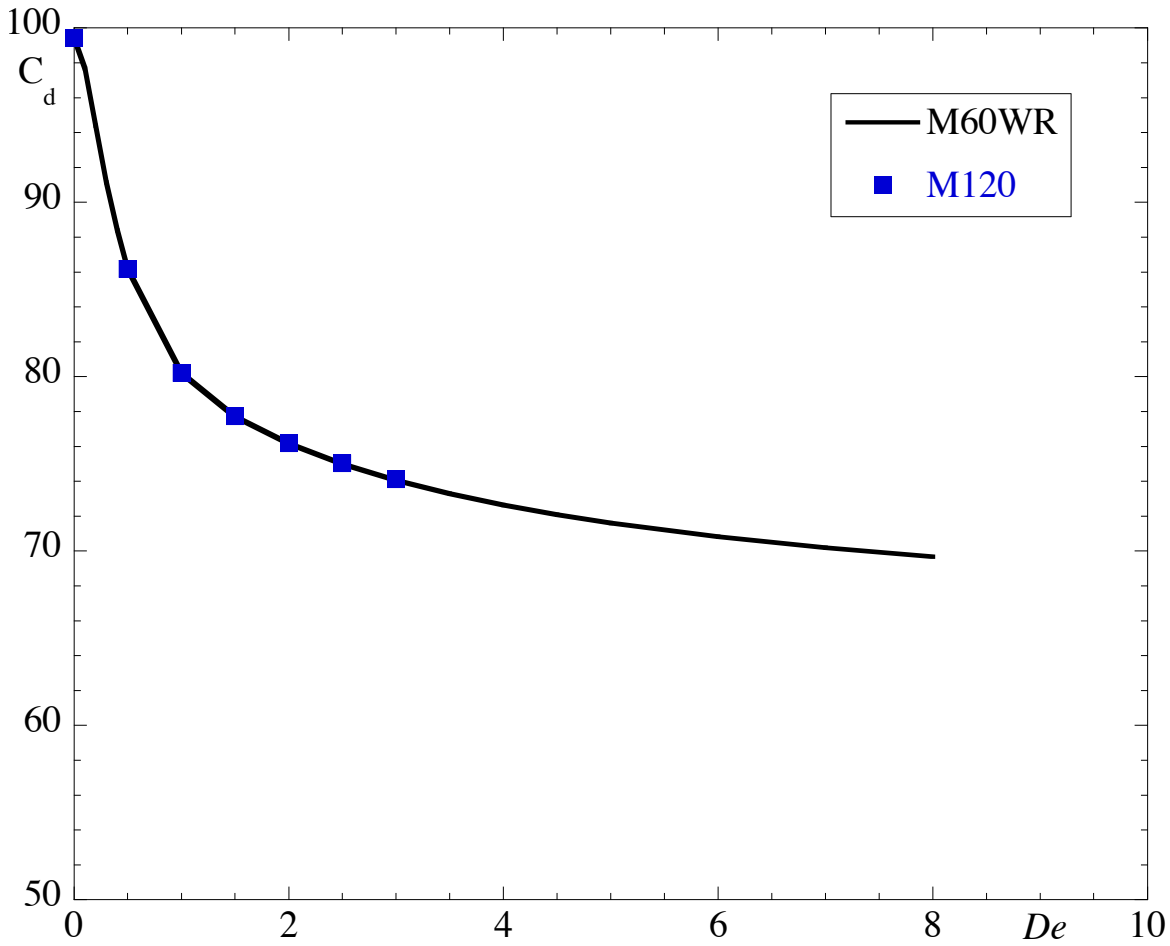


Results: PTT (ξ=0) ε=0.02 (2)



Results: Giesekus (α=0.01)

$$\beta = 0.41$$



Conclusions (1)

- **UCM e Oldroyd-B**
  - **C_d differs from literature (Dou & Phan-Thien)**
  - **No negative wake; near cylinder upstream streamline shift, far cylinder downstream streamline shift**
  - **First T_{xx} peak stabilizes and second peak grows with De until divergence. For large β second peak grows less**

- **FENE-(M)CR**
  - **C_d lower than some literature (Dou & Phan-Thien), but matcheng Kim et al**
  - **L^2=100: Negative wake far from cylinder**
  - **L^2=10: Strong negative wake at rear of cylinder**
  - **Second peak T_{xx} grows and exceeds first peak. First peak decreases with *De***

Conclusions (2)

- **PTT**
  - **C_d slightly lower than literature (ε=0.25) (Dou & Phan-Thien)**
  - **Increasing upstream streamline shift with De and strong negative wake near the cylinder**
  - **Second peak of T_{xx} grows initially and saturates. First peak decreases with *De*— Contrast behaviour seen with other fluids**

Acknowledgements

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FCT (Fundação para a Ciência e Tecnologia) and **FEDER**

Effect of contraction ratio upon viscoelastic flow in contractions: the axisymmetric case

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Objectives

- Study numerically the flow of Oldroyd-B and PTT fluids in axisymmetric contractions
- Analyse the influence of contraction ratio, $CR = R_1/R_2$
- Find scaling rules for X_R, Ψ_R and C
- Compare with similar results for planar contractions

Motivation

Nigen & Walters (JNNFM vol. 102):

"The preoccupation with the 4:1 contraction is now known to be misguided and many new and fascinating flow features are encountered beyond that ratio ..."

Equations

Continuity: $\nabla \cdot \boldsymbol{u} = 0$

Momentum: $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \nabla \cdot \left[\eta_s \left(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}} \right) \right]$

Constitutive Equation:

$$\left[1 + \frac{\lambda \varepsilon}{\eta_P} \mathrm{tr}(\boldsymbol{\tau}) \right] \boldsymbol{\tau} + \lambda \left[\frac{\partial \boldsymbol{\tau}}{\partial t} + \nabla \cdot \boldsymbol{u}\boldsymbol{\tau} \right] = \eta_P \left(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}} \right) + \lambda \left(\boldsymbol{\tau} \cdot \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}} \cdot \boldsymbol{\tau} \right)$$

$\varepsilon > 0 \rightarrow$ Linear PTT model ($\varepsilon = 0.25$)

$\varepsilon = 0 \rightarrow$ Oldroyd-B model

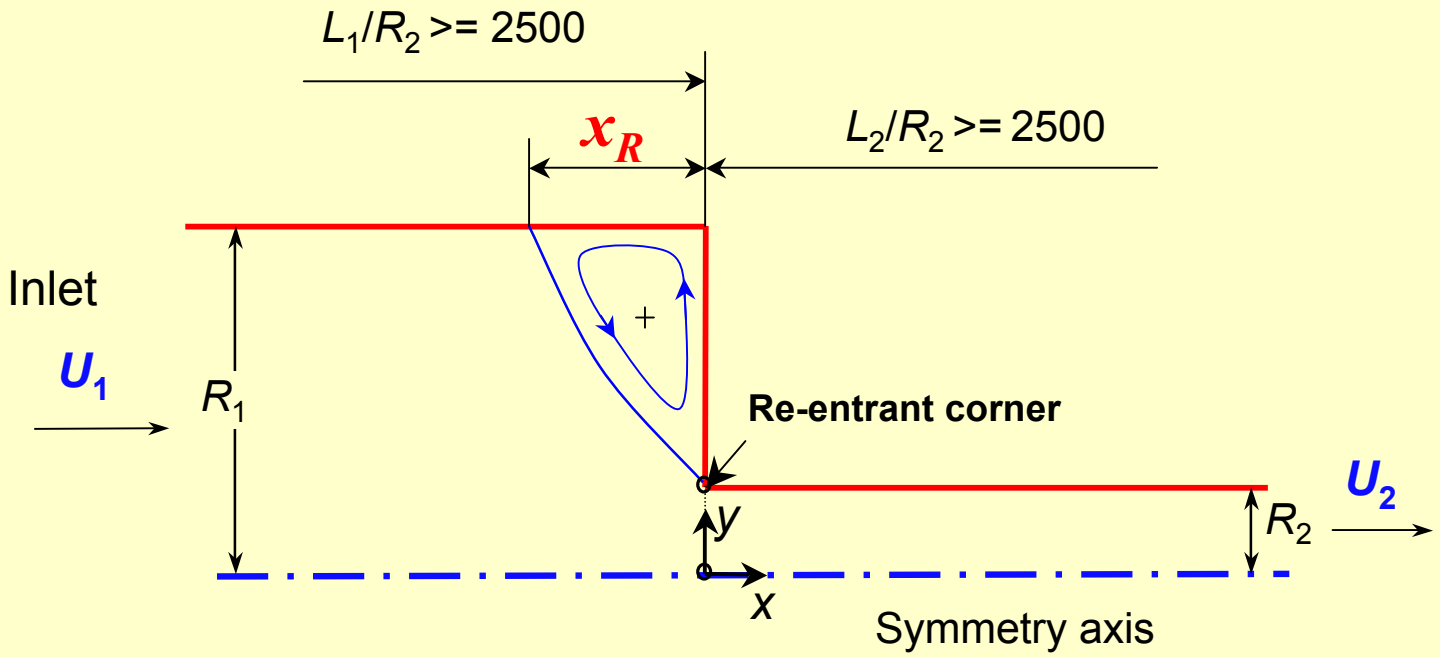
$$\beta = \frac{\eta_S}{\eta_S + \eta_P} = \frac{1}{9}$$

(Solvent viscosity ratio)

Finite Volume Method properties

- Generalised Non-Orthogonal Coordinates
- Block structured meshes
- Collocated meshes
- Time marching algorithm (1st order implicit Euler method)
- SIMPLEC algorithm (minor modifications due to the presence of constitutive eq.)
- CUBISTA High-Resolution Scheme (convective fluxes)

Specification of problem and Computational Meshes

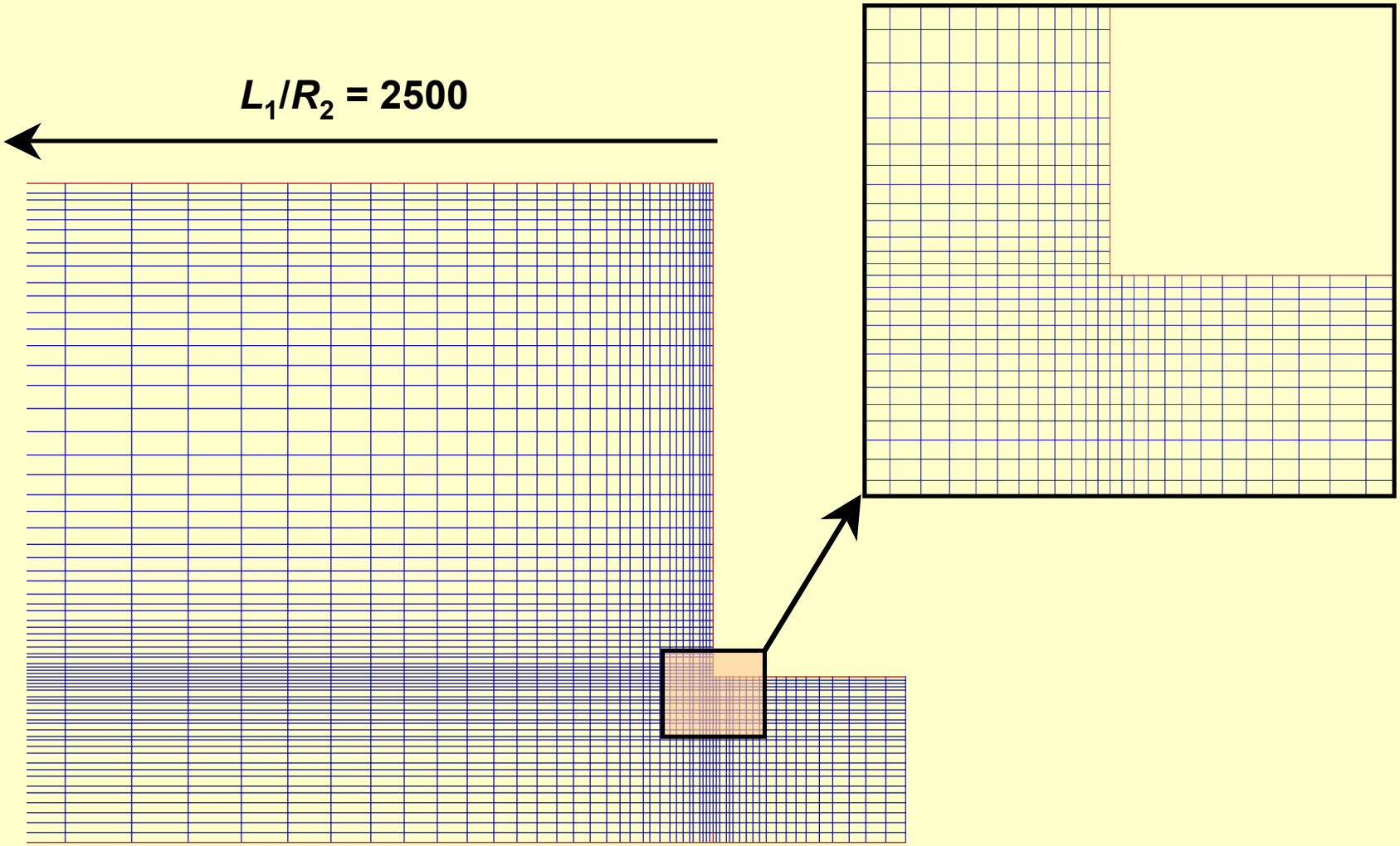


$Re = 0$ (creeping flow)

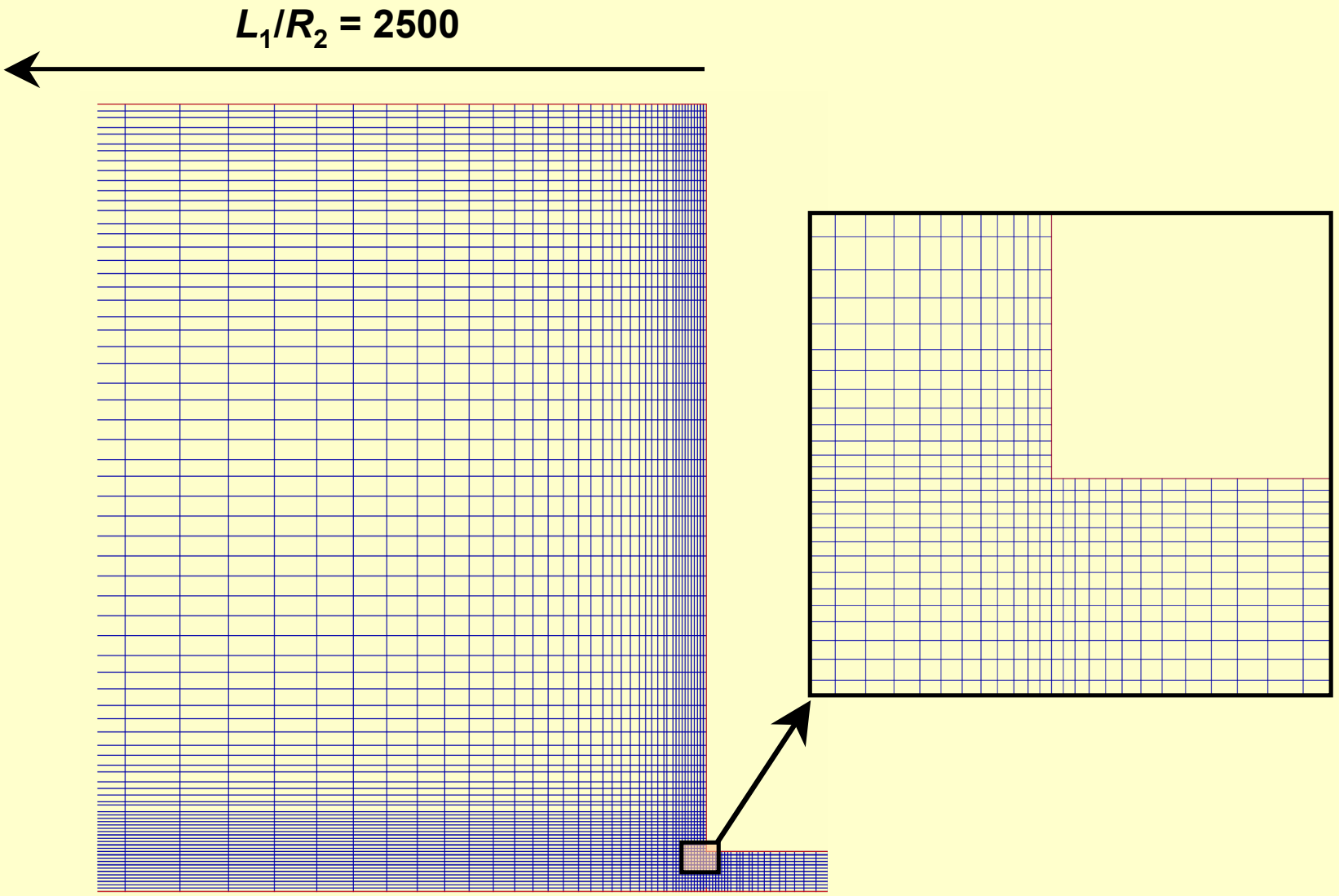
$$De = \frac{\lambda U_2}{R_2}$$

CR	L_1/R_2	L_2/R_2	NC	DOF	$\Delta x_{min}=\Delta y_{min}$
4	2500	2500	8 980	53 880	0.020
10	2500	2500	10 420	62 520	0.020
20	2500	2500	11 956	71 736	0.020
40	2500	2500	15 796	94 776	0.020
100	5000	5000	23 920	143 520	0.020

