FED-Vol. 249

RHEOLOGY AND FLUID MECHANICS OF NONLINEAR MATERIALS — 1999 —

Edited by
D. A. SIGINER

1999 ASME INTERNATIONAL MECHANICAL ENGINEERING CONGRESS AND EXPOSITION

FED-Vol. 249, Rheology and Fluid Mechanics of
Nonlinear Materials — 1999
ASME 1999

FORCED CONVECTION IN CHANNELS WITH VISCOUS DISSIPATION FOR THE SIMPLIFIED PHAN-THIEN—TANNER FLUID

Fernando T. Pinho[1]
Centro de Estudos de Fenómenos de Transporte
Departamento de Engenharia Mecânica e Gestão Industrial
Faculdade de Engenharia, Universidade do Porto
Rua dos Bragas, 4050-123 Porto, Portugal
fpinho@fe.up.pt

Paulo J. Oliveira[1]
Departamento de Engenharia Electromecânica
Universidade da Beira Interior
Rua Marquês D'Ávila e Bolama
6200 Covilhã, Portugal
pjpo@ubi.pt

ABSTRACT

The temperature distribution and the heat transfer coefficient for forced convection in laminar channel flow with viscous dissipation are derived for the simplified Phan-Thien— Tanner fluid with a linear stress coefficient. Fully-developed thermal and hydrodynamic conditions are assumed with a constant wall heat flux imposed on both walls. As a simplifying assumption the effect of temperature variations on the material parameters is neglected.

The results show that, in all circumstances, ie for wall heating and cooling and regardless of the magnitude of viscous dissipation, an increase of fluid elasticity and/or an increase of ε results in enhanced heat transfer. As a beneficial consequence the range of temperatures inside the duct is reduced. There is also a coupling effect of viscous dissipation and fluid elasticity: heat transfer enhancement by fluid elasticity is stronger in the presence of a more intense viscous dissipation. For positive wall heat fluxes, ie wall cooling, whenever the Brinkman number exceeds a threshold value, the viscous dissipation overcomes the wall cooling effect and the fluid heats up longitudinally. Fluid elasticity delays this critical Brinkman number to higher values.

KEYWORDS: Phan-Thien—Tanner fluid, convective heat transfer, channel flow, viscous dissipation

1. INTRODUCTION

In polymer processing, operations involving extrusion often use dies with a geometric shape similar to that of a channel, Tadmor and Gogos (1979). In this industrial process, polymer melts flow at high temperatures and thus knowledge of temperature distributions and heat transfer coefficients are of great importance to improve the design of dies. This is particularly so because the quality of the final product depends on the ability to avoid the appearance of hot spots or instabilities during the manufacturing process. Hot spots are prone to occur because the low thermal conductivity of polymers causes non-uniform temperature variations; such temperature non-uniformities also result from viscous heating and variation of property values with temperature.

The Phan-Thien—Tanner (PTT) constitutive equation is frequently used to model the behaviour of polymer melts and concentrated polymer solutions, Larson (1988). It was derived from considerations of network theory by Phan-Thien and Tanner (1977).

Its simplified version, called shortly SPTT, is written as

$$Y(\mathrm{tr}\tau, T)\tau + \lambda \overset{\nabla}{\tau} = 2\eta \mathbf{D} \tag{1}$$

where $\overset{\nabla}{\tau}$ stands for Oldroyd's upper convected derivative of the stress tensor τ

$$\overset{\nabla}{\tau} = \frac{D\mathbf{u}}{Dt} - \tau . \nabla\mathbf{u} - \nabla\mathbf{u}^{\mathrm{T}} . \tau \tag{2}$$

In Eq. (1) λ represents a relaxation time, η is a viscosity coefficient equal to the product of the relaxation time by the relaxation modulus λG, $\mathbf{D}$ is the rate of strain tensor and $\mathbf{u}$ is the velocity vector. According to Phan-Thien (1978), the stress-coefficient function Y assumes an exponential form which is linearised in this work for simplicity. The linearisation of function Y is acceptable whenever the amount of extensional deformation is limited (weak flows), following the flow classification procedure of Tanner (1985). According to this classification the plane Poiseuille flow is a weak flow and so the general exponential form of the stress coefficient function can be approximated by its linearised form, Eq. (3)

$$Y(tr\tau) = 1 + \frac{\varepsilon\lambda}{\eta}\mathrm{tr}\tau \tag{3}$$

Here, tr τ is the trace of the stress tensor τ and ε is a free parameter related to the extensional properties of the fluid; it imposes an upper limit to the elongational viscosity which is proportional to the inverse of ε. The upper-convected Maxwell model is recovered when $\varepsilon = 0$, giving an unbounded elongational viscosity in simple extensional flow. Parameter ε may have an

1 correspondence can be sent to either author

influence on the shear properties as well, imparting shear-thinning to the fluid provided its value is not too small (Phan-Thien, 1978 has shown no effect of ε when it is of the order of 10^{-2}).

The analytical hydrodynamic solution of the flow of a simplified PTT fluid, with a linearised stress coefficient, in a pipe and a channel is given in Oliveira and Pinho (1999). Here, we extend that analysis to one of the corresponding heat transfer channel flow problems and derive expressions for the temperature distribution and heat transfer coefficient. In particular, a closed form solution for the fully-developed heat transfer problem with a constant wall heat flux is derived and discussed. Considering the high viscosities of real polymer melts and the large velocity gradients found in industrial processes, the issue of viscous dissipation is taken into account except for the effect on the material parameters, which are assumed to be independent of temperature. It is remarked that for similar problems involving inelastic non-Newtonian fluids there is already a wealth of knowledge, in particular for fluids obeying the power law model (see, for instance, Irvine and Karni, 1987).

