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Multi-unit Auctions With Asymmetric Bidders

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IAT4EB Workshop, ECAI 2010
Lisbon 17/8/2010

ALADDIN
autonomous learning agents for decentralised data and information networks

Introduction

- Overarching Goal:
  - To design autonomous agents that would be able to represent humans in online auctions
- Tools:
  - Use game theoretic solution concepts to determine good strategies
  - Gradually be able to analyze more complex settings, which consider together all (or most) of the features which are important to the strategy selection

Solution Concepts

- Dominant Strategy:
  - This is the best strategy (response) to all possible opponent strategies
  - Very strong solution concept

- Bayes-Nash Equilibrium:
  - This is the best strategy (response) to the equilibrium opponent strategies
  - Difficult optimization problem: Need to find a fixed point in the strategy space

Problem Setting for Multi-unit Auctions

- **N** bidders, each wanting 1 item
- Valuations are private information which is i.i.d. drawn from known distribution **F(u)**
- **m** identical items for sale in 1 auction
- Each bidder maximizes own utility:
  - Risk neutral agents, i.e. profit equals utility
  - Utility: U_i = (own profit)
- No Reserve value, and infinite budget
- Uniform pricing rule for winners:
  - Auction closes immediately (1 round of bids)
  - m^{th} price, or $(m+1)^{th}$ price

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  - Agents can have any risk attitude function *u(x)*
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Problem S… f…
Multi-uni…

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  - Risk neutral agents, i.e. profit eq… utility
  - Utility: $U_i = (1-\alpha) \cdot (\text{agent_profit}) - \alpha \cdot (\text{opponent_profit})$
- No Reserve value, and infinite budget
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The parameter α is called the spite coefficient and it determines the relative weight assigned to minimizing the opponents profit as opposed to maximizing its own.

Classic case: α=0

Assymetric Bidders

- Not all bidders are created equal!
- Reasons for having different models, such as company size, corporate profile etc.

Assymetric Bidders

1. Not all bidders have the same utility function $u_\alpha(x)$ and distribution of valuations $F_\alpha(v)$
2. Not all bidders have the same competition factor α

- Cases examined:
  - Each bidder can have any one of a number of models, each with a certain probability $h(\alpha)$, which is know a priori; only the bidder knows its own model α
  - All the opponent models are known

Computing the Equilibria

System of Differential Equations

- In each of these cases we need to solve a λ×λ system of differential equations of the form:

$$\frac{\Psi_{j,1}\left(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)),\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right)}{g'_{\alpha_1}(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)))}+\ldots+\frac{\Psi_{j,\lambda}\left(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)),\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right)}{g'_{\alpha_\lambda}(g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i)))}=$$

$$\Psi_{j,0}\left(g_{\alpha_j}(v_i),g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i))\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right),\forall j=1,\ldots,\lambda$$

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z z

Final System of Differential Equations to Be Solved

- System to be solved:

$$g_i'(u) = F_i(g_i(u), g_1^{-1}(g_i(u)), \ldots, g_i^{-1}(g_i(u)), \ldots, g_L^{-1}(g_i(u)))$$

u

for all i=1,…,L

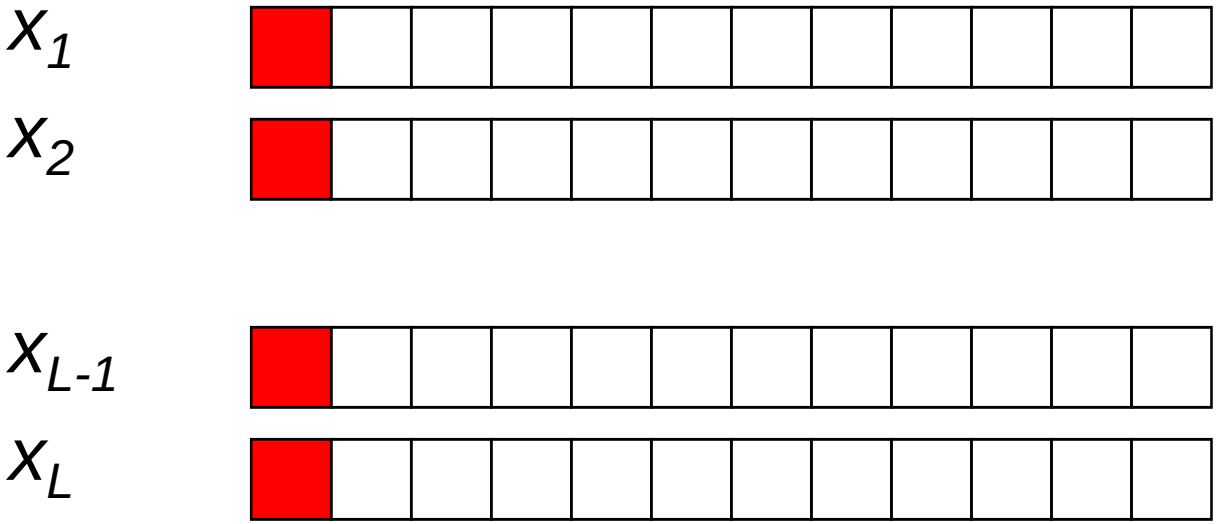
Boundary condition:

$$g_i(R)=R$$

Usual case solved by solvers

- System solved by most solvers (e.g. Runge-Kutta):

$$g_i'(u) = F_i(u, g_1(u), \ldots, g_L(u))$$



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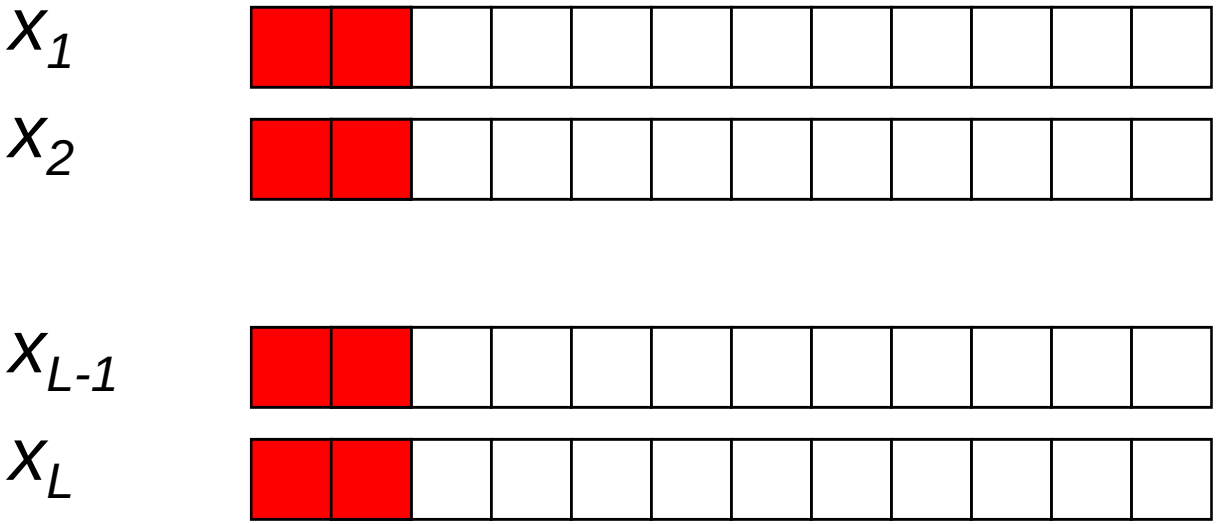