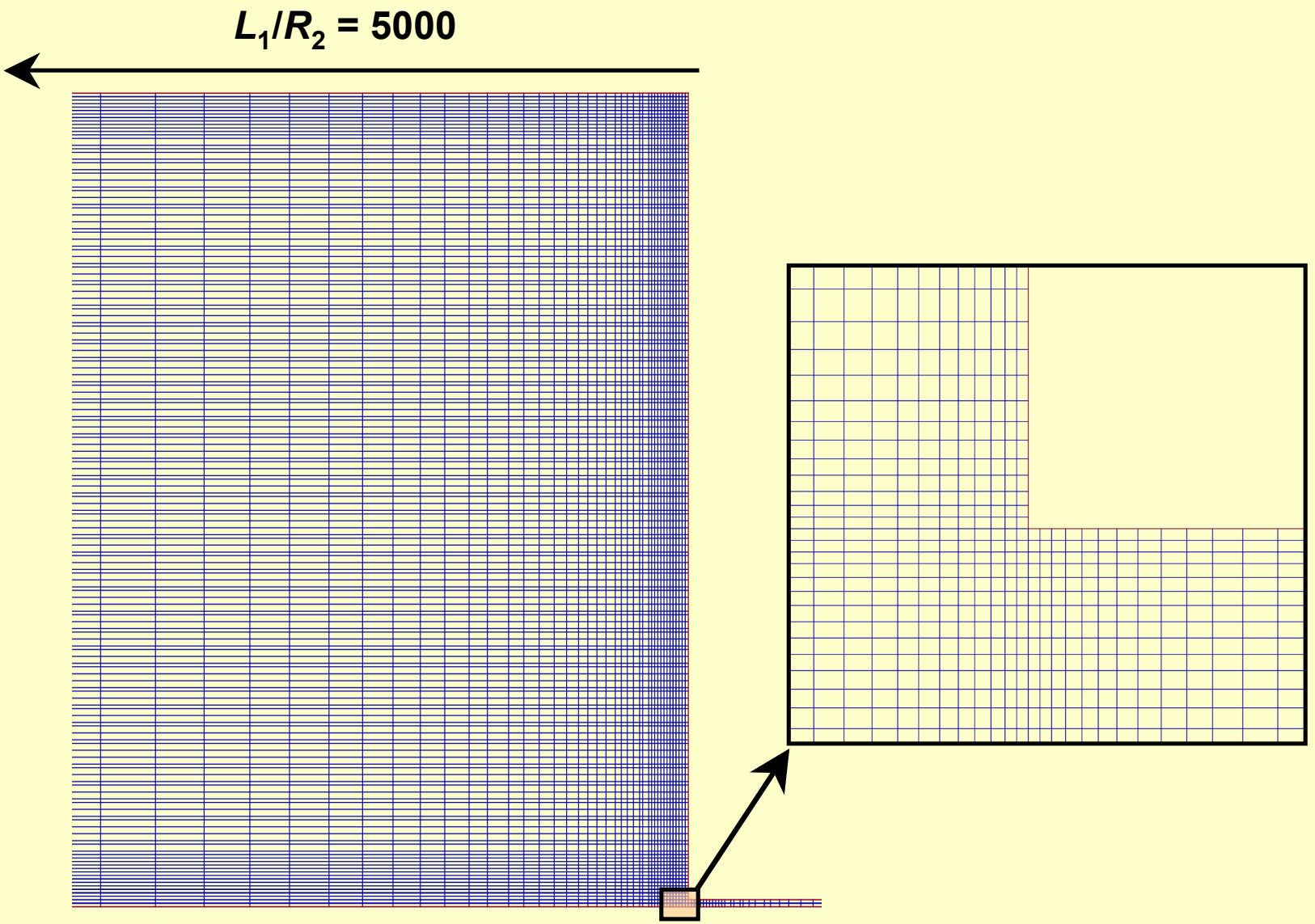
Computational Meshes (*CR* = 4)

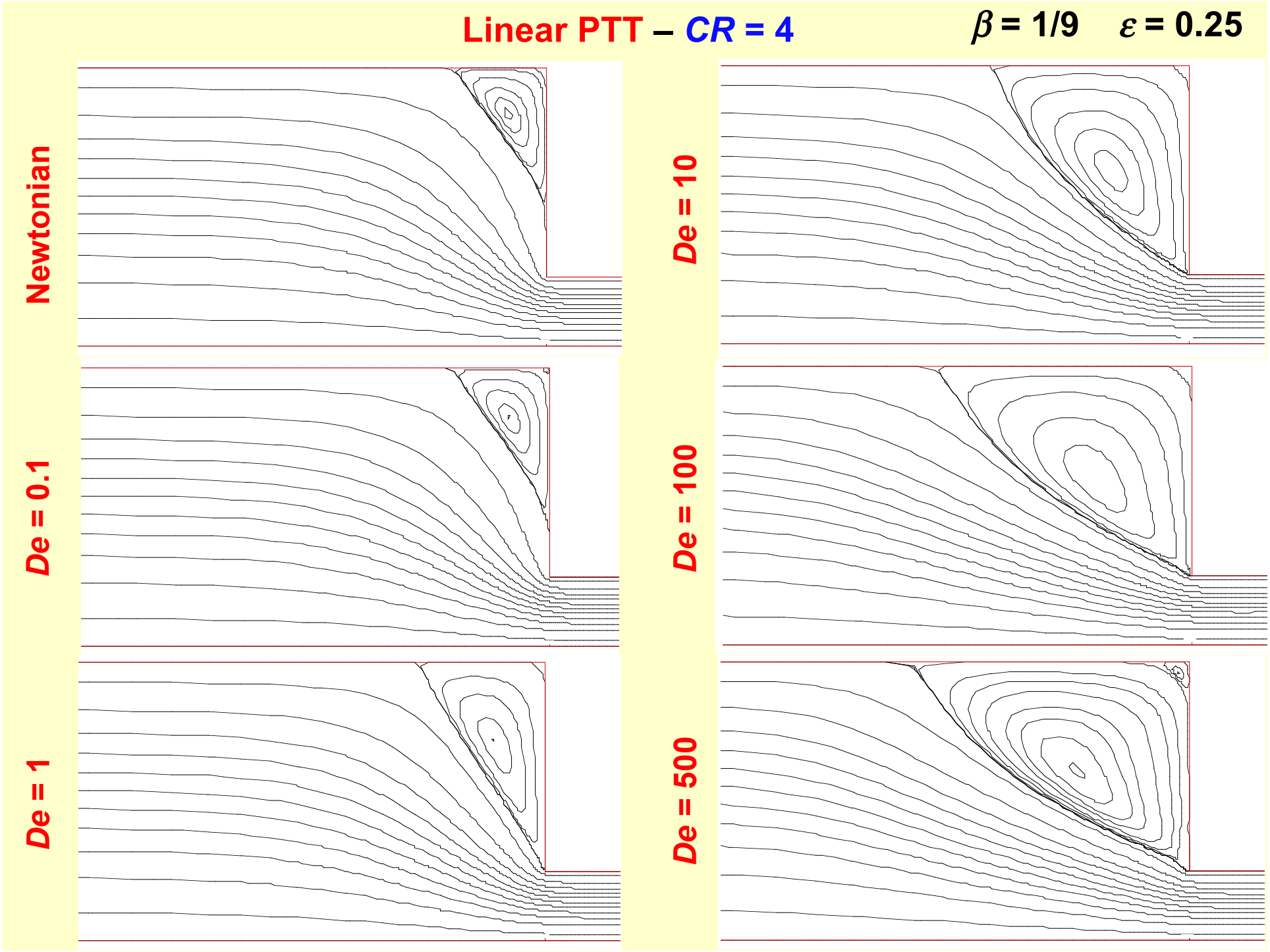


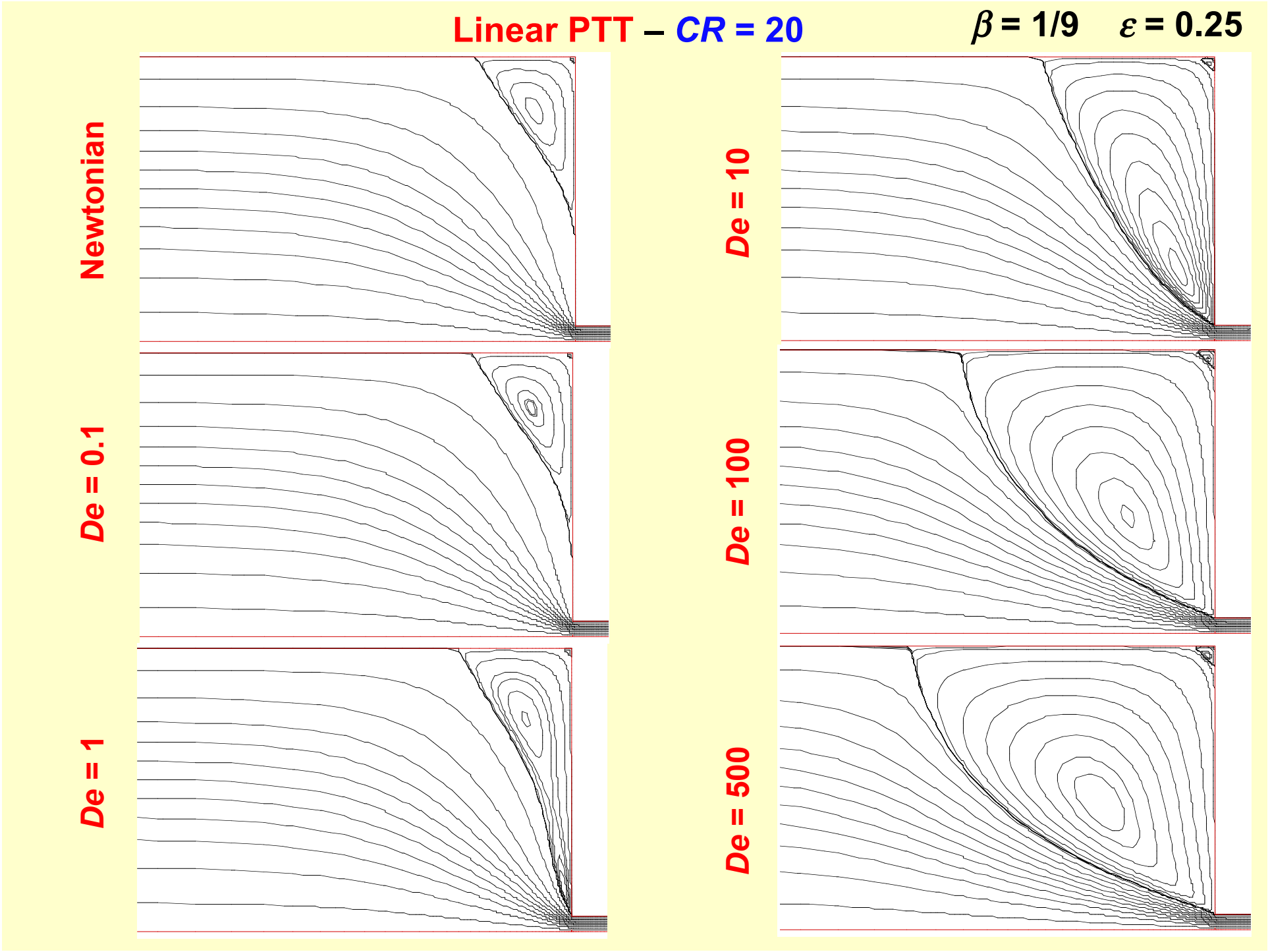
Computational Meshes (*CR* = 20)



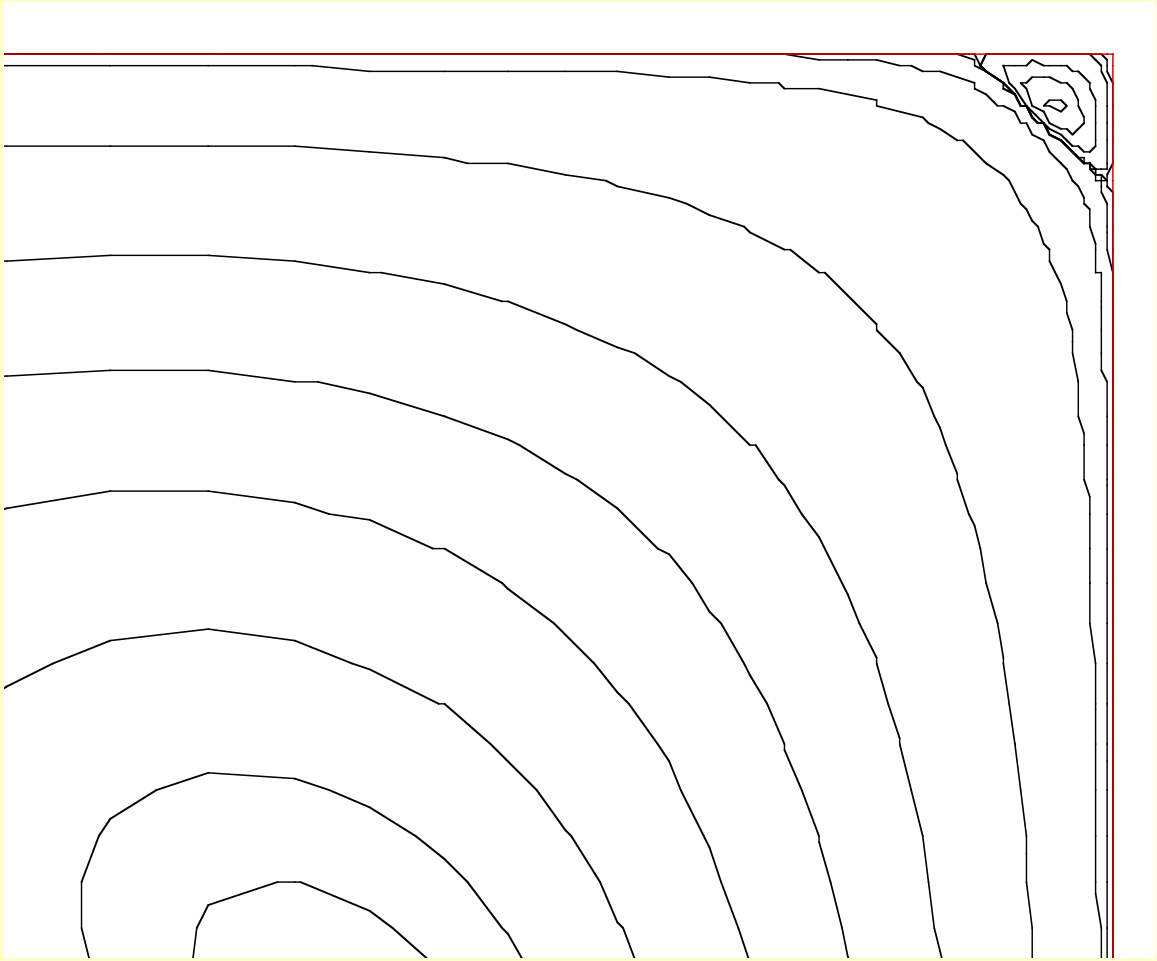
Computational Meshes (*CR* = 100)




Linear PTT – CR = 4
β = 1/9 ε = 0.25
Newtonian
De = 10
De = 0.1
De = 100
De = 1
De = 500

