Development Length in Planar Channel Flows of Newtonian Fluids Under the Influence of Wall Slip

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This technical brief presents a numerical study regarding the required development length ($L = L_{df}/H$) to reach fully developed flow conditions at the entrance of a planar channel for Newtonian fluids under the influence of slip boundary conditions. The linear Navier slip law is used with the dimensionless slip coefficient $k_t = k_t(1/u_t/H)$, varying in the range $0 < k_t < 1$. The simulations were carried out for low Reynolds number flows in the range $0 < Re < 100$, making use of a rigorous mesh refinement with an accuracy error below 1%. The development length is found to be a nonmonotonic function of the slip velocity coefficient, increasing up to $k_t \approx 0.1 - 0.4$ (depending on $Re$) and decreasing for higher $k_t$. We present a new nonlinear relationship between $L$, $Re$, and $k_t$ that can accurately predict the development length for Newtonian fluid flows with slip velocity at the wall for $Re$ of up to 100 and $k_t$ up to 1. [DOI: 10.1115/1.4007383]

1 Introduction

The relevance of the development length is well known in engineering. The assumption that the flow is fully developed in regions where it remains under strong influence of the inlet boundary conditions can seriously underestimate the design of flow systems and incorrectly assume specific velocity profile shapes leading to wrong conclusions in the interpretation of data. Another relevant aspect in system flow design is the fact that some flows of Newtonian fluids in microchannels exhibit slip velocity at the walls, specially if they are hydrophobic, as shown in several experimental [1–5] and numerical [6] investigations. A detailed review of experiments on Newtonian fluids showing the existence of slip velocity is given by Neto et al. [7]. Correlations to predict the development length for Newtonian fluid flows, as a function of the Reynolds number and under no-slip boundary conditions, are available in the literature. Recent accurate correlations [8–10] indicate that the development length varies nonlinearly with the Reynolds number, while experimental data of flows in microchannels with a rectangular cross section at low Reynolds numbers [11] showed shorter developing lengths compared to conventional correlations for 2D channel flows. To the best of our knowledge, there is no literature on development lengths for Newtonian fluids in the presence of wall slip. The inclusion of slip boundary conditions in the modeling process is very important, mainly due to the emergence of industrial micro-and nanotechnologies using Newtonian fluids that exhibit wall slip [7,12]. This justifies the present contribution, where a numerical study is presented on the required development lengths for Newtonian fluid flow in planar channels under the influence of slip boundary conditions using the linear Navier slip law [13], with the dimensionless slip coefficient $k_t$ varying in the range $0 < k_t < 1$ ($k_t = 1$ corresponds to a significant slip close to a plug velocity profile, and these high slip lengths can be found in experimental results due to the presence of gaseous material at the interface [7]).

2 Equations, Numerical Analysis, and Geometry

It is assumed that this internal flow is two-dimensional, incompressible, laminar, isothermal, and steady. The governing equations for such flow conditions are the continuity

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

and the momentum

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \mathbf{\tau} \quad (2)$$

equations, where $\mathbf{u}$ is the velocity vector, $p$ is the pressure, $\rho$ is the fluid density, and $\mathbf{\tau}$ is the Newtonian extra-stress tensor, which is given by

$$\mathbf{\tau} = \mu \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) = 2\mu \mathbf{D} \quad (3)$$

where $\mathbf{D}$ is the symmetric rate of deformation tensor and $\mu$ is the dynamic viscosity.

The channel geometry and Cartesian coordinate system are represented in Fig. 1. A uniform velocity profile, $U$, is imposed at the inlet of the planar channel, with all other variables set to zero. Vanishing streamwise gradients are applied to all variables at the outlet plane, except for the pressure, which is linearly extrapolated to the outlet from the two nearest upstream cells. At the wall, the usual no-slip boundary condition was replaced by a wall slip law, in this case, the linear Navier slip law [13],

$$u_{w,i} = -k_t \tilde{u}_{i,w} \quad (4)$$

where $u_{w,i} = u_{w,i}/U$ is the dimensionless slip velocity, $k_t$ is the dimensionless slip coefficient that allows one to control the
intensity of the slip velocity, and $\tau_{yy,w} = \tau_{yy,w}(H/U\mu)$ stands for the dimensionless wall shear stress. Equation (4) states that the tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shear stress opposite direction, both variables calculated at the wall. Hence, the Navier tangent velocity vector points in the tangent shea
and Barber and Emerson [18] and is given by

\[ \frac{L}{C_{20}} = \frac{1 + 1.35 \bar{k}_{sl} \bar{C}_{l}^{0.5} + 0.28 \bar{k}_{sl} \bar{C}_{l}^{0.5} \cdot 0.01 \bar{k}_{sl} \bar{C}_{l}^{0.5}}{1 + 1.35 \bar{k}_{sl} \bar{C}_{l}^{0.5} + 0.01 \bar{k}_{sl} \bar{C}_{l}^{0.5}} \left[ \left( 0.631^{1.8} + (0.047 \bar{R}_{e})^{1.8} \right)^{1/3} \right] \]  

(5)

for \( 0 \leq \bar{R}_{e} \leq 100 \) and \( \bar{k}_{sl} \leq 1 \).