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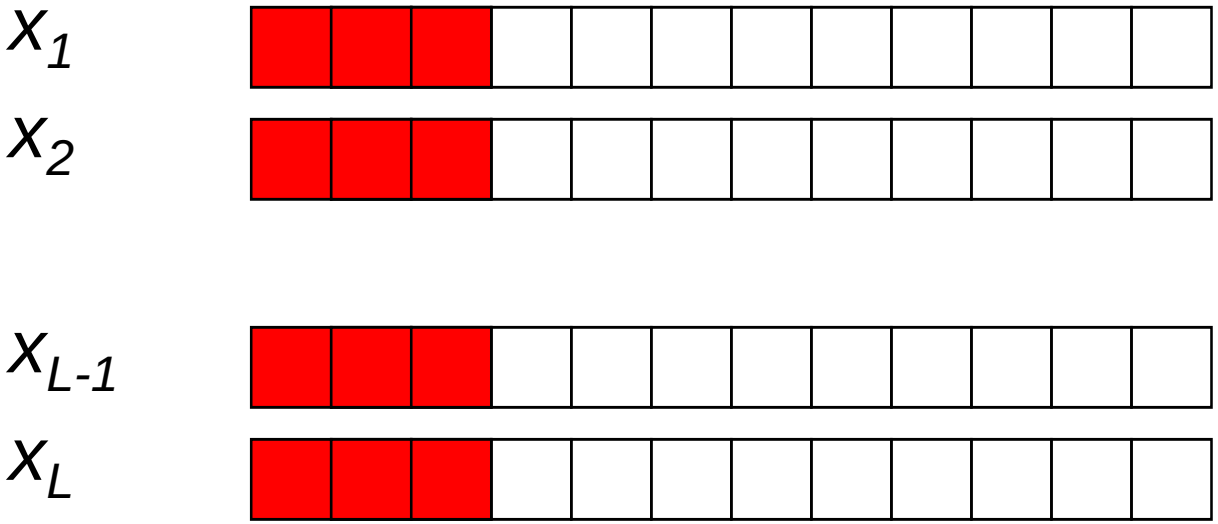

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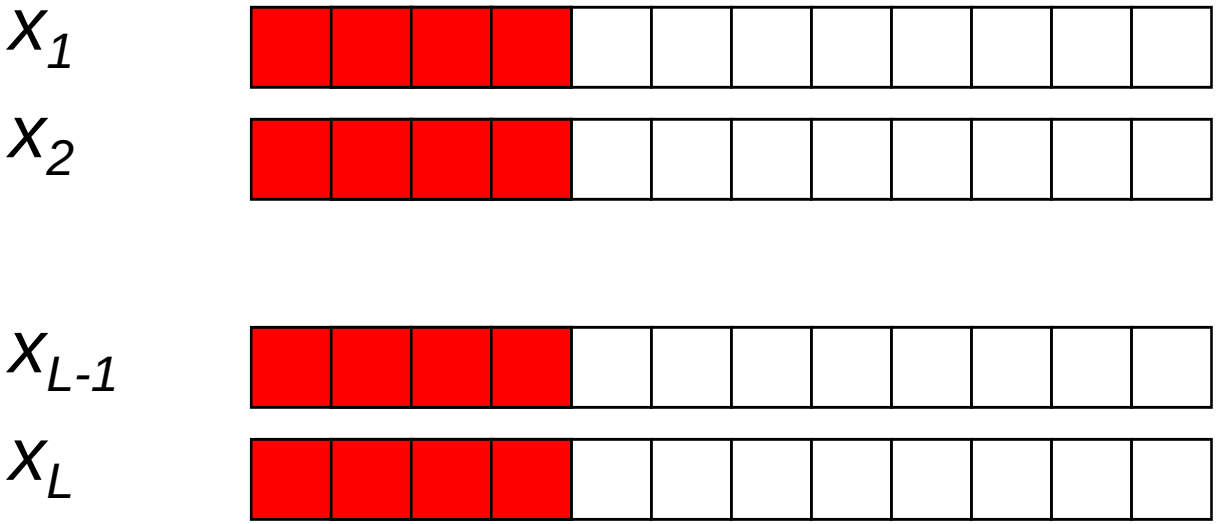
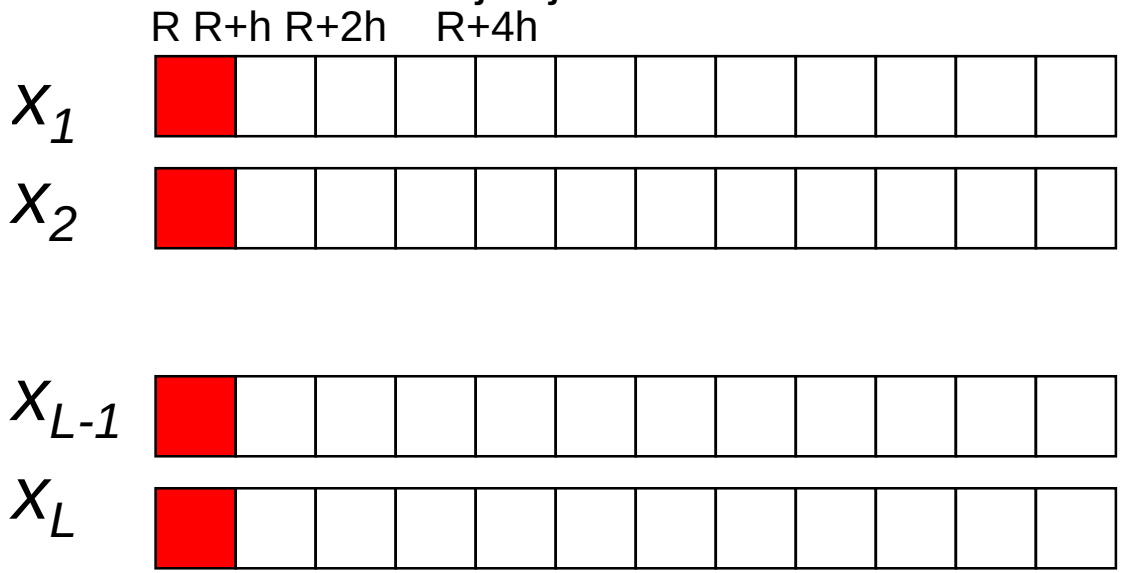