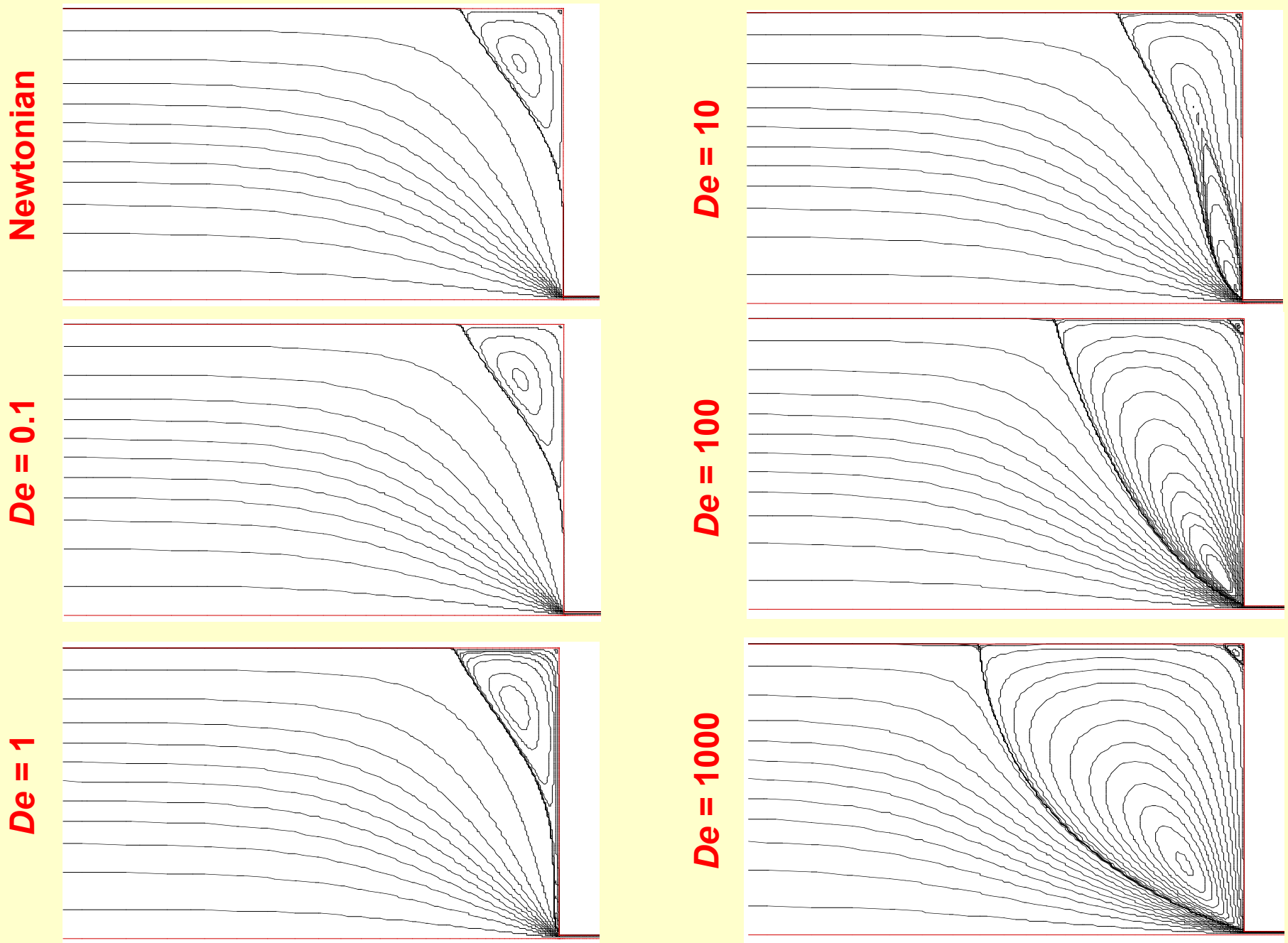

Linear PTT – CR = 20
β = 1/9 ε = 0.25
Newtonian
De = 0.1
De = 1
De = 10
De = 100
De = 500

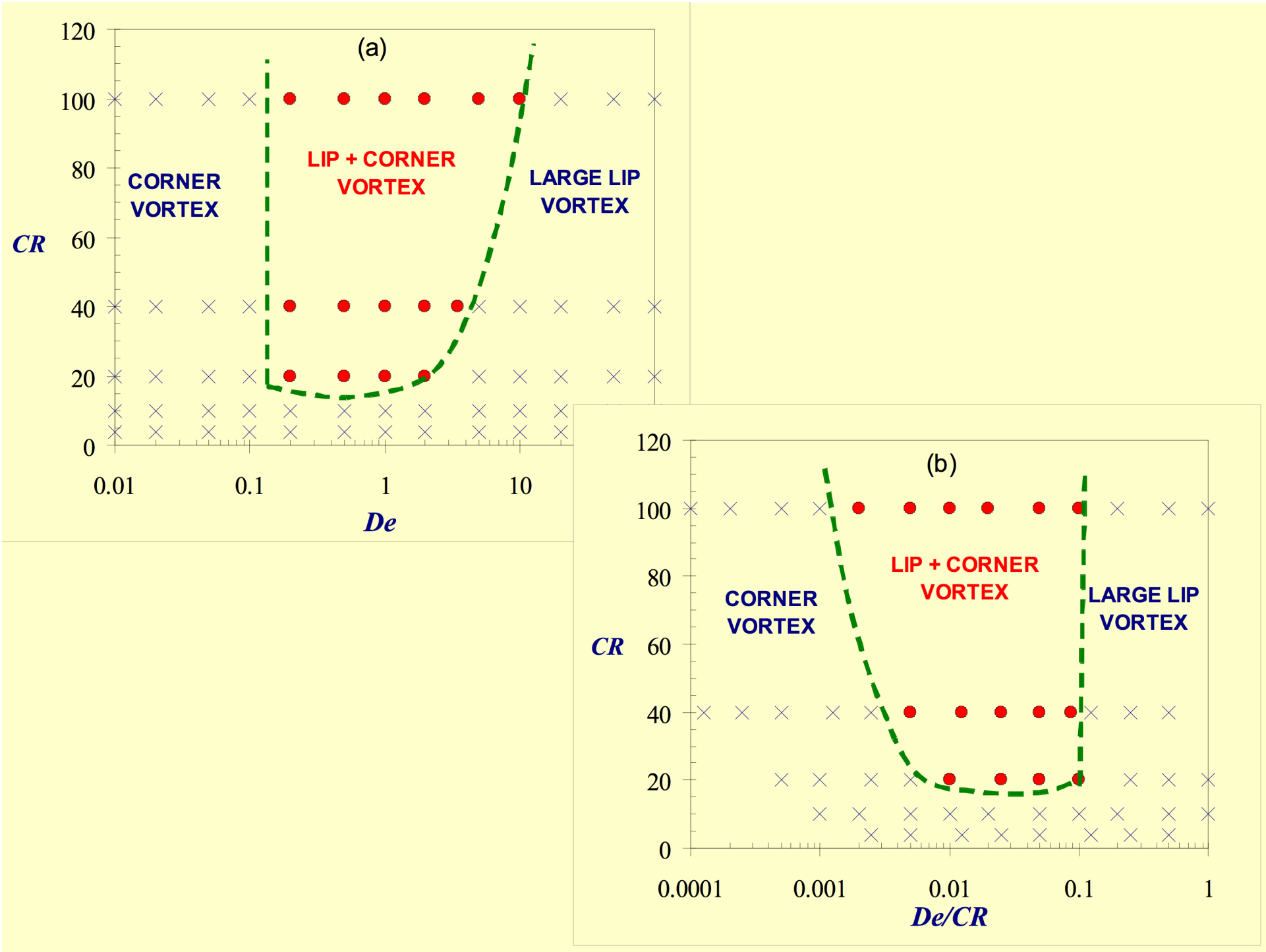
Secondary flow

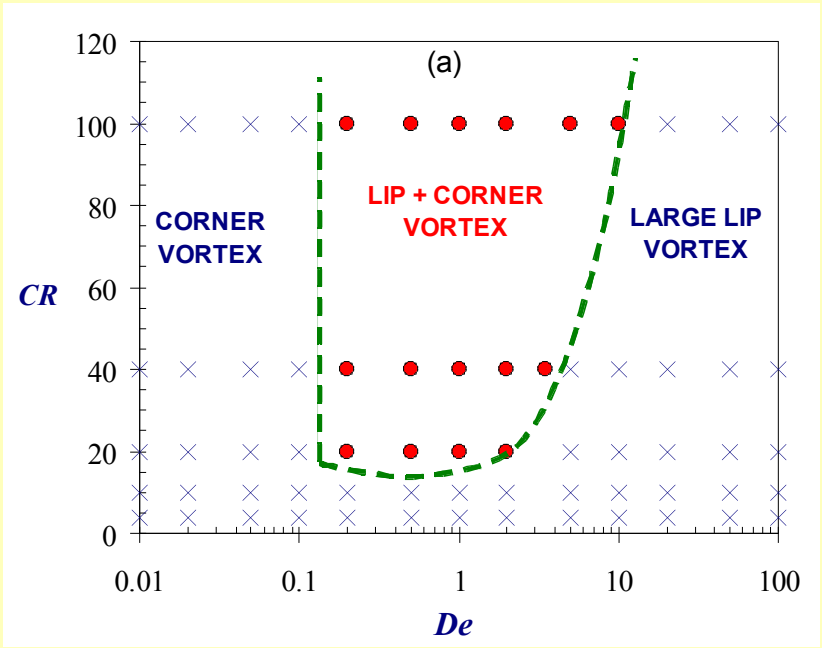


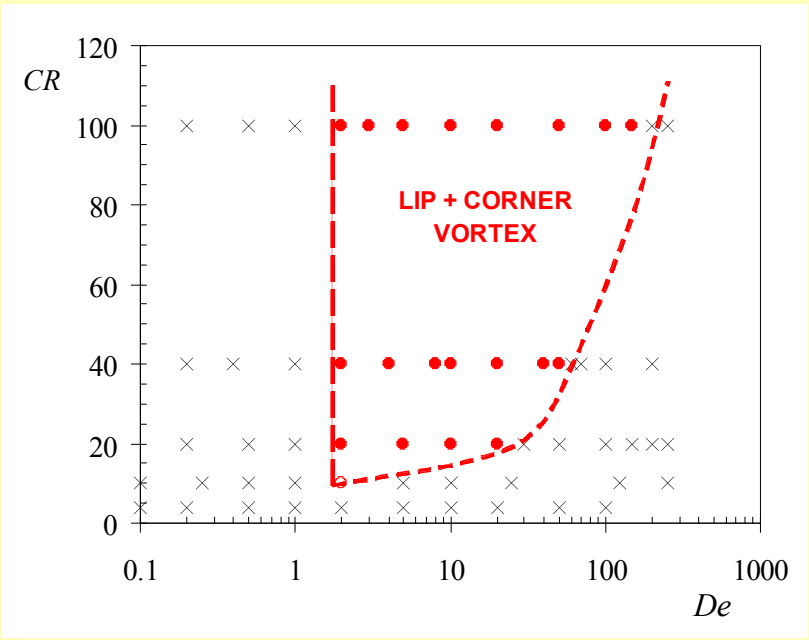
Linear PTT – *CR* = 100

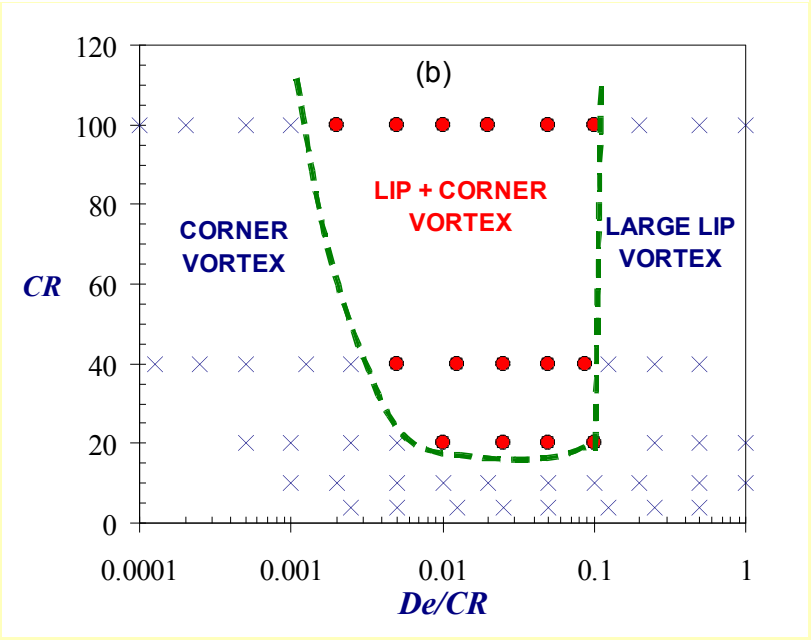
$\beta = 1/9 \quad \varepsilon = 0.25$

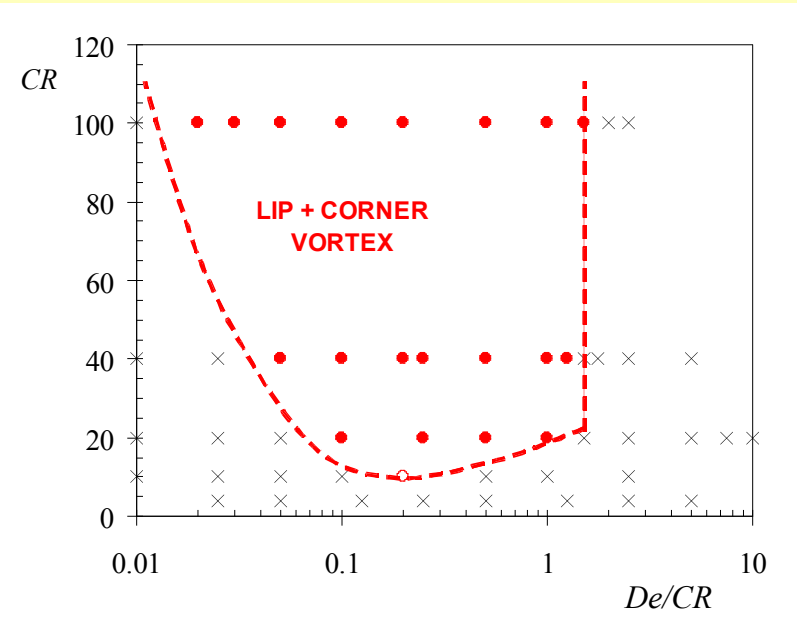



(a)
120
100
80
60
40
20
0
CR
0.01
0.1
1
10
De
CORNER VORTEX
LIP + CORNER VORTEX
LARGE LIP VORTEX
(b)
120
100
80
60
40
20
0
CR
0.0001
0.001
0.01
0.1
1
De/CR
CORNER VORTEX
LIP + CORNER VORTEX
LARGE LIP VORTEX