In the next section the problem is formulated and the solution of the corresponding hydrodynamic isothermal problem will be presented. Then follows the presentation of the analytical solution of the heat transfer problem. A discussion of the results with emphasis on the effects of fluid elasticity and viscous dissipation on the relevant quantities will be presented before the closure of the paper.

2. FORMULATION OF THE PROBLEM

The flow in the plane channel is considered to be fully-developed both thermally and hydrodynamically. It is also assumed that the flow is steady, laminar and has constant properties, i.e., no dependence of the fluid properties and model parameters on temperature will be considered. A constant heat flux is imposed at the wall, ($y = H$, with H the channel half-width), and $y = 0$ is the centreline where symmetry conditions apply.

The equation to be solved is the energy transport equation with provision for viscous dissipation, which can be simplified into, after invoking fully-developed conditions,

$$k\frac{\partial^2 T}{\partial y^2}+\Phi=\rho c_p u\frac{\partial T}{\partial x} \tag{4}$$

where k, ρ and c_p stand for the thermal conductivity, density and specific heat, respectively. The temperature T varies with the axial x and transverse y coordinates, u stands for the longitudinal velocity component and Φ is the dissipation function which is generaly defined as

$$\Phi \equiv \tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{5}$$

The thermal boundary conditions are symmetry at the channel mid-plane

$$\left.\frac{\partial T}{\partial y}\right|_{y=0}=0 \tag{6}$$

and a constant wall heat flux,

$$\left.-k\frac{\partial T}{\partial y}\right|_{y=H}=\dot{q}_w \tag{7}$$

which is negative when entering the channel.

For fully-developed Poiseuille flow the dissipation function involves the shear component exclusively, ie.,

$$\Phi=\tau_{xy}\dot{\gamma}_{xy}=\tau_{xy}\frac{du}{dy} \tag{8}$$

with the velocity, shear stress and shear rate profiles taking the forms derived by Oliveira and Pinho (1999) and given in Eqs. (9) to (11), respectively.

$$\frac{u}{\bar{u}}=\frac{3}{2}\frac{\bar{u}_N}{\bar{u}}\left[1-\left(\frac{y}{H}\right)^2\right]\left\{1+9\varepsilon De^2\left(\frac{\bar{u}_N}{\bar{u}}\right)^2\left[1+\left(\frac{y}{H}\right)^2\right]\right\} \tag{9}$$

$$\frac{\tau_{xy}}{3\eta\,\bar{u}/H}=-\frac{\bar{u}_N}{\bar{u}}\left(\frac{y}{H}\right) \tag{10}$$

$$\frac{\dot{\gamma}_{xy}}{3\bar{u}/H}=-\frac{\bar{u}_N}{\bar{u}}\left(\frac{y}{H}\right)\left\{1+18\varepsilon De^2\left(\frac{\bar{u}_N}{\bar{u}}\right)^2\left(\frac{y}{H}\right)^2\right\} \tag{11}$$

The non-dimensional group $De=\lambda\bar{u}/H$ is the Deborah number, which represents the ratio of a fluid characteristic time to the flow characteristic time, and is a measure of elasticity in the flow. It is also based on the cross-sectional average velocity $\bar{u}$ for the PTT fluid. $\bar{u}_N$ is the average velocity for a Newtonian fluid flowing under the same pressure gradient dp/dx

$$\bar{u}_N\equiv\frac{-(dp/dx)H^2}{3\eta} \tag{12}$$

and the ratio of both was shown by Oliveira and Pinho (1999) to be given by:

$$\frac{\bar{u}_N}{\bar{u}}=\frac{432^{1/6}\left(\delta^{2/3}-2^{2/3}\right)}{6b^{1/2}\delta^{1/3}} \quad \text{with } \delta=\left(3^3b+4\right)^{1/2}+3^{3/2}b^{1/2}$$

$$\text{and } b=\frac{54}{5}\varepsilon De^2 \tag{13-a}$$

For the following derivations it will be advantageous to use a modification of b, which we will designate as a, and is defined below.

$$a\equiv 9\varepsilon De^2\left(\frac{\bar{u}_N}{\bar{u}}\right)^2 \tag{13-b}$$

This non-dimensional parameter is a measure of both the extensional and the elastic properties of the fluid.

3. ANALYTICAL SOLUTION

The following description follows the procedure outlined by Holman (1981) for the corresponding Newtonian pipe flow case.

The constant wall flux boundary condition (Eq. 7) implies that the cross-section average temperature ($\bar{T}$) must increase longitudinally at a constant rate. Together with the condition of thermal fully-developed flow, that implies a constant longitudinal gradient of temperature $\partial T/\partial x$.