Modified solver

$g_i'(u) = F_i(g_i(u), g_1^{-1}(g_i(u)), \ldots, g_i^{-1}(g_i(u)), \ldots, g_L^{-1}(g_i(u)))$

- Compute the next value of *j* with the minimum current value $g_j(x_j)$



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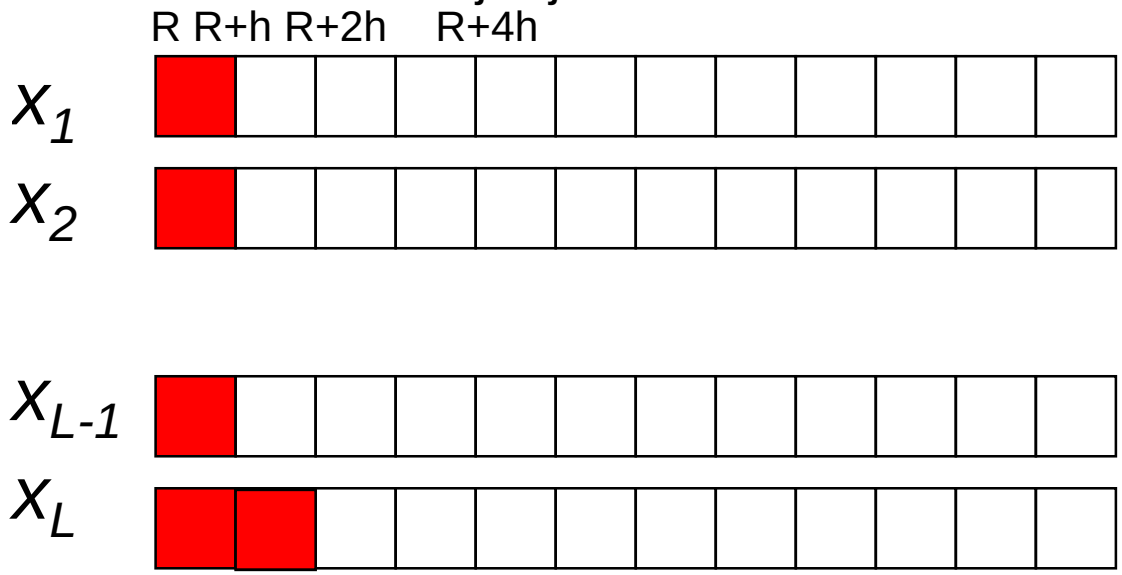
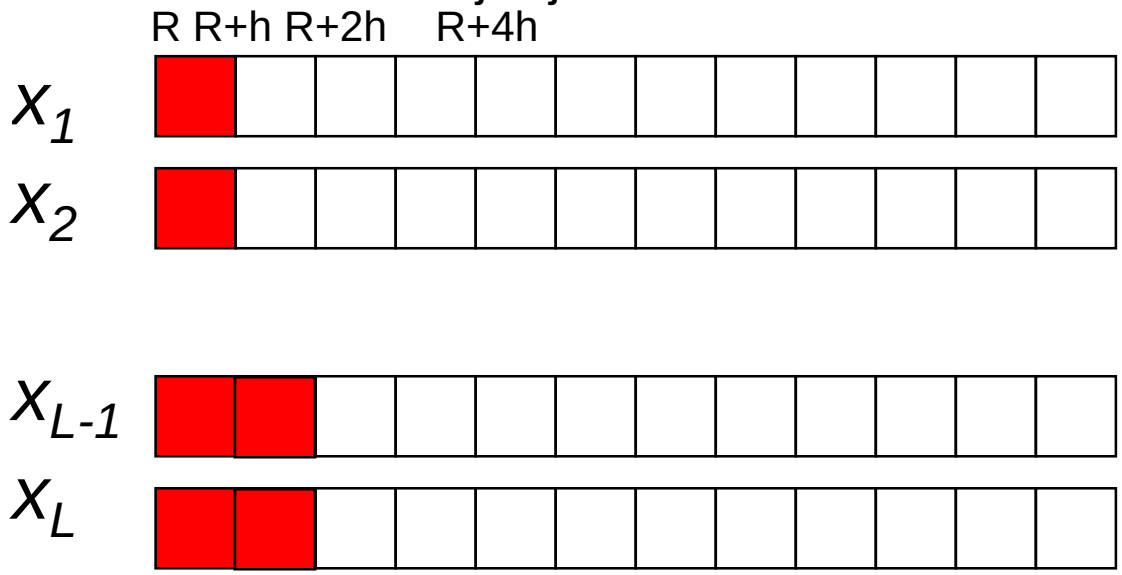