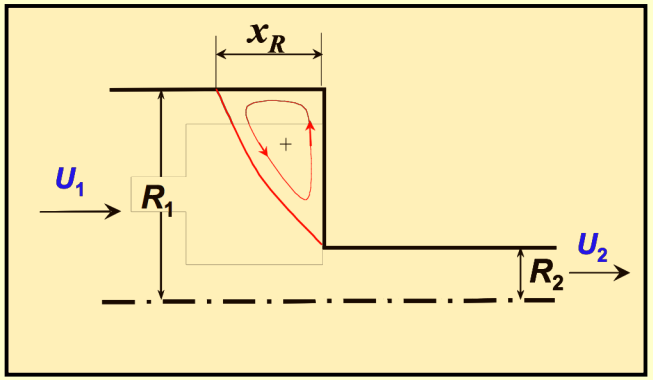


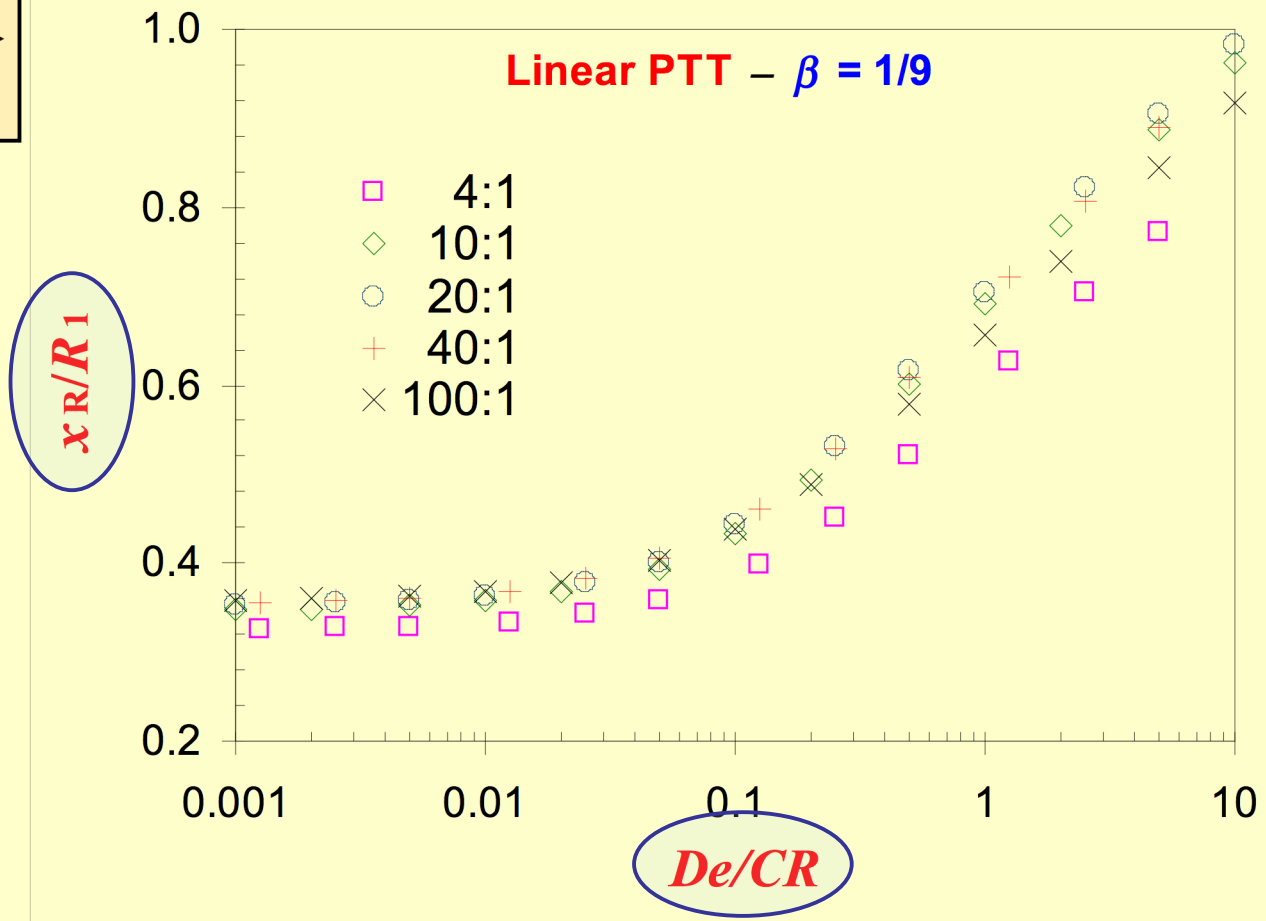


AXISYMMETRIC

PLANAR

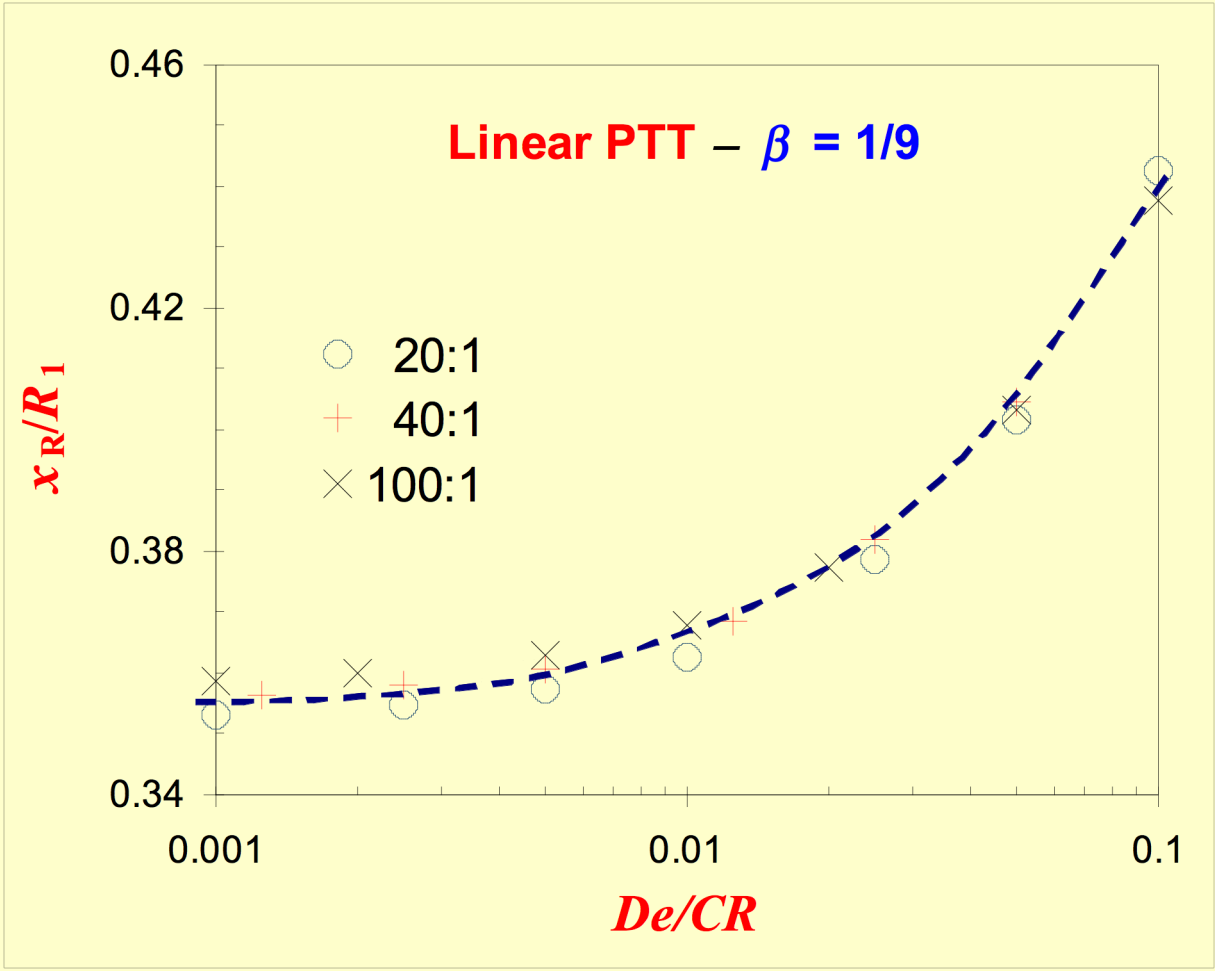
Corner vortex size





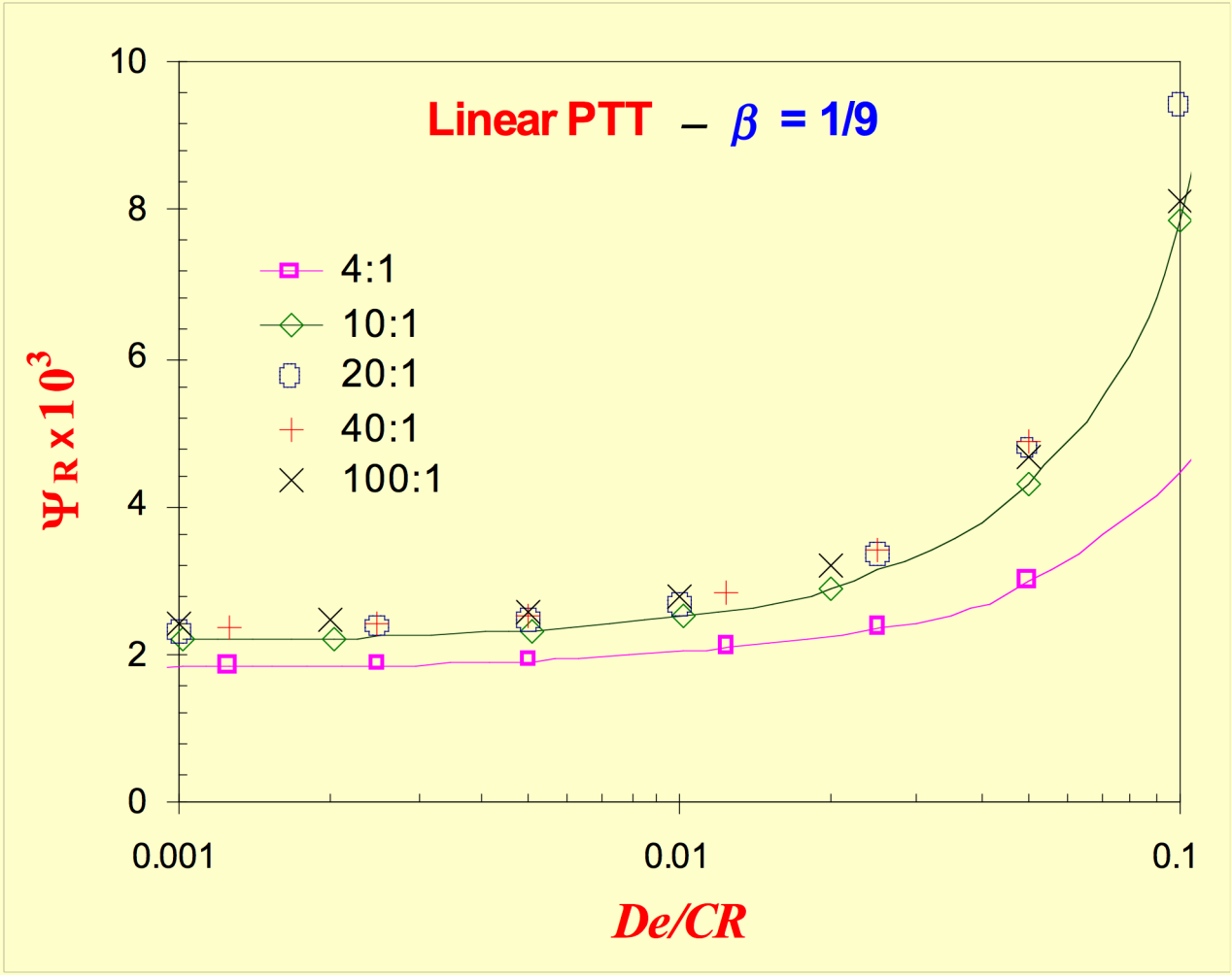
Corner vortex size

Difference < 2 %

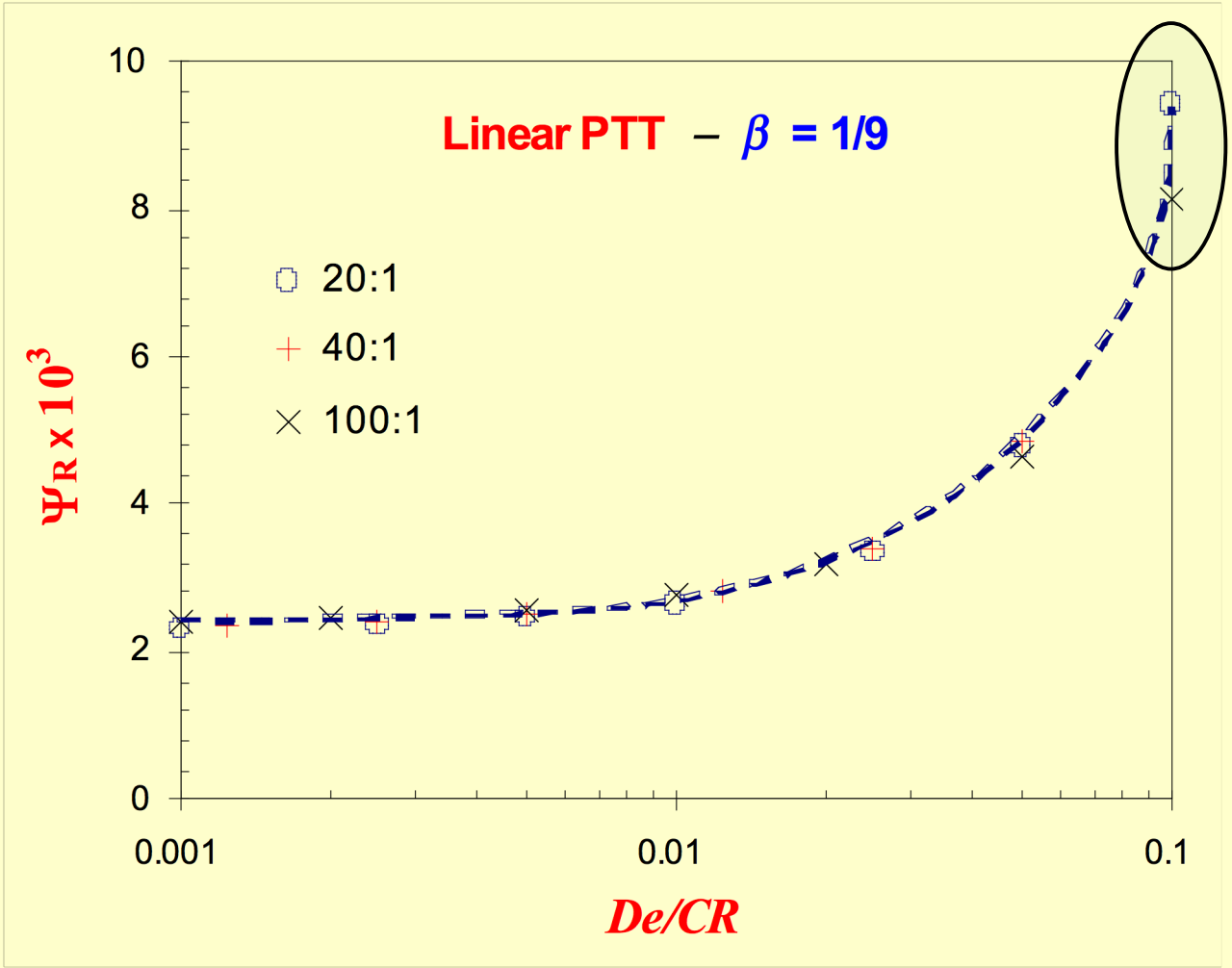


Corner vortex size (x_R / R_1) depends on *De*/*CR* for large *CR*

Corner vortex intensity



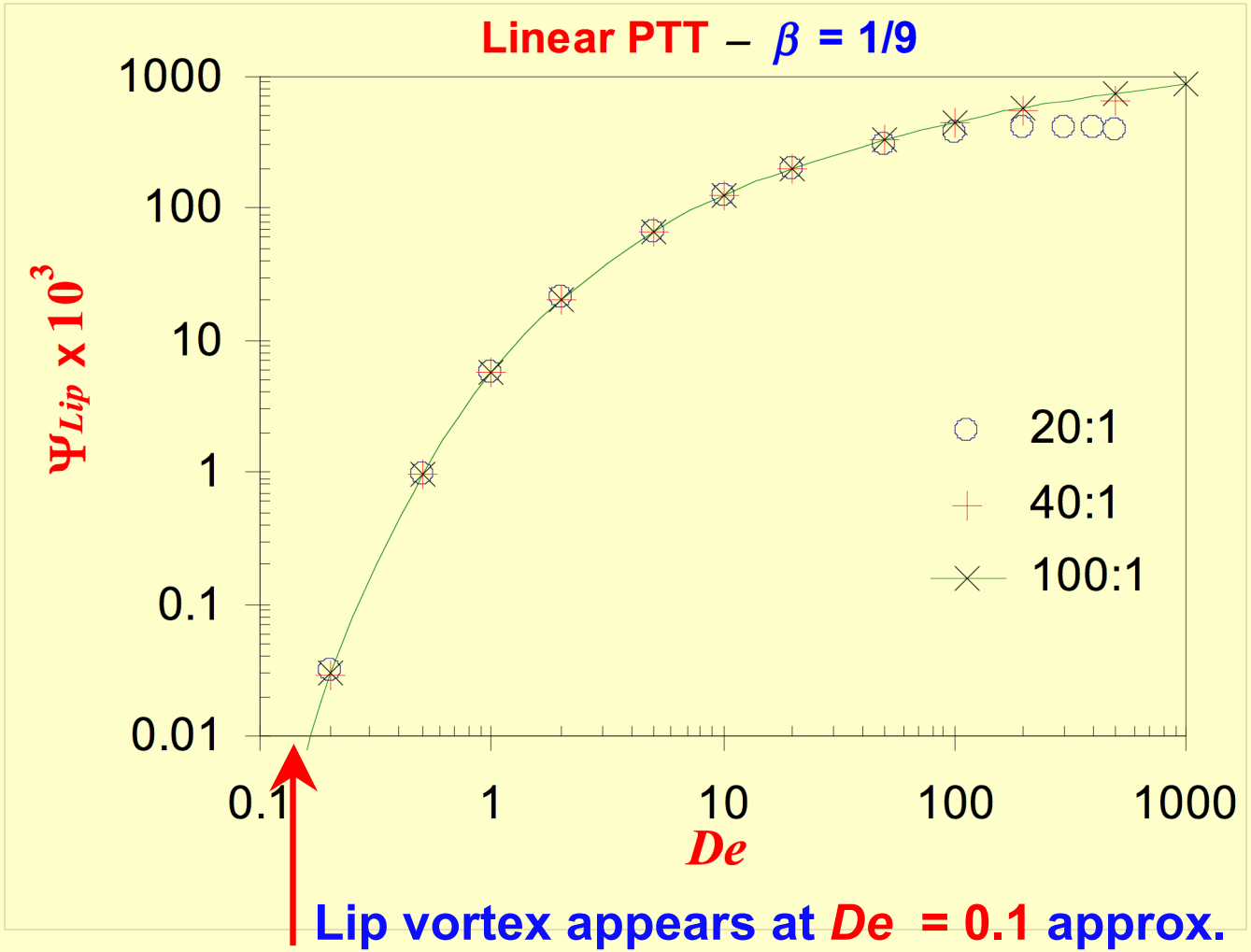
Corner vortex intensity



Merging of corner and lip vortex

(*De / CR* = 0.1)