Eqs. (8) to (11) are back-substituted into the energy equation, which can then be integrated a first time. The axisymmetry boundary condition (6) is applied next to solve for the first constant of integration leading to the transverse distribution of the gradient of temperature

$$\frac{\partial T}{\partial y}=\frac{3}{2}\frac{H\overline{u_N}}{\alpha}\frac{dT}{dx}\left[(1+a)\frac{y}{H}-\frac{1}{3}\left(\frac{y}{H}\right)^3-\frac{a}{5}\left(\frac{y}{H}\right)^5\right]$$
$$-\frac{9\eta\overline{u_N}^2}{kH}\left[\frac{1}{3}\left(\frac{y}{H}\right)^3+\frac{2a}{5}\left(\frac{y}{H}\right)^5\right] \quad (14)$$

where the thermal properties have been compacted into the definition of the thermal diffusivity

$$\alpha\equiv\frac{k}{\rho c_p}$$

Eq. (14) is integrated next and the second boundary condition is applied indirectly: instead of using Eq. (7) immediately, it is more convenient to introduce the centreline temperature T_c which is later related to the wall heat flux through the definitions of $\dot{q}_w$ and the bulk temperature $\overline{T}$. Thus, the temperature distribution resulting from the integration of Eq. (14) becomes

$$T-T_c=\frac{3}{2}\frac{\overline{u_N}H^2}{\alpha}\frac{dT}{dx}\left[\frac{1+a}{2}\left(\frac{y}{H}\right)^2-\frac{1}{12}\left(\frac{y}{H}\right)^4-\frac{a}{30}\left(\frac{y}{H}\right)^6\right]$$
$$-\frac{9\eta\overline{u_N}^2}{k}\left[\frac{1}{12}\left(\frac{y}{H}\right)^4+\frac{a}{15}\left(\frac{y}{H}\right)^6\right] \quad (15)$$

The wall temperature is easily obtained from Eq. (15) after setting $T=T_w$ at $y/H=1$

$$T_w-T_c=\frac{3\overline{u_N}H^2}{4\alpha}\frac{dT}{dx}\left[\frac{5}{6}+\frac{14}{15}a\right]-\frac{9\eta\overline{u_N}^2}{k}\left[\frac{1}{12}+\frac{a}{15}\right] \quad (16)$$

Note that in Eqs. (14) to (16) the second term on the right-hand-side arises from the viscous dissipation function in the energy equation, and the elasticity/extensional capacity of the fluid is buried in parameter a, as defined at the end of the previous section (Eq. 13-b).

The heat transfer coefficient (h) is defined as usual

$$\dot{q}_w\equiv h\left(\overline{T}-T_w\right) \quad (17)$$

where the cross-section average temperature, also called bulk temperature, is obtained from

$$\overline{T}\equiv\frac{\int_0^H uTdy}{\int_0^H udy} \quad (18)$$

The denominator of Eq. (18) represents the volumetric flow rate per unit side length $H\overline{u}$. Oliveira and Pinho (1999) derived analytically an useful expression for the bulk velocity

$$\overline{u}=\overline{u_N}\left(1+\frac{6}{5}a\right) \quad (19)$$

to be used in the ensueing analysis. Note, however, that a still depends on $\overline{u}$. Upon substitution and integration of Eq. (18), we obtain the expression for the bulk temperature

$$\overline{T}-T_c=\frac{\frac{9\overline{u_N}H^2}{20\alpha}\frac{dT}{dx}\left[\frac{108}{231}a^2+\frac{145}{189}a+\frac{13}{42}\right]-\frac{27\eta\overline{u_N}^2}{2k}\left[\frac{12}{3465}a^2+\frac{1}{105}a+\frac{1}{210}\right]}{1+\frac{6}{5}a} \quad (20)$$

Eqs. (7) and (17) are combined to solve for the heat transfer coefficient

$$h=\frac{-k\left(\frac{\partial T}{\partial y}\right)_{y=H}}{\overline{T}-T_w} \quad (21\text{-a})$$

which is calculated next and presented in non-dimensional form as a Nusselt number where D_H is the hydraulic diameter in this case equal to $4H$

$$Nu\equiv\frac{D_H h}{k}=\frac{4Hh}{k} \quad (21\text{-b})$$

After performing the necessary substitutions in Eq. (21) we obtain the Nusselt number

$$Nu=\frac{\left[\frac{4\overline{u_N}H^2}{\alpha}\frac{dT}{dx}-\frac{12\eta\overline{u_N}^2}{k}\right]\left[1+\frac{6}{5}a\right]^2}{\frac{\overline{u_N}H^2}{\alpha}\frac{dT}{dx}\left[\frac{1212}{1925}a^2+\frac{116}{105}a+\frac{17}{35}\right]-\frac{\eta\overline{u_N}^2}{k}\left[\frac{1296}{1925}a^2+\frac{48}{35}a+\frac{24}{35}\right]} \quad (22)$$

Eq. (22) is not in a convenient format because it is written in terms of dimensional quantities. In the absence of viscous dissipation, equivalent to setting $\eta=0$, Eq. (22) would depend exclusively on a, which is a dimensionless measure of elasticity. The viscous dissipation is conveniently accounted for by the dimensionless Brinkman number and in this work we use the modified version defined in Shah and London (1978) for a prescribed heat flux:

$$Br\equiv\frac{\eta\overline{u}^2}{D_H\dot{q}_w} \quad (23)$$

Using Eq. (19) we get a relationship between Br and $\eta\overline{u_N}^2$

$$\eta\overline{u_N}^2=\frac{4H\dot{q}_w Br}{\left[1+\frac{6}{5}a\right]^2} \quad (24)$$

Since $\dot{q}_w=-k\left.\frac{\partial T}{\partial y}\right|_{y=H}=\left[\frac{3\eta\overline{u_N}^2}{H}-\frac{k\overline{u_N}H}{2\alpha}\frac{dT}{dx}\right]\left[1+\frac{6}{5}a\right]$, and using Eq. (24), we relate the modified Brinkman number to the longitudinal temperature gradient

$$\frac{\overline{u_N}H}{\alpha}\frac{dT}{dx}=\frac{\dot{q}_w}{k}\frac{\left[12Br-1-\frac{6}{5}a\right]}{\left[1+\frac{6}{5}a\right]^2} \quad (25)$$