We quantified the error between the computed and the correlation values as the average of the relative errors (for each slip coefficient and Reynolds number), and an average error below 1% was obtained.

In Fig. 3, we plot the profiles of the dimensionless streamwise velocity component along the axial direction at several transverse positions for two distinct values of the slip coefficient (\( \bar{k}_{sl} = 0.0001 \) and 0.1). We can observe in both cases that the dimensionless velocity increases as we move towards the symmetry axis (\( y = 0 \)). The conservation of mass, together with the fact that the wall slip velocity increases with the slip coefficient, forces the centerplane velocity to decrease inversely with \( \bar{k}_{sl} \). From the inset in Fig. 3, we can also see that the development length for \( \bar{k}_{sl} = 0.1 \) is longer than the development length for \( \bar{k}_{sl} = 0.0001 \). For \( \bar{k}_{sl} = 0.0001 \), the dimensionless axial velocity component presents a pronounced local maximum close to the channel wall that increases as the Reynolds number increases. For higher values of \( \bar{k}_{sl} \), we found that the introduction of the slip velocity tends to suppress the appearance of this near-wall velocity overshoot. To better understand this behavior, we also plotted the transverse profiles of the dimensionless streamwise velocity for various positions along the channel, shown in Fig. 6, for creeping flow conditions (\( \bar{k}_{sl} \geq 0 \)) and \( \bar{R}_{e} = 100 \). When the contribution from the slip velocity is negligible (\( \bar{k}_{sl} = 0.0001 \)), the development of the axial velocity profiles is not purely convex and shows a local minimum on the symmetry axis and a local maximum near the walls. These overshoots are generated as a result of the abrupt fluid deceleration happening near the wall at the inlet that happens faster than diffusion is able to transport momentum to the centerplane. As slip increases, this deceleration effect is reduced and the local maximum disappears (for the full slip condition, there is no fluid deceleration). A more in-depth description of this velocity overshoot for nonslip conditions is reported in Darbandi and Schneider [19]. Comparing Figs. 6(a) and 6(b), we can conclude

![Fig. 3](Image)

**Fig. 3** Variation with \( \bar{R}_{e} \) of the difference in \( L \) of Eq. (5) relative to the no-slip case results of Durst et al. [8] as a function of the slip coefficient \( \bar{k}_{sl} \).

![Fig. 4](Image)

**Fig. 4** Nonlinear functional correlations for \( L = f(\bar{R}_{e}, \bar{k}_{sl}) \) for channel flows.

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<th>( \bar{R}_{e} )</th>
<th>( \bar{k}_{sl} = 0.0001 )</th>
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**Table 3** Development length (\( L \)) obtained for different values of the slip coefficient (\( \bar{k}_{sl} \)) and different Reynolds numbers (\( \bar{R}_{e} \)) for the mesh M2 (the extrapolated values are in bold).
that, for small slip velocities, the overshoots are present, even when inertia is negligible, with their magnitudes (measured by \( \frac{u_{\text{max}} - u_{\text{sym}}}{u_{\text{sym}}} \), where \( u_{\text{max}} \) is the local maximum velocity in the profile and \( u_{\text{sym}} \) is the corresponding velocity at the symmetry axis) increasing with inertia. For all the simulations, the maximum overshoot magnitude is attained for \( Re = 100 \) with a value of 15.83% (close to 15.8% obtained with \( kl = 0 \) by Darbandi and Schneider [19]). For higher values of the slip coefficient, the appearance of the velocity overshoots is almost suppressed, as also observed in Figs. 6(a) and 6(b), where we can see that, for inertialess conditions, there is no overshoot (since \( Re \) is the ratio between diffusive and advective time scales, the effect of viscosity is transmitted to the whole channel very quickly when \( Re = 0 \)), while for higher Reynolds number (\( Re = 100 \)), a very small overshoot is present only for \( kl = 0.1 \). This can be explained as a result of a smaller deceleration effect of the fluid elements near the channel walls due to the slip condition, allowing both the convection and diffusion to transport momentum to the centerplane. The velocity at the centerplane is also affected (and indirectly the development length), as observed in the dimensionless velocity profiles along the centerplane for different slip coefficients and \( Re \to 0 \), plotted in Fig. 7, showing smaller \( L \) for \( kl > 0 \).

4 Conclusions

We conducted a detailed and systematic numerical investigation of the development length in planar channel flows of Newtonian fluids under laminar flow conditions and under the presence of hydrodynamic wall slip. We show that a judicious choice of mesh refinement and highly accurate numerical methods allow the prediction of highly accurate development length values. A new nonlinear correlation for \( L(kl, Re) \) is proposed, which shows good accuracy over the range \( kl \leq 1 \) and \( Re \leq 100 \). This nonlinear correlation predicts a nonmonotonic behavior between the wall slip coefficient and the development length, with the development length increasing up to \( kl \approx 0.1 \) and \( Re \approx 0.4 \) for creeping and high inertia (\( Re = 100 \)) flows, respectively, and decreasing for higher values of \( kl \).

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