Lip vortex intensity

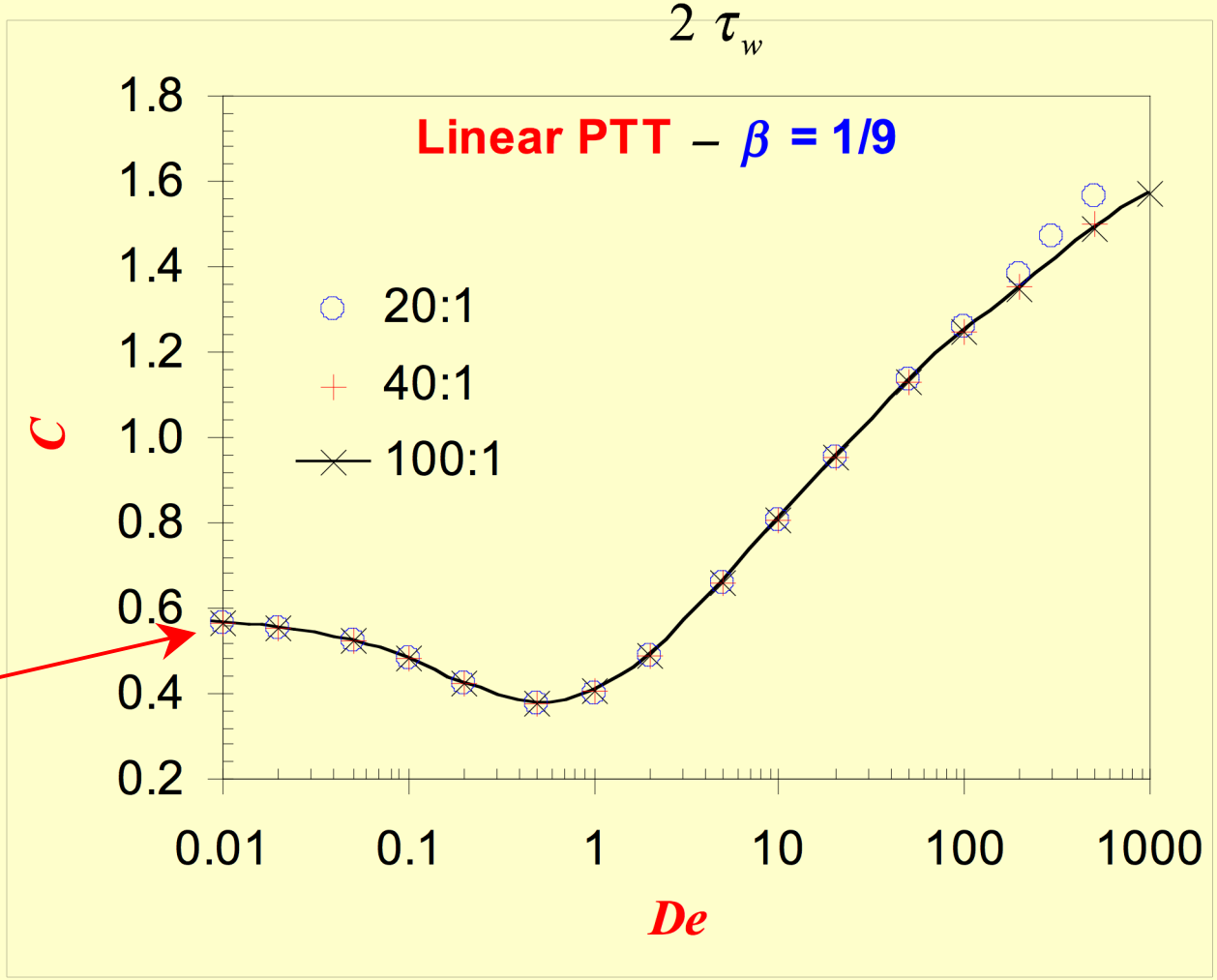


Lip vortex appears at *De* = 0.1 approx.

Lip vortex scales with *De*

Couette correction

$$C = \frac{\text{extra pressure drop}}{2\,\tau_w}$$



$C = 0.589$ (Sampson's prediction)