Finally, we can back-substitute Eqs. (24) and (25) into Eq. (22) to arrive at a more useful expression for the Nusselt number, based exclusively on non-dimensional parameters,

$$Nu=\frac{4\left[1+\frac{6}{5}a\right]^3}{\left[1+\frac{6}{5}a\right]\left[\frac{1212}{1925}a^2+\frac{116}{105}a+\frac{17}{35}\right]-Br\left[\frac{1872}{385}a^2+\frac{816}{105}a+\frac{108}{35}\right]} \quad (26)$$

As a useful check we see that Eq. (26) reduces to well-known Newtonian solutions: for no elasticity ($a=0$) and negligible viscous dissipation ($Br=0$) it reduces to Nu=8.235 (Holman, 1981), but for non-negligible dissipation Eq. (26) becomes

$$Nu = \frac{140}{17-108Br} \tag{27}$$

an expression in agreement with a Newtonian fluid derivation (Schlichting, 1968).

The temperature profile $T(y)$ may be cast into a standard non-dimensional form using the usual definition

$$\theta(y) \equiv \frac{T(y)-T_w}{\bar{T}-T_w} \tag{28}$$

which, together with the use of the modified Brinkman number, is given by

$$\theta(y) = \frac{F_{cr}\left[1+\frac{6}{5}a\right]\left\{\frac{1+a}{2}\left(\frac{y}{H}\right)^2 - \frac{1}{12}\left(\frac{y}{H}\right)^4 - \frac{a}{30}\left(\frac{y}{H}\right)^6 - \frac{5}{12} - \frac{7}{15}a\right\}}{24Br\left[\frac{144}{1925}a^2+\frac{16}{105}a+\frac{8}{105}\right] - F_{cr}\left[\frac{808}{1925}a^2+\frac{232}{315}a+\frac{102}{315}\right]} - \frac{24Br\left[1+\frac{6}{5}a\right]\left\{\frac{1}{12}\left(\frac{y}{H}\right)^4 + \frac{a}{15}\left(\frac{y}{H}\right)^6 - \frac{1}{12} - \frac{a}{15}\right\}}{24Br\left[\frac{144}{1925}a^2+\frac{16}{105}a+\frac{8}{105}\right] - F_{cr}\left[\frac{808}{1925}a^2+\frac{232}{315}a+\frac{102}{315}\right]} \tag{29-a}$$

(Note that in Eq. (29-a) the function $F_{cr} \equiv 12Br - 1 - \frac{6}{5}a$ has been used simply for compactness)

However, as will be shown in the discussion section, the standard dimensionless temperature defined by Eq. (28) is not convenient when viscous dissipation is present and there is some advantage in basing the temperature scale on the imposed heat flux giving the following expressions for the various relevant temperature differences:

$$\frac{\left(T(y)-T_c\right)k}{\dot{q}_w H} = 12Br\frac{\left[\frac{3(1+a)}{4}\left(\frac{y}{H}\right)^2 - \frac{3}{8}\left(\frac{y}{H}\right)^4 - \frac{a}{4}\left(\frac{y}{H}\right)^6\right]}{\left[1+\frac{6}{5}a\right]^2} - \frac{\left[\frac{3(1+a)}{4}\left(\frac{y}{H}\right)^2 - \frac{1}{8}\left(\frac{y}{H}\right)^4 - \frac{a}{20}\left(\frac{y}{H}\right)^6\right]}{\left[1+\frac{6}{5}a\right]} \tag{29-b}$$

$$\frac{\left(T_w-T_c\right)k}{\dot{q}_w H} = \frac{Br\left[\frac{9}{2}+6a\right]}{\left[1+\frac{6}{5}a\right]^2} - \frac{\left[\frac{5}{8}+\frac{7}{10}a\right]}{\left[1+\frac{6}{5}a\right]} \tag{30}$$

$$\frac{\left(\bar{T}-T_c\right)k}{\dot{q}_w H} = Br\frac{\left[\frac{99}{54}+\frac{127}{35}a+\frac{180}{77}a^2\right]}{\left[1+\frac{6}{5}a\right]^3} - \frac{\left[\frac{81}{385}a^2+\frac{29}{84}a+\frac{39}{280}\right]}{\left[1+\frac{6}{5}a\right]^2} \tag{31}$$

$$\frac{\left(\bar{T}-T_w\right)k}{\dot{q}_w H} = \frac{3}{2}\frac{\left[\frac{808}{1925}a^2+\frac{232}{315}a+\frac{102}{315}\right]}{\left[1+\frac{6}{5}a\right]^2} - Br\frac{\left[\frac{1872}{385}a^2+\frac{272}{35}a+\frac{108}{35}\right]}{\left[1+\frac{6}{5}a\right]^3} \tag{32}$$

The viscous dissipation contribution to the temperature profile for a Newtonian fluid is also in agreement with the results in Schlichting (1968).

4. DISCUSSION OF RESULTS

Improved understanding of the heat transfer phenomenon can be gained from previous knowledge of the hydrodynamic problem. To this aim, transverse velocity profiles are plotted in Fig. 1 as a function of ε and De. The plot includes the Newtonian velocity profile and it is clear that both ε and De increase the intensity of shear-thinning. Shear-thinning velocity profiles are characterised by an increase in wall shear rates and the reduction of centreline velocities, thus higher momentum and energy diffusion is expected to take place in the wall region.