$g_L(R+h) < g_j(R+h)$

Modified solver

$g_i'(u) = F_i(g_i(u), g_1^{-1}(g_i(u)), \ldots, g_i^{-1}(g_i(u)), \ldots, g_L^{-1}(g_i(u)))$

- Compute the next value of *j* with the minimum current value $g_j(x_j)$

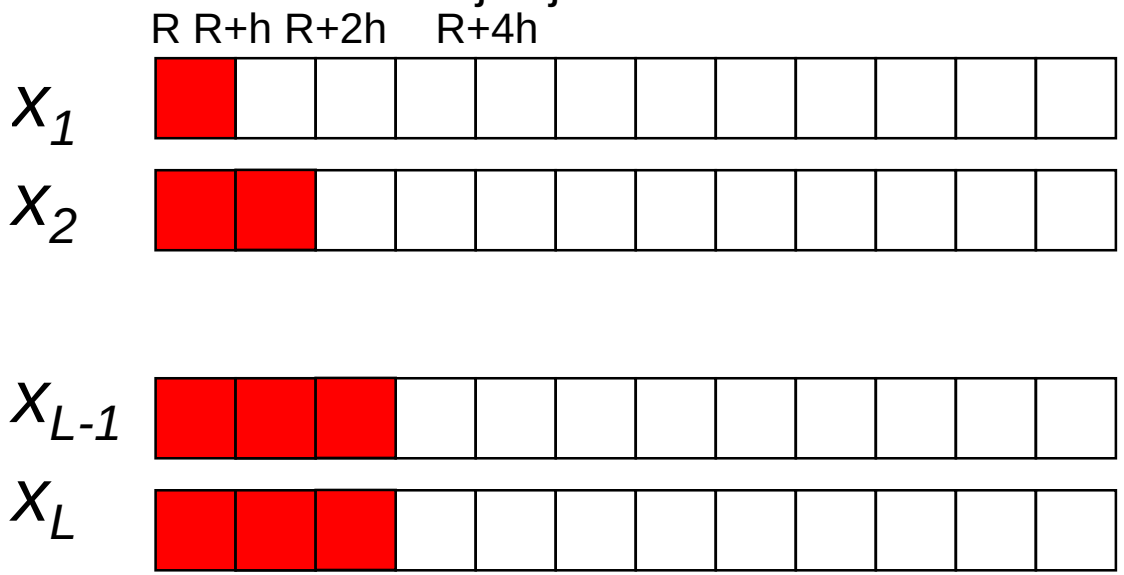


$g_{L-1}(x_{L-1}) < g_j(x_j)$

Modified solver

$g_i'(u) = F_i(g_i(u), g_1^{-1}(g_i(u)), \ldots, g_i^{-1}(g_i(u)), \ldots, g_L^{-1}(g_i(u)))$