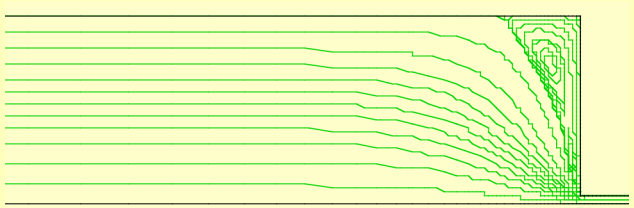
Couette correction scales with *De*

Scaling of streamlines

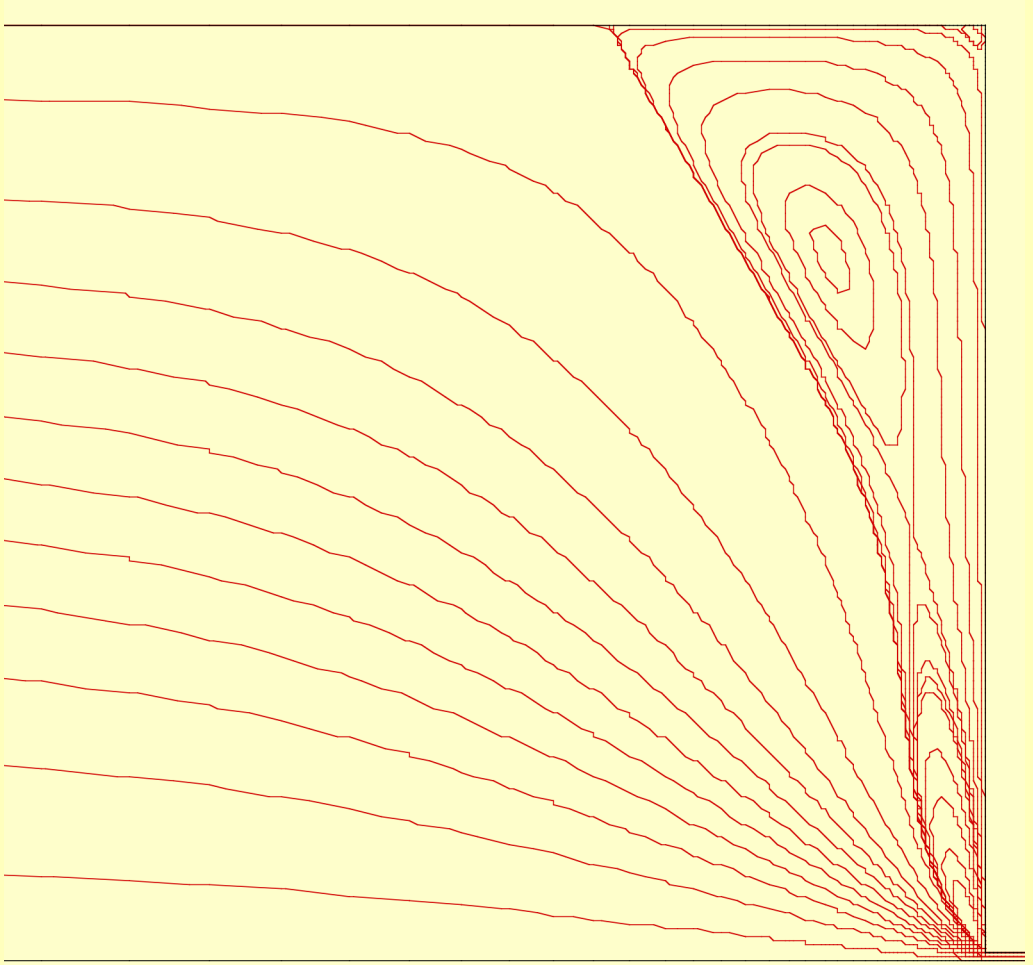
Linear PTT – *De/CR* = 0.05

TRUE SIZES

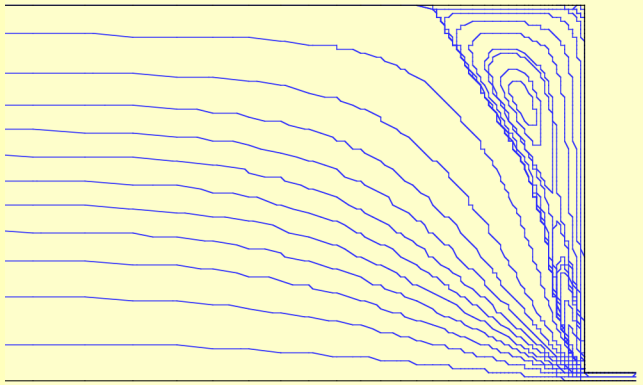
CR = 20 *De* = 1



CR = 100 *De* = 5



CR = 40 *De* = 2



Scaling of streamlines

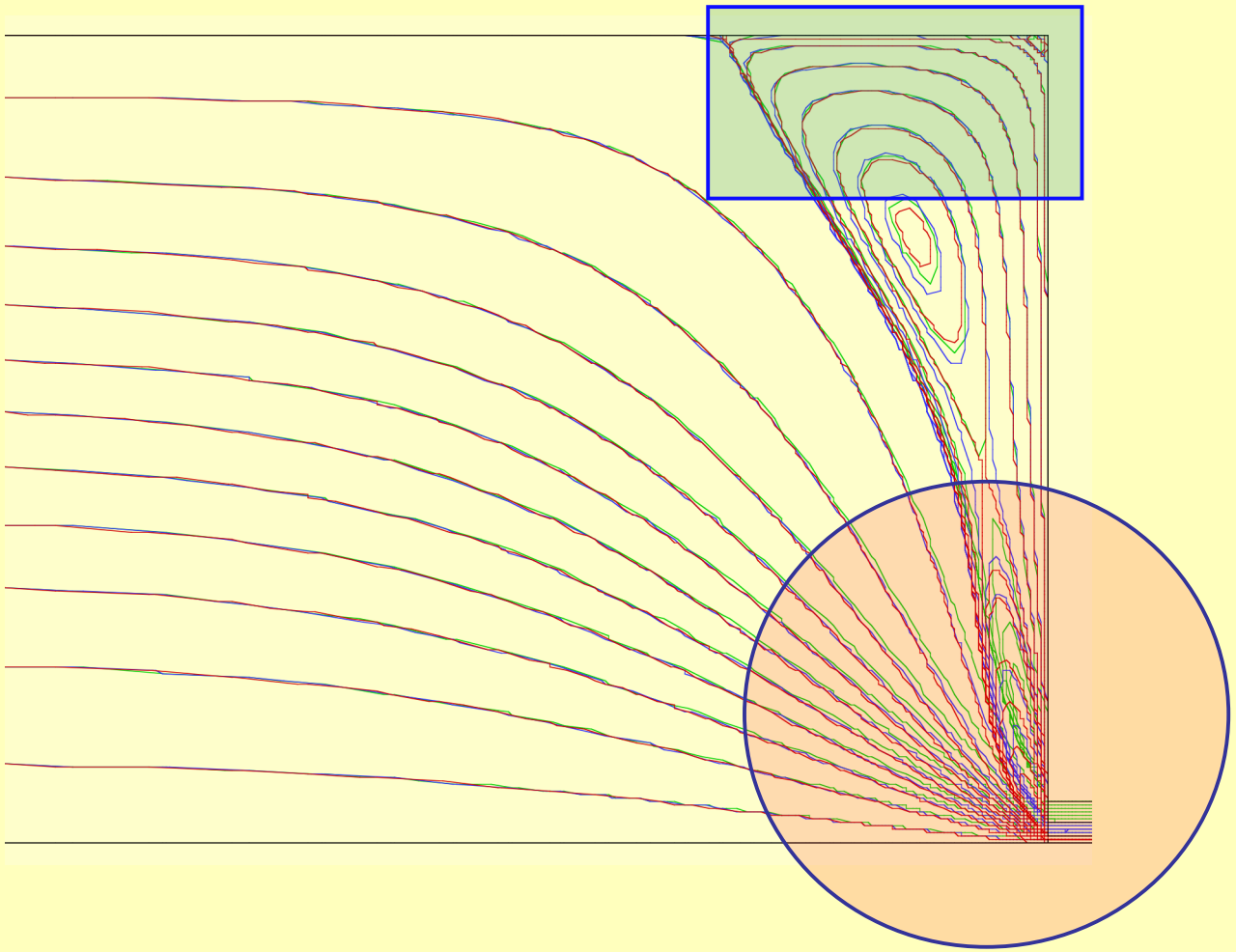
SCALING with *CR*

CR = 20 *De* = 1

CR = 40 *De* = 2

CR = 100 *De* = 5

De / CR = 0.05

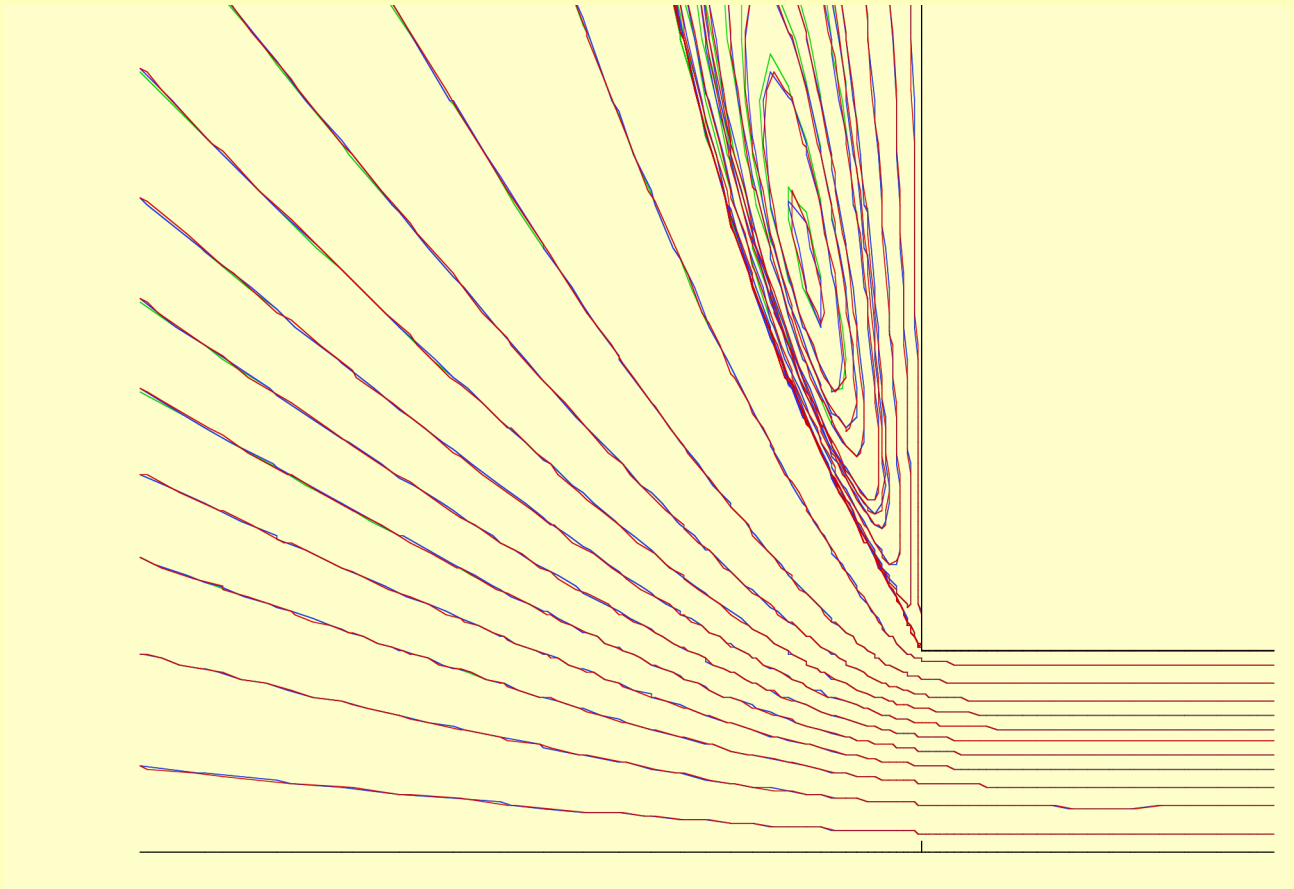


NOTE: When dimensions are scaled with *CR*, corner vortex streamlines collapse

Scaling of streamlines

Lip vortex – no scaling needed

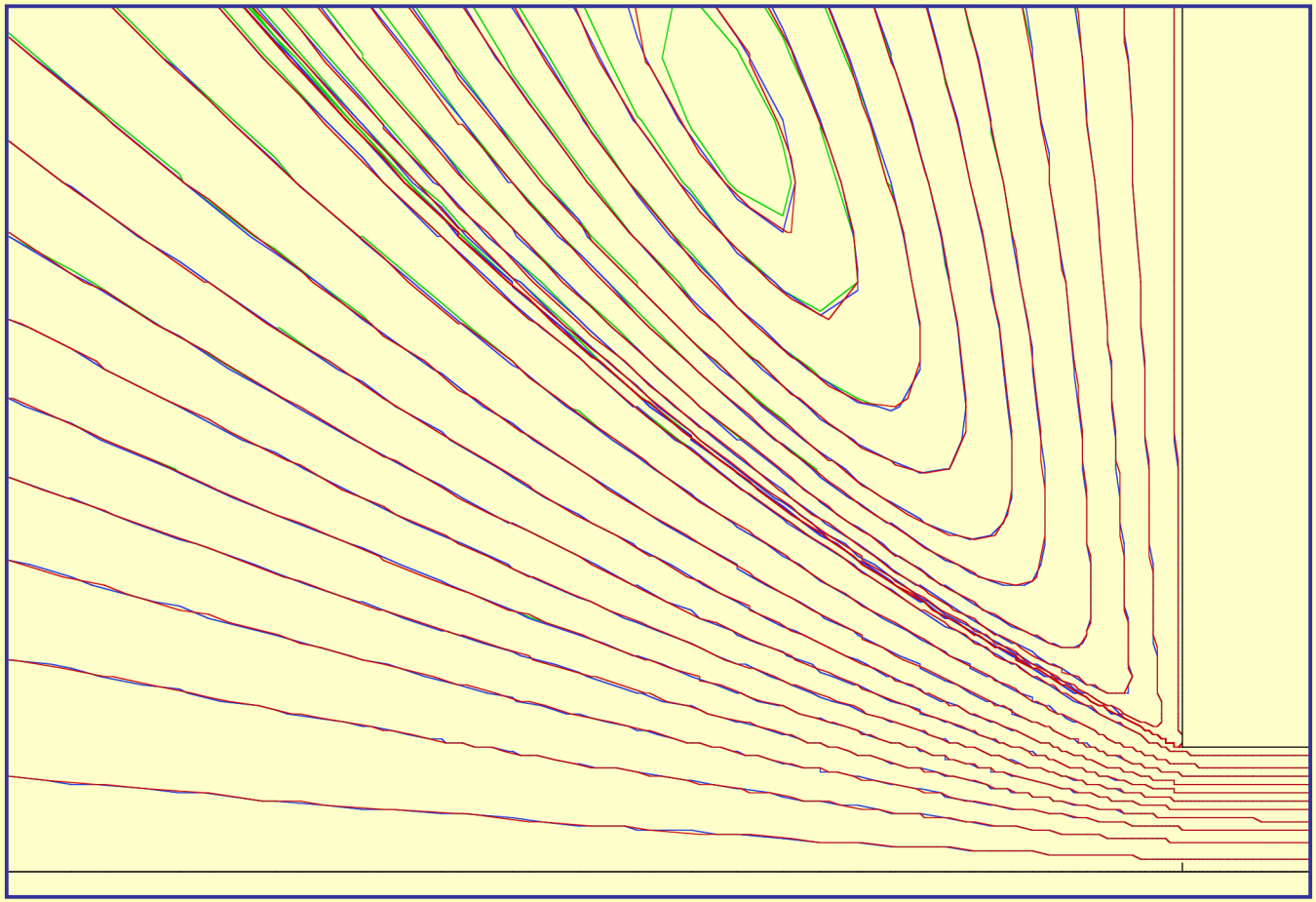
Linear PTT ($De = 1$) CR = 20, 40, 100



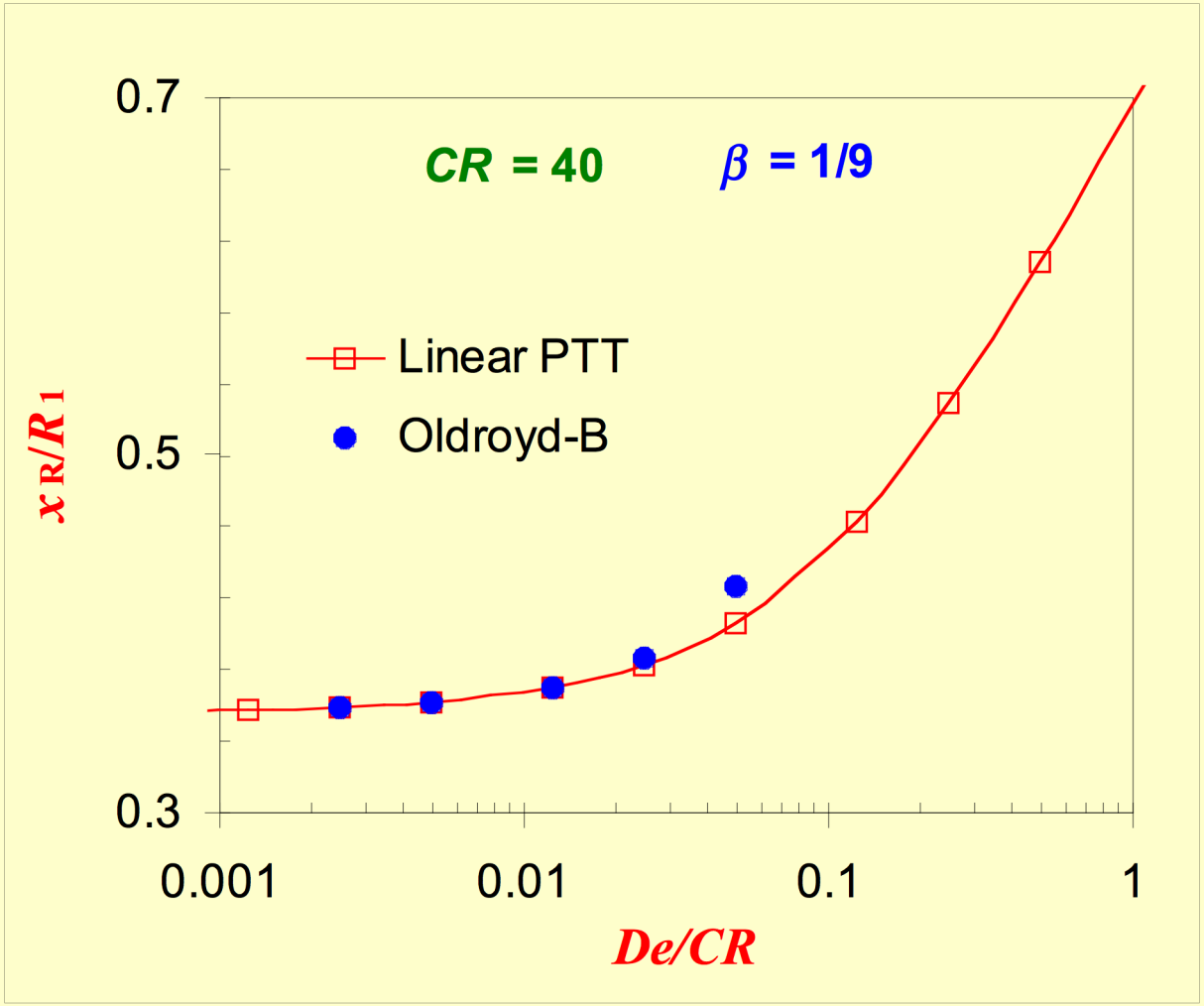
Scaling of streamlines

Lip vortex – no scaling needed

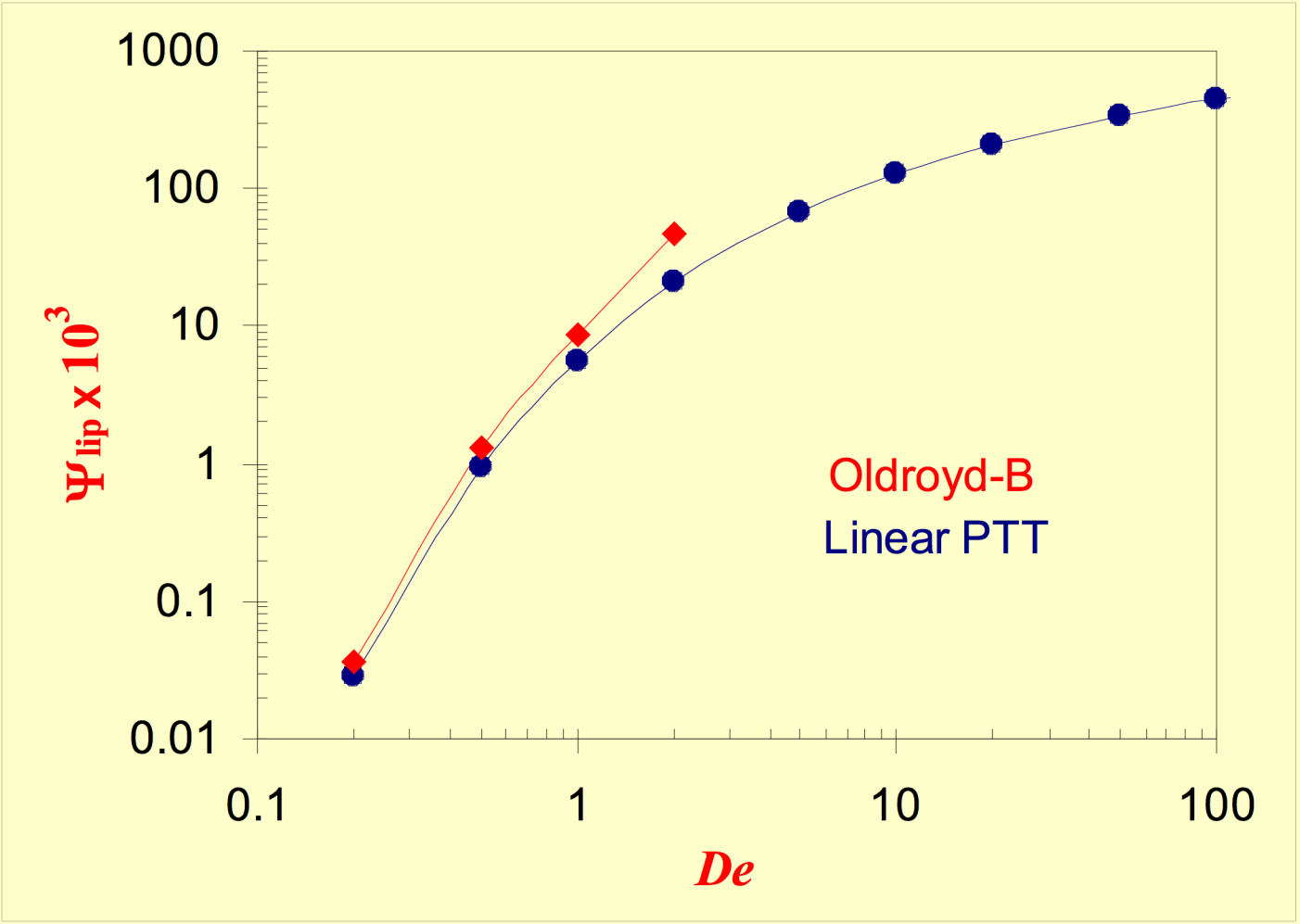
Linear PTT *De* = 20 *CR* = 20, 40, 100



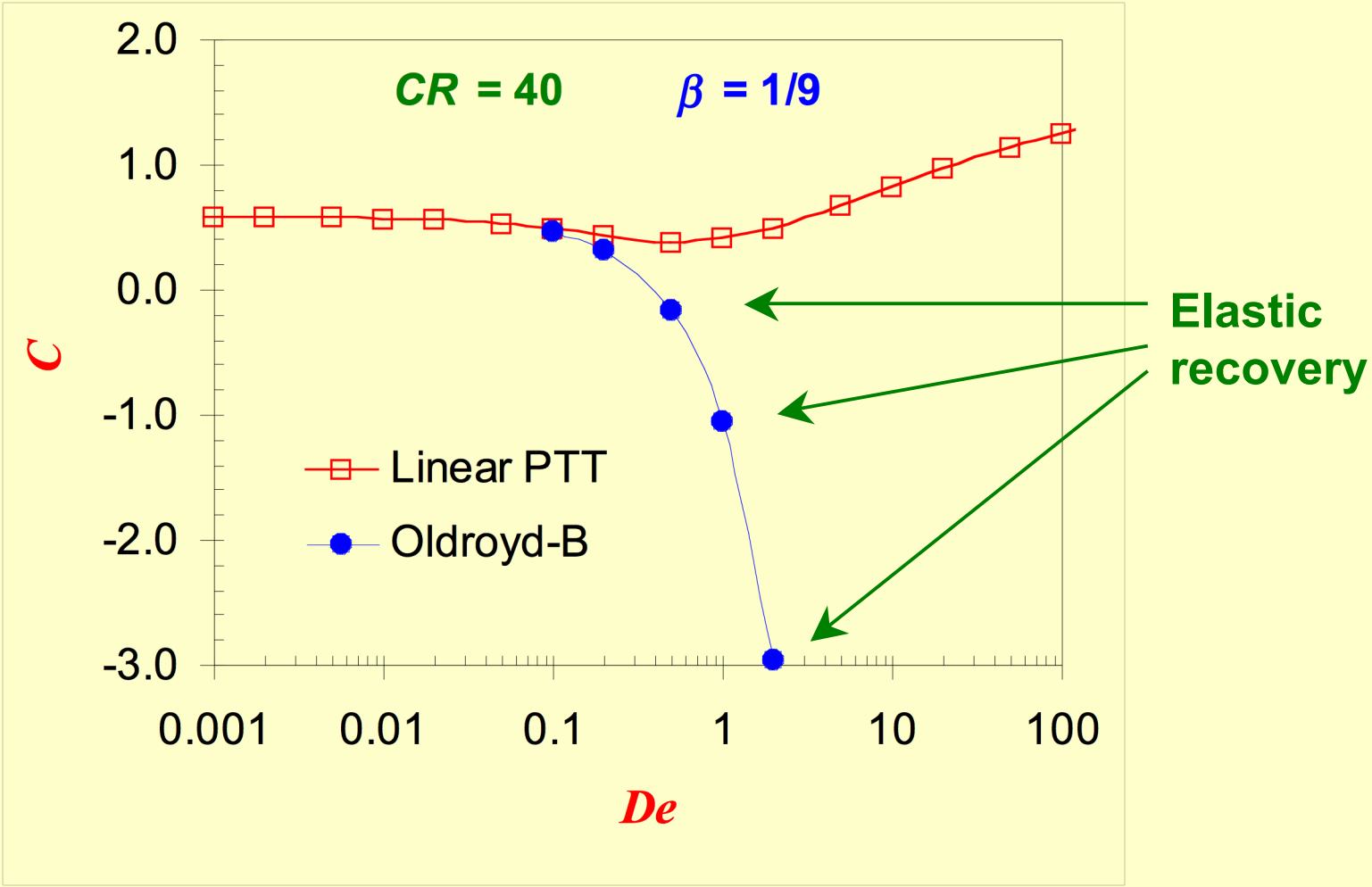