Concerning the heat transfer problem, it must be remarked that in the absence of viscous dissipation the solution is independent of whether there is wall cooling or heating. However, viscous dissipation always contributes to internal heating of the fluid, hence the solution will differ according to the direction of the imposed heat flux at the wall.

The discussion will address first the situation where both viscous dissipation and wall flux contribute to fluid heating and will proceed then to analyse the wall cooling situations.

<u>Negative heat flux at the wall (Wall heating)</u>

In agreement with the definition for the direction of the wall heat flux, Eq. (7), a negative value of $\dot{q}_w$ implies that heat is being supplied across the walls into the fluid and Eq. (23) requires that $Br < 0$. Simultaneously, Eq. (25) implies a positive longitudinal gradient of temperature ($dT/dx > 0$), i.e. the fluid is being heated. Fig. 2 shows the variation of the Nusselt number with the Deborah number, the modified Brinkman number and the extensional parameter ε of the constitutive equation. The Nusselt number is shown to vary between two asymptotes, corresponding to low and high Deborah numbers, respectively, but with the lower asymptote strongly dependent on the value of the Brinkman number. The asymptotic Nusselt number at high Deborah, on the other hand, is independent of the Brinkman number, although viscous dissipation tends to delay the establishment of the asymptotic Nu values to higher De numbers. Note, however, that in practice the Deborah number does not attain such high values as those in Fig. 1, and is usually smaller than 10 to 100.

For an elastic fluid devoided of extensional characteristics ($\varepsilon = 0$), Nu remains equal to the corresponding value at $De = 0$, even for $De > 0$. In the absence of viscous dissipation effects, the Nusselt number increases from the Newtonian value of 8.235 to the asymptotic value of 9.149, which is independent of the value of ε

and De. As the magnitude of the Brinkman number increases, the Nusselt number at low De decreases and, simultaneously, the role of elasticity is enhanced via both the effect of De and ε. Note also that the relative variation of the Nusselt number with the Deborah number becomes more pronounced, as can be observed in Fig. 3. In this figure, the ratio of Nu to the corresponding value Nu_0 (Nu for $De = 0$) is shown as a function of De for different values of Br. A threefold increase of Nusselt number is seen to occur for Brinkman numbers as low as -1 at a Deborah number of 10.

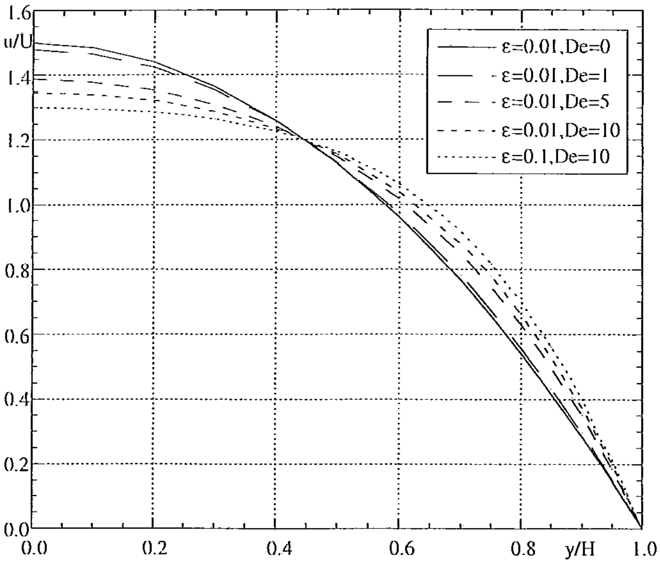

Figure 1- Transverse profiles of axial velocity as a function of ε and the Deborah number De.

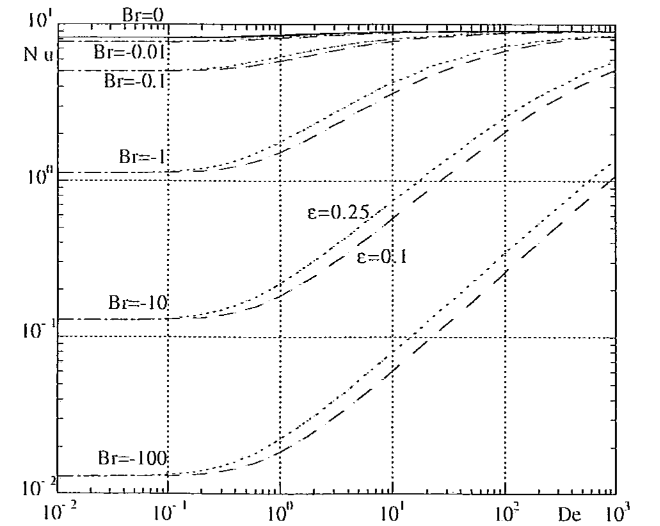

Figure 2) Variation of the Nusselt number as a function of the Deborah number, Brinkman number and ε, in the case of wall heating. Due to $\dot{q}_w$ convention, Br is negative for wall heating.

The decrease in Nusselt number with viscous dissipation is a consequence of the increased temperature range within the pipe and namely of the difference $T_w - \overline{T}$ for a situation of constant wall heat flux. For the same reason an increase in Nusselt number by elasticity is accompanied by a reduction in temperature differences within the pipe. This reduction of the temperature differences stems from an improved heat transfer in the wall region due to the higher velocity gradients that are caused by higher values of the Deborah number and/or ε, ie, it is a typical consequence of shear-thinning behaviour.