- Compute the next value of *j* with the minimum current value $g_j(x_j)$



After several steps

System of Differential Equations

- In each of these cases we need to solve a λ×λ system of differential equations of the form:

$$\frac{\Psi_{j,1}\left(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)),\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right)}{g'_{\alpha_1}(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)))}+\ldots+\frac{\Psi_{j,\lambda}\left(g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i)),\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right)}{g'_{\alpha_\lambda}(g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i)))}=$$

$$\Psi_{j,0}\left(g_{\alpha_j}(v_i),g_{\alpha_1}^{-1}(g_{\alpha_j}(v_i))\ldots,g_{\alpha_\lambda}^{-1}(g_{\alpha_j}(v_i))\right),\forall j=1,\ldots,\lambda$$

z z

Simplifying them

$$\frac{\Psi_{j,1}\left(g_{\alpha_1}^{-1}(z),\ldots,g_{\alpha_\lambda}^{-1}(z)\right)}{g'_{\alpha_1}(g_{\alpha_1}^{-1}(z))}+\ldots+\frac{\Psi_{j,\lambda}\left(g_{\alpha_1}^{-1}(z),\ldots,g_{\alpha_\lambda}^{-1}(z)\right)}{g'_{\alpha_\lambda}(g_{\alpha_\lambda}^{-1}(z))}=$$

$$\Psi_{j,0}\left(z,g_{\alpha_1}^{-1}(z),\ldots,g_{\alpha_\lambda}^{-1}(z)\right),\forall j=1,\ldots,\lambda$$

Simplifying them

- Now this is a system we can solve:
  - Linear system : decompose derivatives
  - Use standard solvers

$$\zeta'_{\alpha_1}(z)\Psi_{j,1}\Big(\zeta_{\alpha_1}(z),\ldots,\zeta_{\alpha_\lambda}(z)\Big)+\ldots+\zeta'_{\alpha_\lambda}(z)\Psi_{j,\lambda}\Big(\zeta_{\alpha_1}(z),\ldots,\zeta_{\alpha_\lambda}(z)\Big)=$$
$$\Psi_{j,0}\Big(z,\zeta_{\alpha_1}(z),\ldots,\zeta_{\alpha_\lambda}(z)\Big),\forall j=1,\ldots,\lambda$$

Asymmetric Risk Attitudes and Valuations

- m^{th} price auction – Unknown opponent models

Theorem 1 *In the case of an mth price sealed-bid auction with N participating bidders, in which each bidder i is interested in purchasing one unit of the good for sale with inherent utility (valuation) for that item equal to v_i, which is drawn from $F_{\alpha_i}(v)$, and has a risk attitude described by utility function $u_{\alpha_i}()$, both of which describe his model α_i (where α_i are i.i.d. random variables drawn from distribution $h(\alpha)$), strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\forall x, \alpha_i : (N-m) \sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} F'_\alpha(\zeta_\alpha(x))\zeta'_\alpha(x)h(\alpha) = \quad (6)$$

$$\frac{u'_{\alpha_i}(\zeta_{\alpha_i}(x)-x)}{u_{\alpha_i}(\zeta_{\alpha_i}(x)-x)-u_{\alpha_i}(0)} \sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} F_\alpha(\zeta_\alpha(x))h(\alpha)$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L$ for all i such that $v_i^L = \min_j\{v_j^L\}$. There are λ possible bidder models characterized by parameter $\alpha = \alpha_1, \ldots, \alpha_\lambda$.

Asymmetric Risk Attitudes and Valuations

- m[th] price auction – Known opponent models

Theorem 2 *Consider the same setting as that of theorem 1, with the difference that the models α_i of all bidders are now common knowledge. Then, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \zeta'_{\alpha_j}(x) F'_{\alpha_j}(\zeta_{\alpha_j}(x)) \sum_{s\in P_{m-1,N}^{-\{i,j\}}} \Big(\prod_{\substack{\mu\notin s \\ \mu\neq i,j}} F_{\alpha_\mu}(\zeta_{\alpha_\mu}(x)) \prod_{\mu\in s} (1 - F_{\alpha_\mu}(\zeta_{\alpha_\mu}(x))) \Big) =$$

$$\frac{u'_{\alpha_i}(\zeta_{\alpha_i}(x) - x)}{(u_{\alpha_i}(\zeta_{\alpha_i}(x) - x) - u_{\alpha_i}(0))} \sum_{s\in P_{m-1,N}^{-\{i\}}} \Big(\prod_{\substack{j\notin s \\ j\neq i}} F_{\alpha_j}(\zeta_{\alpha_j}(x)) \prod_{j\in s} (1 - F_{\alpha_j}(\zeta_{\alpha_j}(x))) \Big) \tag{19}$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L$ for all i such that $v_i^L = \min_j\{v_j^L\}$.