Comparison between rheological models



Comparison between rheological models

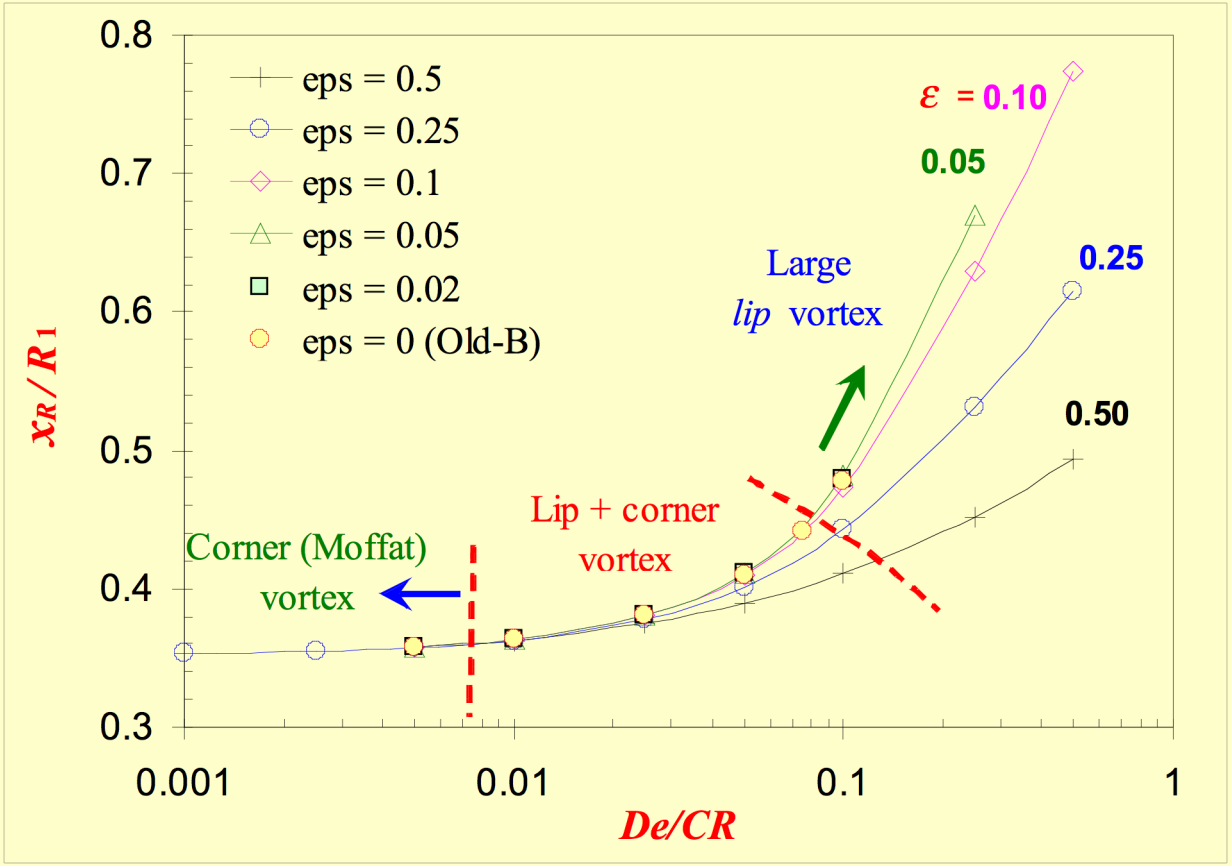


Comparison between rheological models



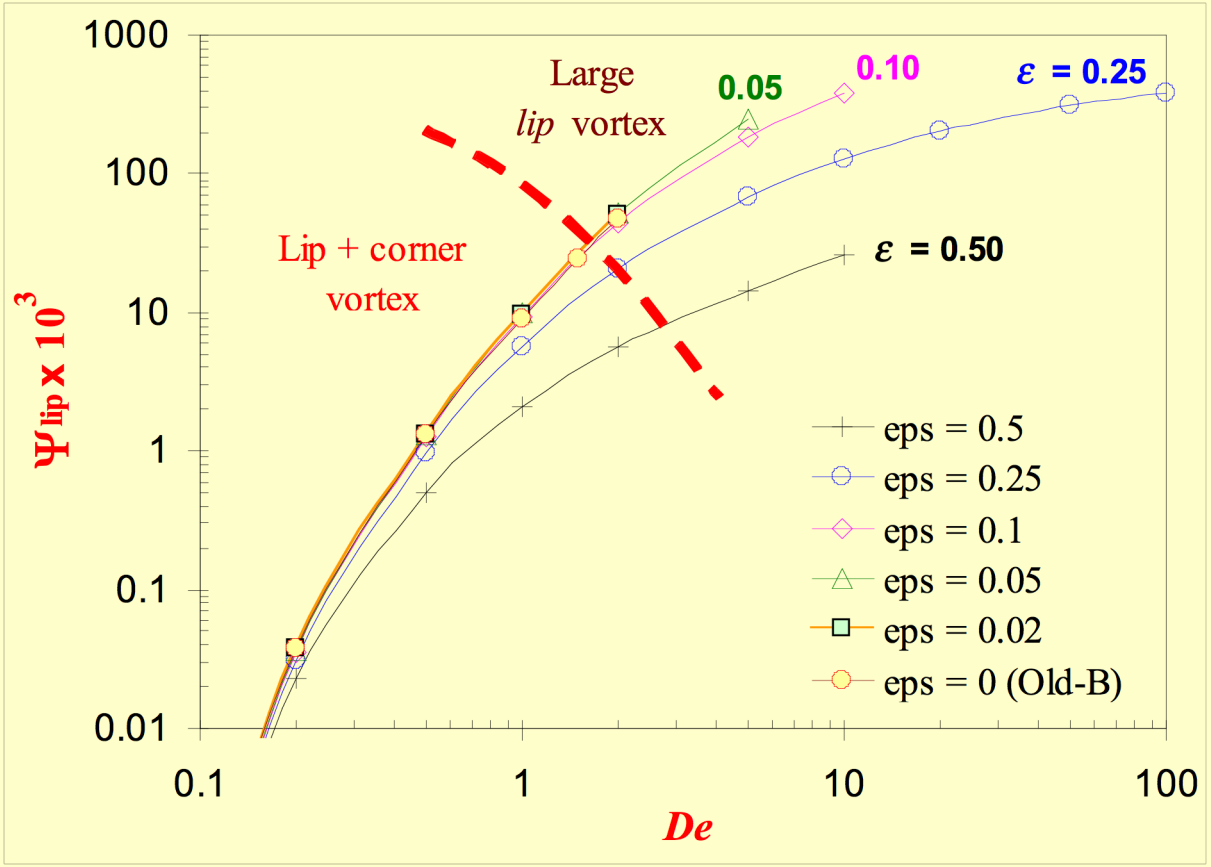
Comparison between rheological models

CR = 20



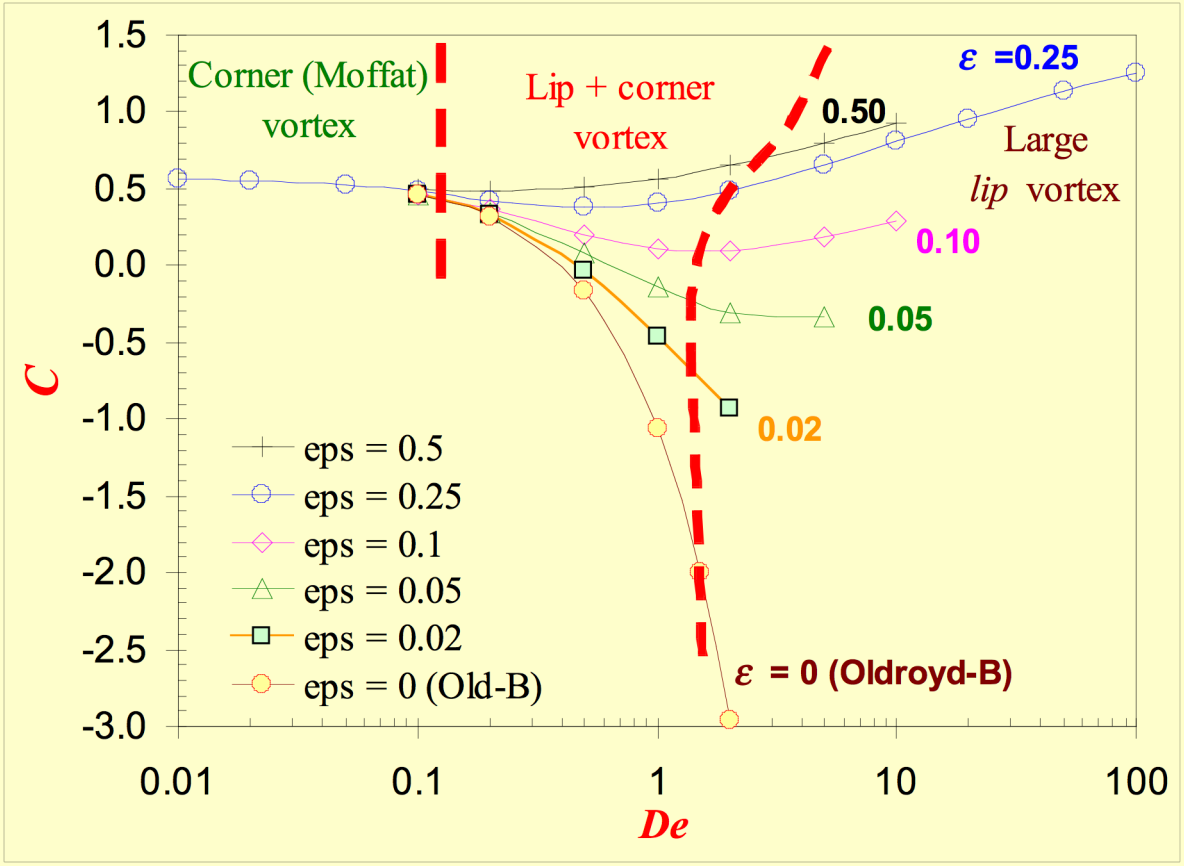
Comparison between rheological models

CR = 20

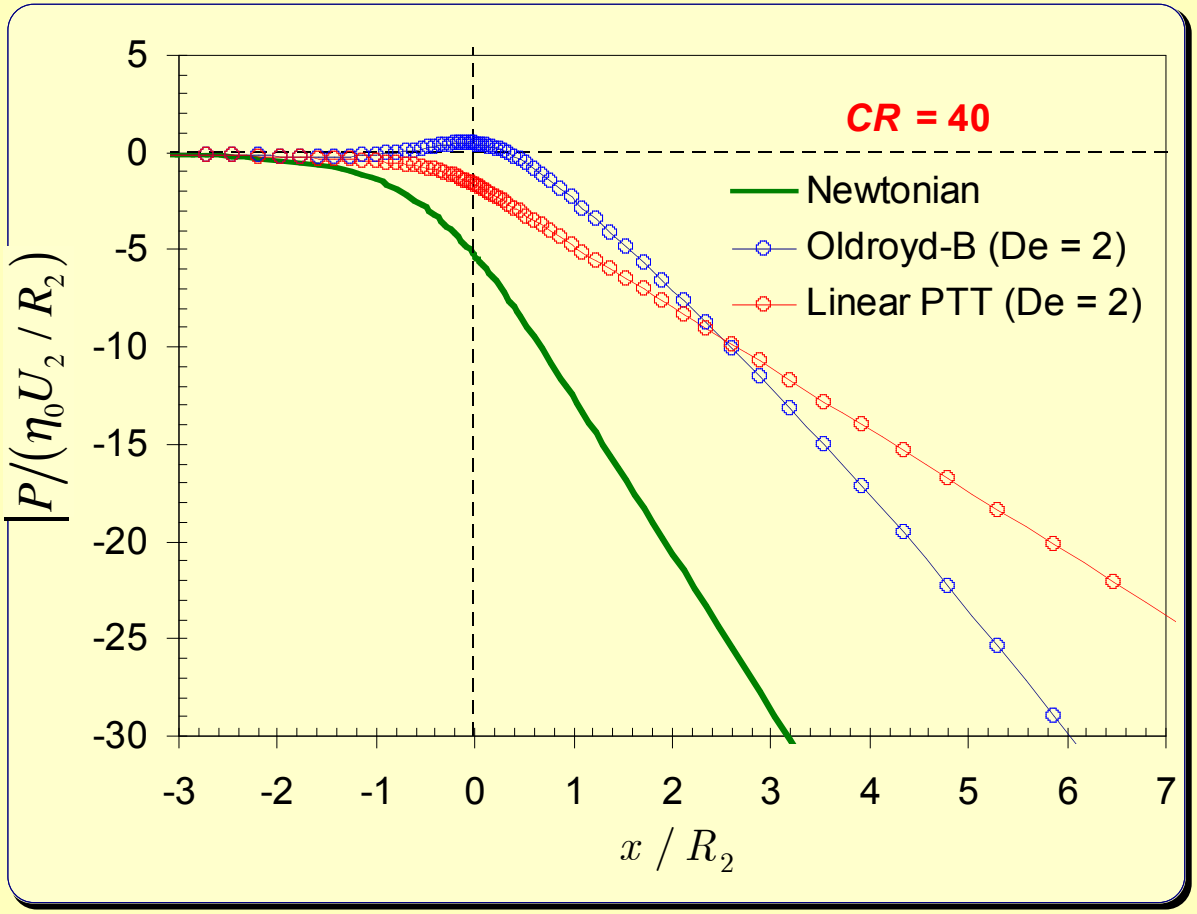


Comparison between rheological models

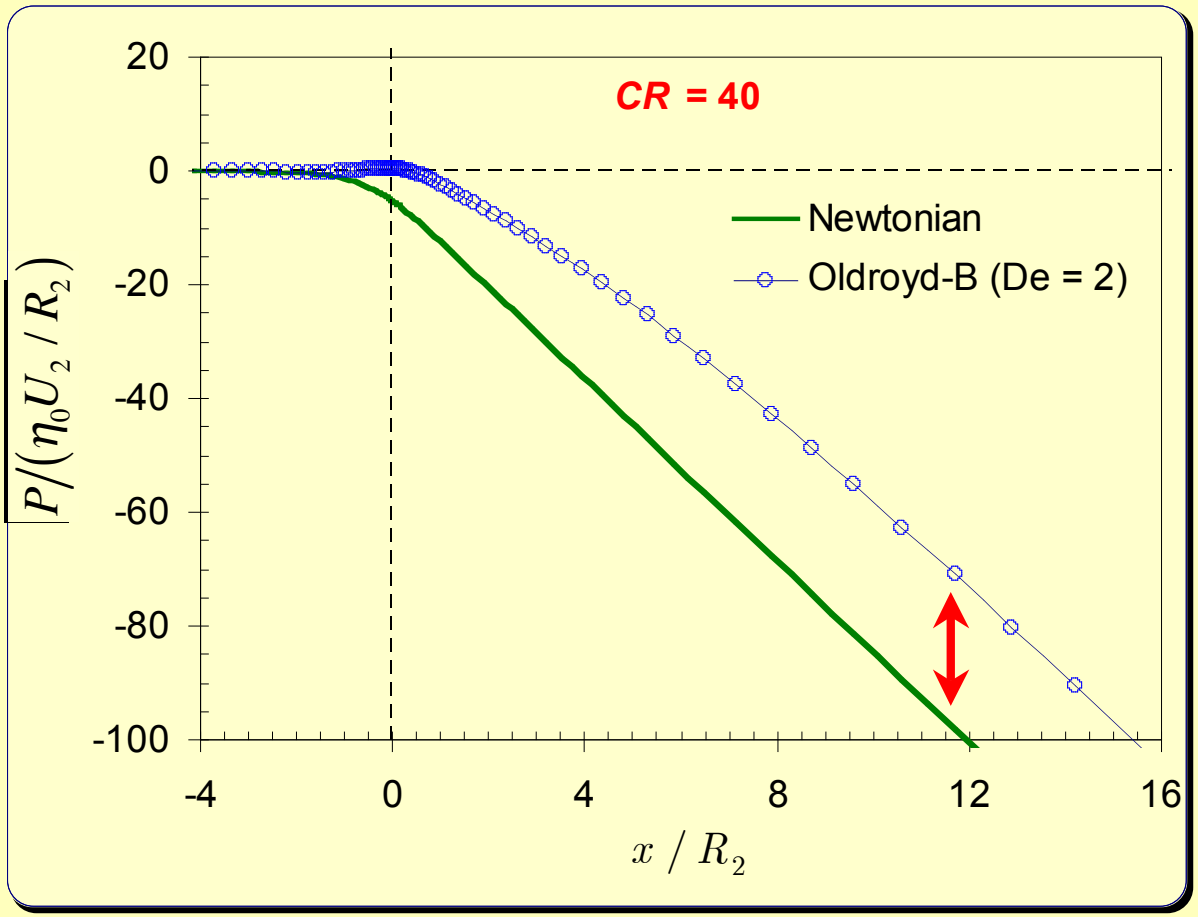
$CR = 20$ **$\beta = 1/9$**



Pressure profiles along centerline

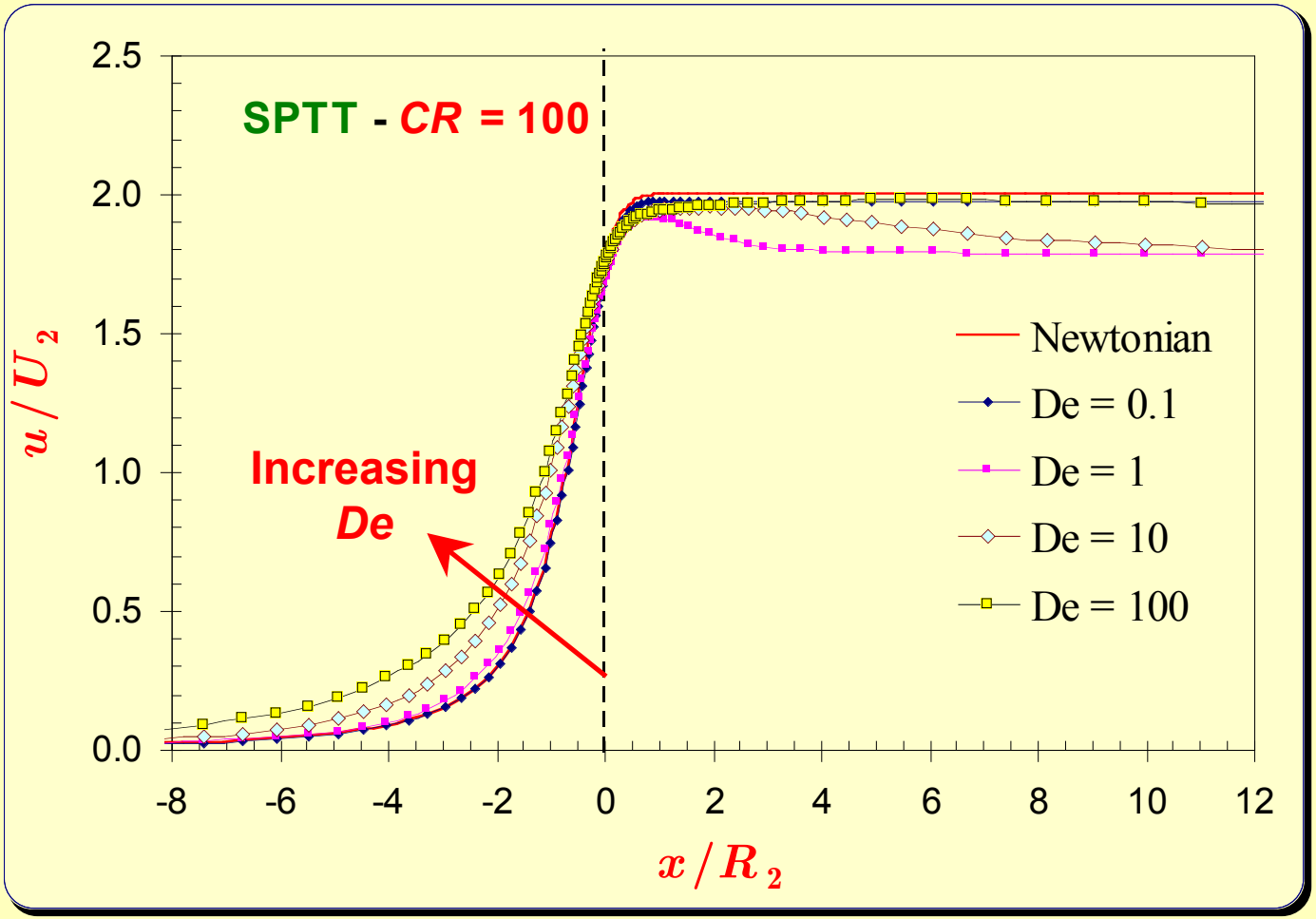


Pressure profiles along centerline



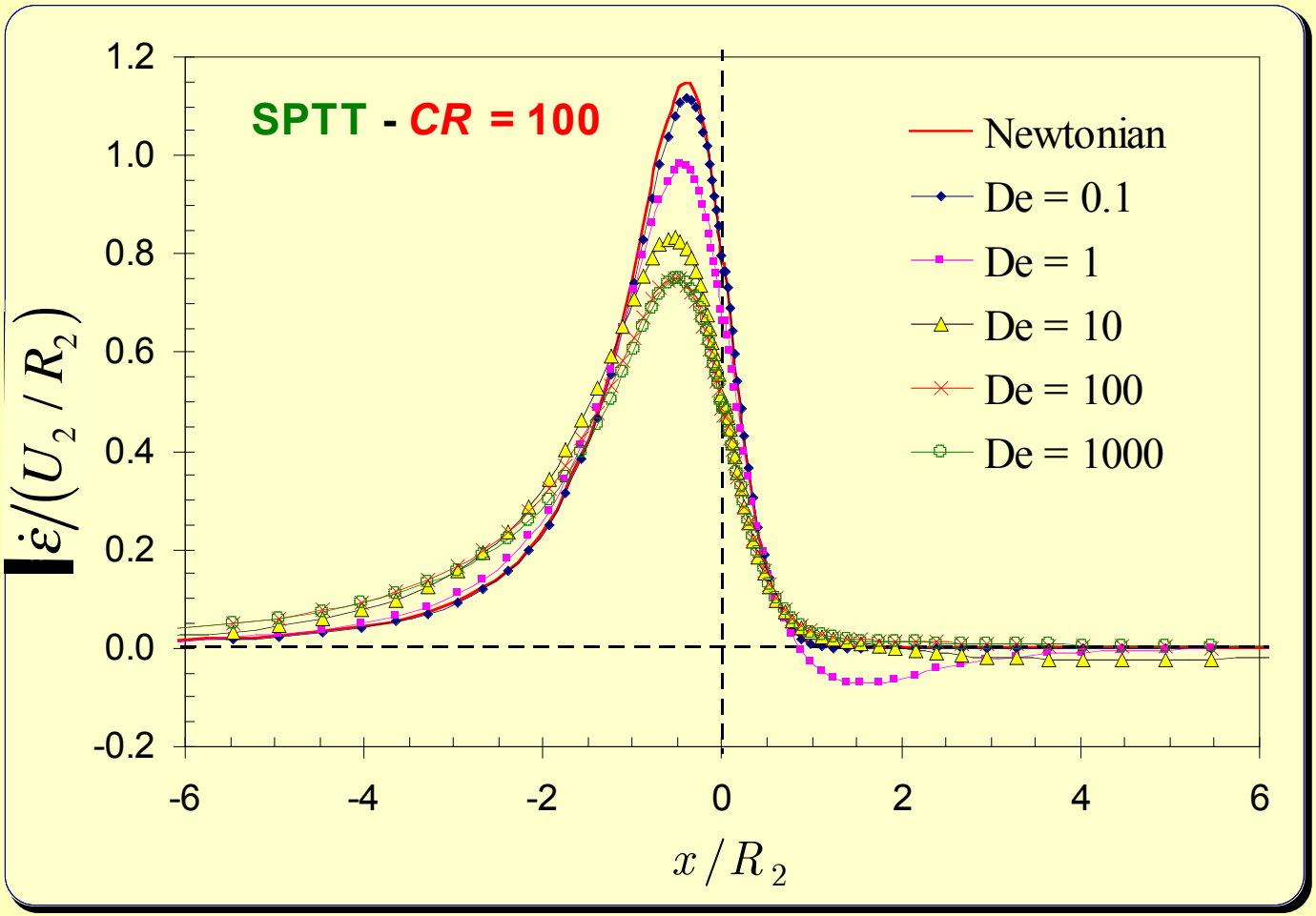
Velocity profiles along centerline

Linear PTT – $\beta = 1/9$, $\varepsilon = 0.25$



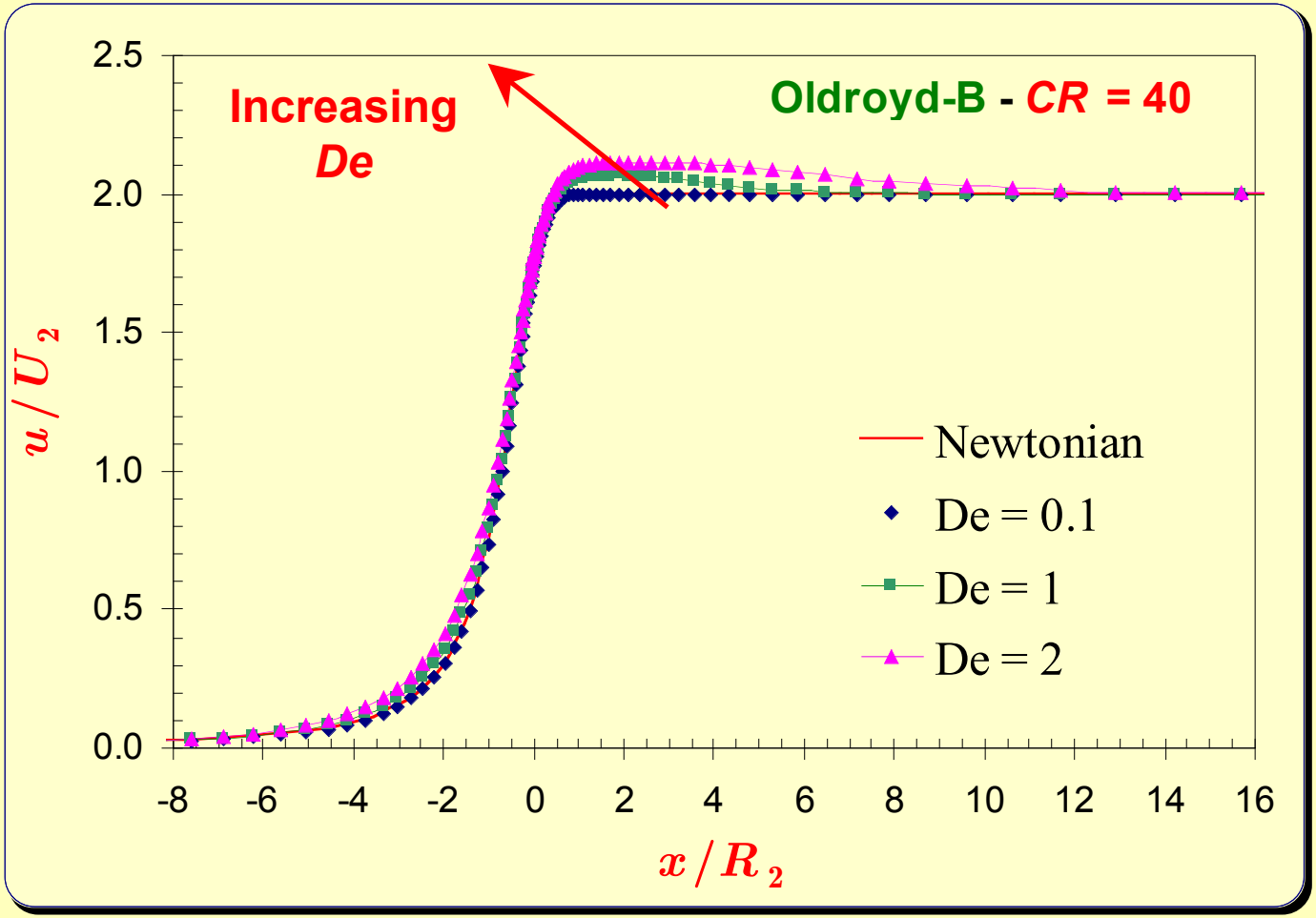
Strain rate along centerline

PTT – β = 1/9, ε = 0.25



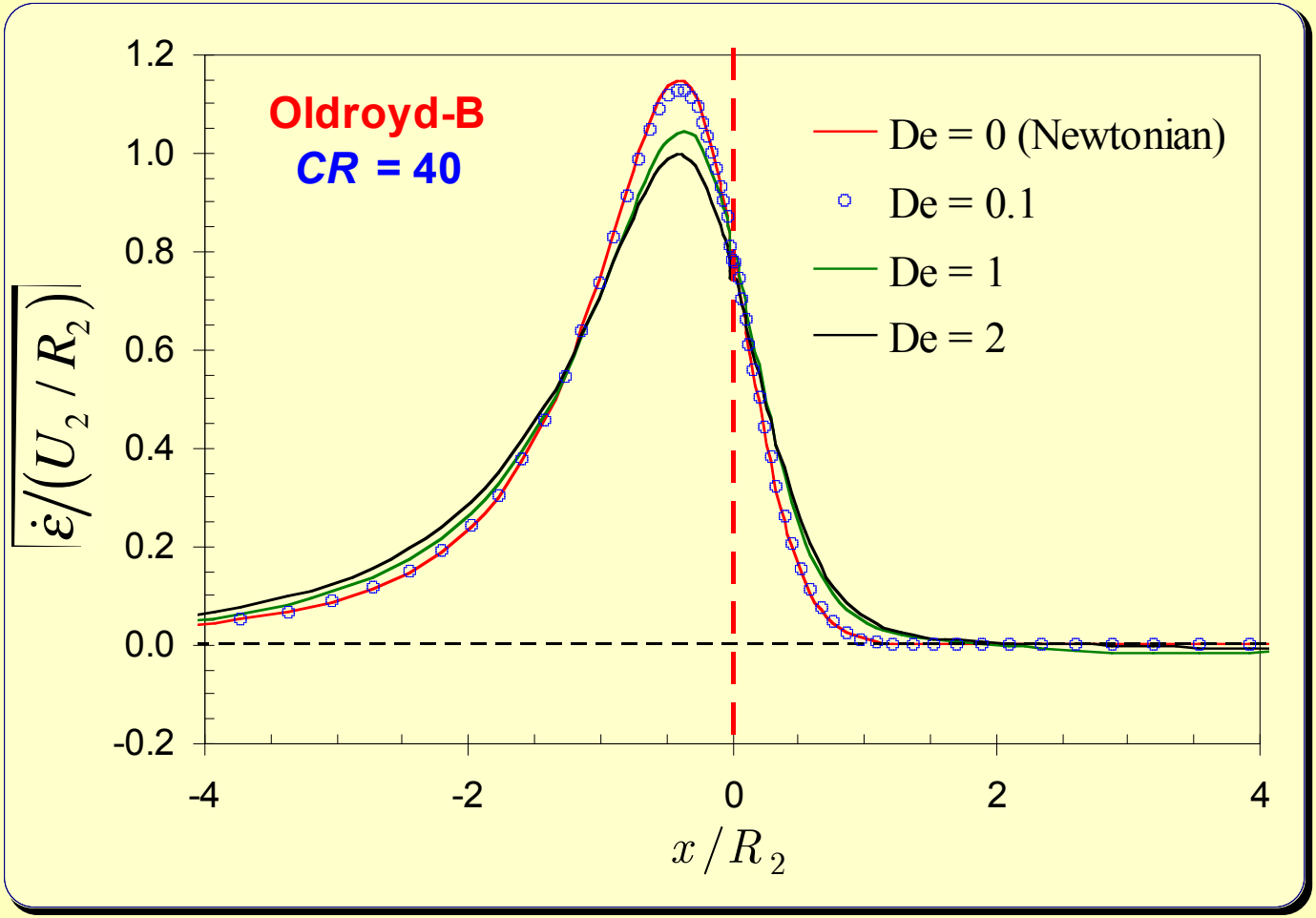
Velocity profiles along centerline