The effect of ε is similar to that of the Deborah number as can be confirmed in the transverse profiles of the normalised temperatures in Fig. 4 for ε varying in the range 0 to 0.25. This is to be expected since elasticity is quantified by function a which couples the effects of ε and De into a single parameter. It is not apparent from Eq. (13-b) how a will vary when ε and De are increased but the results in Oliveira and Pinho (1999) show that a does indeed increase with $\sqrt{\varepsilon De}$. One should also keep in mind that an increase in ε is related to a reduction of the extensional viscosity of the fluid and not with a reduction of shear elasticity, this latter measured by the shear relaxation time of the fluid. In this respect, these results are very important because they show that an elastic fluid with extensional characteristics (measured by ε) have heat transfer characteristics in fully developed duct flow which are substantially different from those of an elastic fluid without extensibility. Indeed, if we take a fluid obeying the upper convected Maxwell equation, which has no capacity for extension ($\varepsilon = 0$ hence $a = 0$) but is clearly elastic, we see that its heat transfer characteristics are coincident with those for a Newtonian fluid.

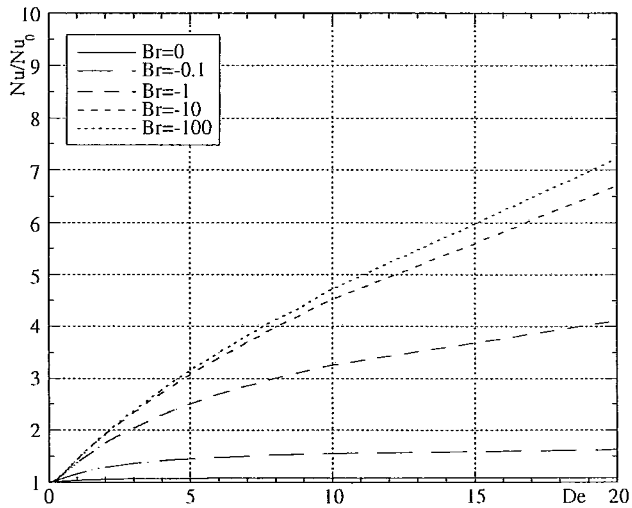

Figure 3) Relative variation of the Nusselt number as a function of the Deborah number and Brinkman number for wall heating and ε=0.1 ($Nu_0 = Nu$ at $De = 0$). Due to $\dot{q}_w$ convention, Br is negative for wall heating.

The standard way of making temperature non-dimensional, based on Eq. (28) and examplified in Fig. 4, is not appropriate for the situation of imposed heat-flux because the temperature scale $\Delta T = \overline{T} - T_w$ varies with the relevant parameters and may lead to misinterpretation of the corresponding variation of T. For example, in Fig. 4 the gradient of this standard non-dimensional temperature near the wall is seen to vary with elasticity, while the actual temperature gradient is constant for the given $\dot{q}_w$. For a given $\dot{q}_w$ the unknown of the problem is ΔT and it is thus more convenient to define a fixed temperature scale that we take as $\dot{q}_w H/k$. Fig. 5 includes two temperature profiles made non-dimensional with this fixed scale, for a Br=- 1 and Br=-2 (wall heating). Other curves at different values of De and ε, not shown here for reasons of space, make clear the forementioned effects of increased elasticity and reduced dissipation in reducing the range of temperature variation in the channel cross-section and consequently leading to higher Nusselt numbers. With this normalisation of temperature, the slope of the curves near the wall must be equal to -1 for all cases, as is apparent in Fig. 5.

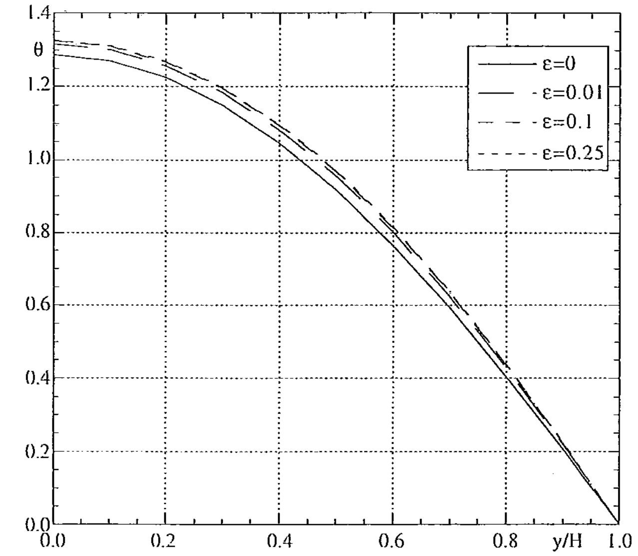

Figure 4) Effect of ε on the radial variation of the standard normalised temperature θ for negligible viscous dissipation (Br=0) and constant De=10.