Asymmetric Competitiveness

- m^{th} price auction – Unknown opponent models

Theorem 3 *In the case of an mth price sealed-bid auction with N participating risk-neutral bidders, in which each bidder i is interested in purchasing one unit of the good for sale with inherent utility (valuation) for that item equal to v_i, and has a competition coefficient α_i, where v_i and α_i are i.i.d. random variables drawn from distributions $F(v)$ and $h(\alpha)$ respectively, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_{\alpha_i}(x) = g_{\alpha_i}^{-1}(x)$ is the solution of the system of differential equations:*

$$\frac{1-\alpha_i m}{N-m} \sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} F(\zeta_\alpha(x))h(\alpha) = \tag{25}$$

$$\sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} \zeta'_\alpha(x)F'(\zeta_\alpha(x))\big(\zeta_\alpha(x) - x - \alpha_i\zeta_\alpha(x) + \alpha_i\zeta_\alpha(x)\big)h(\alpha)$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L, \forall\alpha_i$.

Asymmetric Competitiveness

- $(m+1)^{th}$ price auction - Unknown opp. models

Theorem 4 *In the case of an $(m+1)^{th}$ price sealed-bid auction with N participating risk-neutral bidders, in which each bidder i is interested in purchasing one unit of the good for sale with inherent utility (valuation) for that item equal to v_i, and has a competition coefficient α_i, where v_i and α_i are i.i.d. random variables drawn from distributions $F(v)$ and $h(\alpha)$ respectively, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_{\alpha_i}(x) = g_{\alpha_i}^{-1}(x)$ is the solution of the system of differential equations:*

$$\forall v_i, \alpha_i : -\alpha_i \left(1 - \sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} F(\zeta_\alpha(x))h(\alpha)\right) = \tag{33}$$

$$\sum_{\alpha=\alpha_1,\ldots,\alpha_\lambda} \zeta'_\alpha(x)F'(\zeta_\alpha(x))\left(g_\alpha^{-1}(x) - x - \alpha_i g_\alpha^{-1}(x) + \alpha_i \zeta_\alpha(x)\right)h(\alpha)$$

with boundary conditions: $g_{\alpha_i}(v_i^H) = v_i^H, \forall \alpha_i$.

Asymmetric Competitiveness

- m^{th} price auction – Known opponent models

Theorem 5 *Consider the same setting as that of theorem 3, with the difference that the models α_i of all bidders are now common knowledge. Then, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \zeta'_{\alpha_j}(x)F'(\zeta_{\alpha_j}(x)))(\zeta_{\alpha_i}(x) - x - \alpha_i\zeta_{\alpha_i}(x) + \alpha_i\zeta_{\alpha_j}(x))\cdot$$

$$\sum_{s\in P^{-\{i,j\}}_{m-1,N}} \Big(\prod_{\substack{\mu\notin s \\ \mu\neq i,j}} F(\zeta_{\alpha_\mu}(x)) \prod_{\mu\in s}(1 - F(\zeta_{\alpha_\mu}(x)))\Big) =$$

$$(1-\alpha_i m) \sum_{s\in P^{-\{i\}}_{m-1,N}} \Big(\prod_{\substack{j\notin s \\ j\neq i}} F(\zeta_{\alpha_j}(x)) \prod_{j\in s}(1 - F(\zeta_{\alpha_j}(x)))\Big) \quad (40)$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L, \forall\alpha_i$.