Oldroyd-B – β = 1/9



Strain rate along centerline

Oldroyd-B – β = 1/9



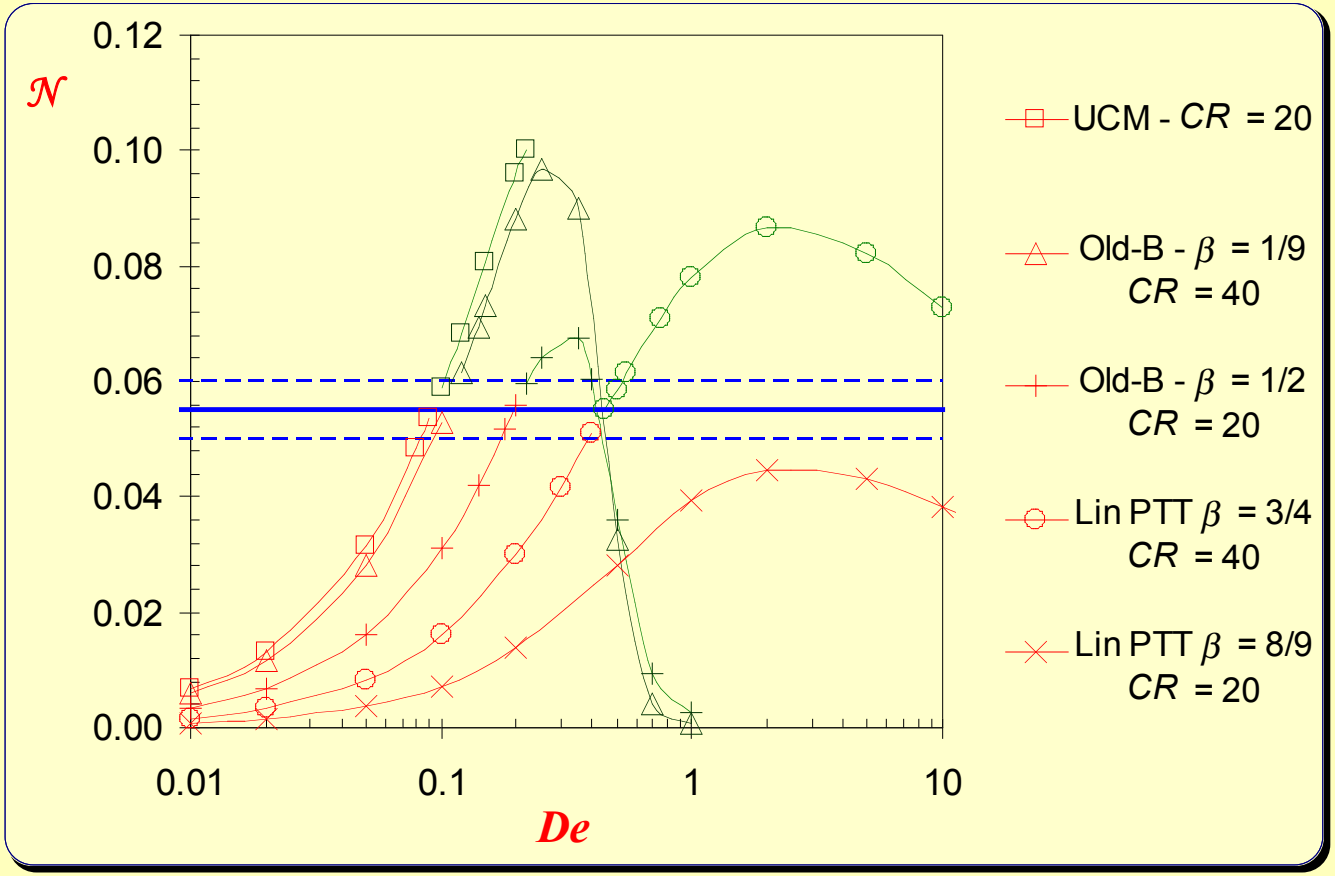
Onset of lip vortex activity – The Normal Stress Ratio

Rothstein & McKinley
JNNFM **98** (2001)

$$\mathcal{N} = \frac{N_1 / \eta_0 \dot{\gamma}}{(\tau_{zz} - \tau_{rr}) / \eta_0 \dot{\varepsilon}} = \frac{Sr(\dot{\gamma})}{Tr(\varepsilon)}$$

evaluated at $\varepsilon = 2\ln(R_1 / R_2)$

$\dot{\gamma} \approx \dot{\varepsilon} = U_2 / R_2$



Conclusions

- Accurate solutions were obtained up to very high Deborah numbers for the PTT model
- Lower accuracy and stability for the Oldroyd-B model
- Corner vortex activity scales with *De*/*CR* for large contraction ratios
- Lip vortex activity and Couette correction scale with *De*
- A map of vortex behaviour was proposed (similar to planar contractions)
- A criteria was proposed to predict the onset of lip vortex for large contraction ratios.