Positive heat flux at the wall (Wall cooling)

A change of the direction of the wall heat flux ($\dot{q}_w > 0$) leads to wall cooling, aimed at reducing the bulk temperature of the fluid. However, viscous dissipation can undermine this process: for low positive values of Br the net result is a consistent decrease in temperature, ie $dT/dx < 0$ (c.f Eq. 25), but when Br exceeds a critical value, the heat generated internally by viscous dissipation will overcome the effect of wall cooling. This critical Brinkman number Br_1 is obtained after equating to zero the gradient of temperature in Eq. (25), to give

$$Br_1 = \frac{1}{12} + \frac{a}{10} \qquad (33)$$

Above this first critical Brinkman number the fluid heats up ($dT/dx > 0$), in spite of the imposed wall cooling, but initially the Nusselt number stays positive. This Nusselt number must be interpreted cautiously because there is a second critical Brinkman number Br_2 defined by

$$Br_2 = \frac{3}{2}\,\frac{\left[1+\frac{6}{5}a\right]\left[\frac{808}{1925}a^2+\frac{232}{315}a+\frac{102}{315}\right]}{\left[\frac{1872}{385}a^2+\frac{272}{35}a+\frac{108}{35}\right]} \qquad (34)$$

At $Br = Br_2$ we have $\overline{T} = T_w$, leading to an undefined Nusselt number since $Nu = 4H\dot{q}_w / \left(k(\overline{T}-T_w)\right)$. This second critical Brinkman number Br_2 is thus a mathematical singularity for the Nusselt number definition here adopted. Above this Brinkman number the Nusselt number switches to negative to express the change in sign of $\overline{T}-T_w$, not a change in the direction of the wall heat flux.

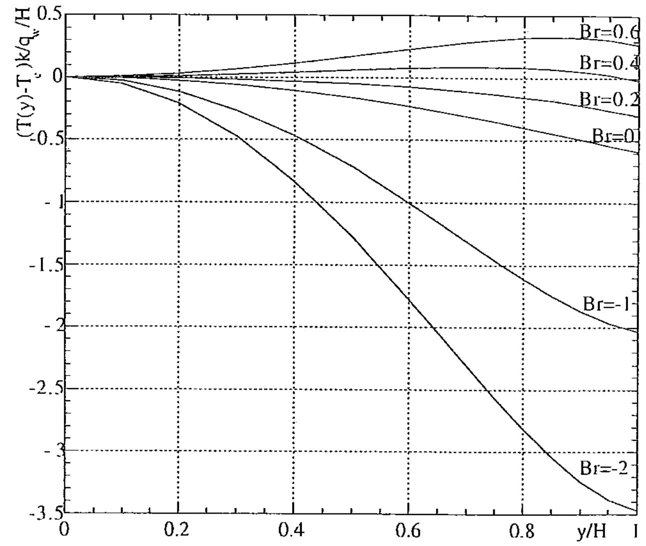

Figure 5) Radial profile of the normalised temperature as a function of the Brinkman number for De= 5 and ε=0.1.

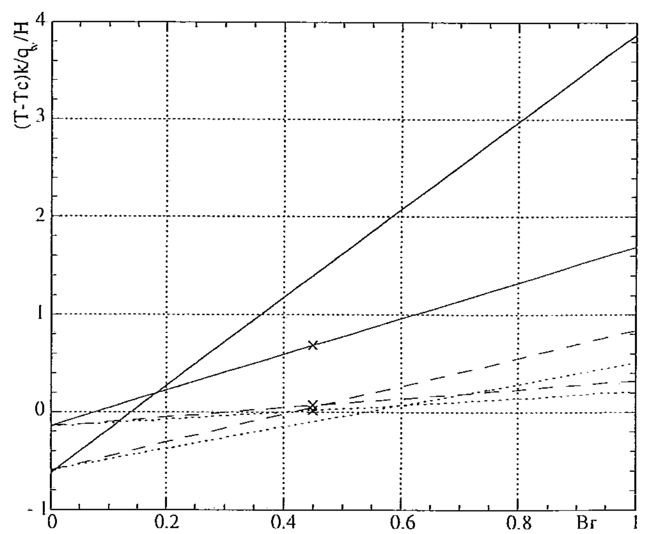

Figure 6) Variation of the normalised bulk (with marker) and wall (no marker) temperatures with the Brinkman number for the cases without (De= 0, solid) and with (De= 5, dashed) elasticity. Two values of ε are considered: 0.1 (long-dash) and 0.25 (short-dash).

The form of the temperature profiles change according to the range of Brinkman number and Fig. 5 shows typical transverse profiles of $T(y)-T_c$ normalised by $\dot{q}_w H/k$ in each of the zones of positive and negative Brinkman number behaviour. For the specific case of Fig. 5, the critical Brinkman numbers are Br_1=0.281 and Br_2= 0.462. Thus, the curves for Br=0.4 and 0.6 exhibit the effect of fluid heating up ($\bar{T} > T_c$) due to the overcoming influence of dissipation although the slope at the wall must always remain equal to -1. The curves are analogous to those of Newtonian fluids for cooled wall boundary layer, and the novelty here is that the boundaries between the various regions change with fluid elasticity (ε and De). This effect, and the role of the material parameter ε will be analysed next.

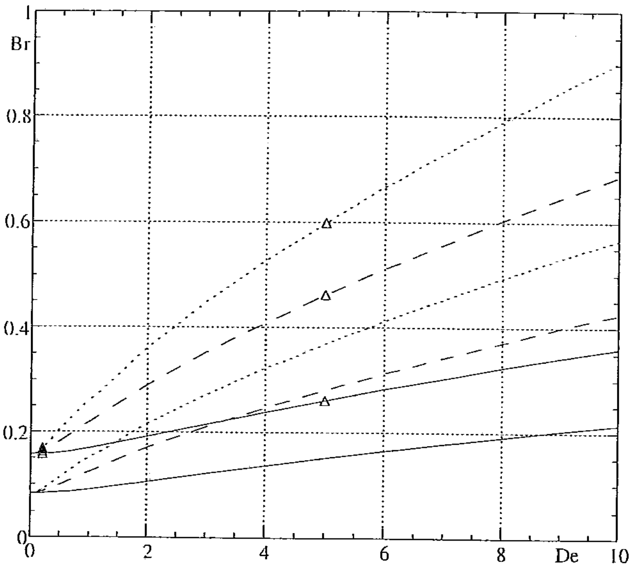

Figure 7) Variation of the critical Brinkman numbers for dT/dx=0 (Br_1: no symbol) and $\bar{T} = T_w$ (Br_2: Δ) with the Deborah number and the material parameter ε. ε= 0.01 (full line); ε= 0.1 (long dashes); ε= 0.25 (short dashes).

Fig. 6 shows the variation of $\frac{(T_w - T_c)k}{\dot{q}_w R}$ and $\frac{(\bar{T} - T_c)k}{\dot{q}_w R}$ (Eqs. (30) and (31), respectively) as a function of the modified Brinkman number for a Newtonian fluid (De= 0) and an elastic fluid (De= 5) with two different values of the extensional parameter (ε= 0.1 and 0.25). The profiles vary linearly with Br and for each pair of values of De and ε, say De=0, the two lines cross at the second critical Brinkman number (Eq. 34). Higher values of the Deborah number and/or of ε always reduce the slope of the curves, with the former parameter having a stronger influence than the latter. Simultaneously, the value of the second critical Brinkman number Br_2 increases: the dashed straight-lines in Fig. 6 cross at Br_2= 0.46 for ε= 0.1 and at Br_2= 0.60 for ε= 0.25. This effect of elasticity on the two critical Brinkman numbers is clarified in Fig. 7 where Br_1 and Br_2 are shown as a function of De for three values of the extensional parameter: ε= 0.01 (small extensional capacity), ε= 0.1 and ε= 0.25 (larger extensional capacity typical of polymer melts).

Both critical Brinkman numbers increase with the Deborah number and the parameter ε, with Br_2 always higher than Br_1. Fluid elasticity thus extends the range of Brinkman numbers over which there is cooling of the fluid when heat is extracted at the wall, which is equivalent to saying that there is a reduction of the effects of viscous dissipation. This occurs because elasticity shifts the region where viscous dissipation is predominant (higher shear rates) towards the wall, thus improving heat-remotion by the imposed wall cooling. This is consistent with the strong reduction in temperature differences observed with elastic fluids (compare curves for T_w and $\bar{T}$ in Fig. 6 at identical conditions). Equivalently, the fluid can sustain higher intensities of viscous dissipation before the reversal of overall fluid "heating/cooling" behaviour.

6. CONCLUSIONS

Temperature distributions and heat transfer coefficients were obtained in the channel flow of a simplified Phan-Thien—Tanner fluid when the stress coefficient assumed a linear form. Viscous dissipation was included but the effect of temperature variations on the material parameters was neglected.

In all circumstances, ie for wall heating and cooling and regardless of the magnitude of viscous dissipation, an increase of fluid elasticity (De) and/or an increase of ε results in enhanced heat transfer. It was found that these effects of fluid extensibility, as measured by the parameter a and indirectly by $\sqrt{\varepsilon De}$, are greatly enhanced by the magnitude of viscous dissipation here quantified by a modified Brinkman number.

For wall cooling and whenever the Brinkman number exceeds a critical threshold value (Br_1 in Eq. (33)), the heat generated by viscous dissipation overcomes the heat removed at the wall and the fluid heats up longitudinally. Fluid elasticity/extensibility delays this critical Brinkman number to higher values.

Purely elastic fluids ($\varepsilon = 0$) have heat transfer characteristics equal to those of Newtonian fluids.

REFERENCES

Holman J. P. , 1981, Heat transfer", 5th ed. New York: McGraw-Hill.

Irvine Jr. T. F. , Karni J. 1987, Non-Newtonian fluid flow and heat transfer". In: Kakaç S, Shah RK, Aung W, editors. Handbook of single-phase convective heat transfer. New York: John Wiley: pp 20.1-20.57.

Larson R. G.,1988, "Constitutive equations for polymer melts and solutions", Boston: Butterworths.

Oliveira P. J. , Pinho F. T. 1999. "Analytical solution for fully-developed channel and pipe flow of Phan-Thien—Tanner fluids" J. Fluid Mech., vol. 387, pp 271-280.

Phan-Thien N., Tanner R. I. , 1977, "A new constitutive equation derived from network theory" J. Non-Newt. Fluid Mech., vol. 2, pp. 353-365.

Phan-Thien N. A., 1978, "A nonlinear network viscoelastic model", J. Rheol., vol. 22, pp. 259-283.

Quinzani L. M. , Armstrong R. C., Brown R. A., 1995, "Use of coupled birefringence and LDV studies of flow through a planar contraction to test constitutive equations for concentrated polymer solutions", J. Rheology, vol. 39, pp. 1201-1228.

Schlichting H., 1968, "Boundary-layer theory" 6th ed. New York: Mc Graw-Hill.

Shah R. K. , London A. L. , 1978, "Laminar flow forced convection in ducts" New York: Academic Press.

Tadmor Z. and Gogos C. G. 1979, "Principles of polymer processing" New York: John Wiley and Sons.

Tanner R. I., 1985, "Engineering Rheology", Oxford: Clarendon Press.