Asymmetric Competitiveness

- $(m+1)^{th}$ price auction - Known opp. models

Theorem 6 *Consider the same setting as that of theorem 4, with the difference that the models α_i of all bidders are now common knowledge. Then, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \zeta'_{\alpha_j}(x)F'(\zeta_{\alpha_j}(x)))(\zeta_{\alpha_i}(x) - x - \alpha_i\zeta_{\alpha_i}(x) + \alpha_i\zeta_{\alpha_j}(x))\cdot$$

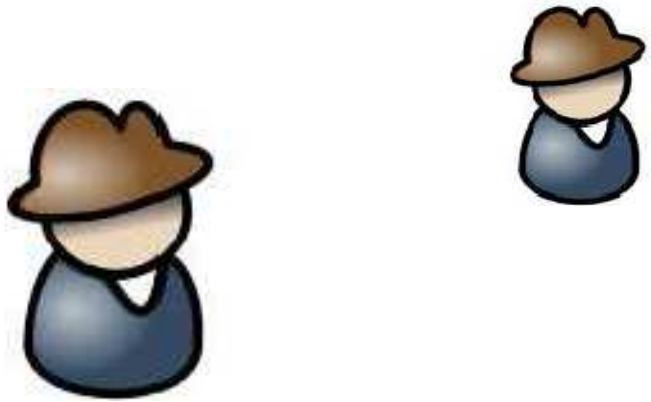
$$\sum_{s\in P^{-\{i,j\}}_{m-1,N}} \Big(\prod_{\substack{\mu\notin s \\ \mu\neq i,j}} F(\zeta_{\alpha_\mu}(x)) \prod_{\mu\in s}(1 - F(\zeta_{\alpha_\mu}(x)))\Big) =$$

$$-\alpha_i m \sum_{s\in P^{-\{i\}}_{m,N}} \Big(\prod_{\substack{j\notin s \\ j\neq i}} F(\zeta_{\alpha_j}(x)) \prod_{j\in s}(1 - F(\zeta_{\alpha_j}(x)))\Big) \qquad (44)$$

with boundary conditions: $g_{\alpha_i}(v_i^H) = v_i^H, \forall\alpha_i.$

Further Asymmetry: Directed Spite/Competition

- A. Sharma & T. Sandholm “Asymmetric Spite in Auctions”, AAAI 2010.
  - Uniform prior, 1 item, 2 bidders
  - New Idea: directed spite



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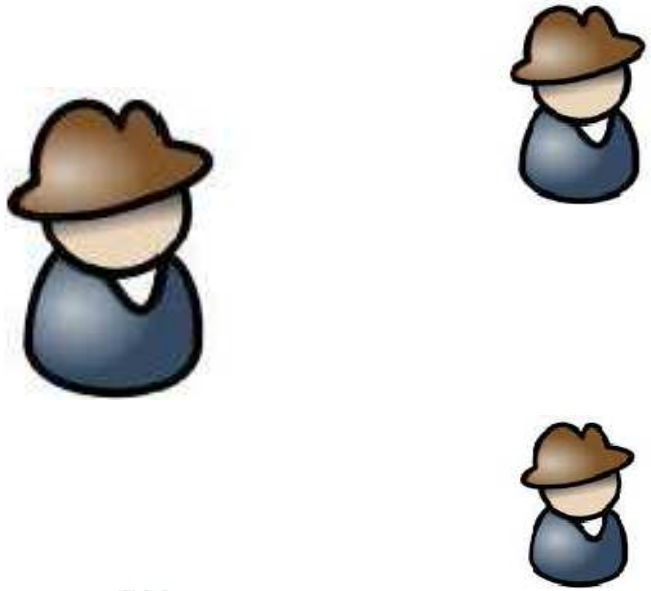


Directed Competitiveness

- m^{th} price auction – Known opponent models

Theorem 7 *Consider the same setting as that of theorem 5, with the difference that the models α_i of all bidders are now a vector of coefficients α_{ij}. Then, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \zeta'_{\alpha_j}(x) F'_{\alpha_j}(\zeta_{\alpha_j}(x)) \big((1-\alpha_{ii})(\zeta_{\alpha_i}(x)-x) + \alpha_{ij}(\zeta_{\alpha_j}(x)-x)\big) \cdot$$

$$\sum_{s\in P^{-\{i,j\}}_{m-1,N}} \Big(\prod_{\substack{\mu\notin s \\ \mu\neq i,j}} F(\zeta_{\alpha_\mu}(x)) \prod_{\mu\in s} \big(1-F(\zeta_{\alpha_\mu}(x))\big)\Big) =$$

$$\sum_{s\in P^{-\{i\}}_{m-1,N}} \big(1-\alpha_{ii}-\sum_{l\in s}\alpha_{il}\big)\Big(\prod_{\substack{j\notin s \\ j\neq i}} F(\zeta_{\alpha_j}(x)) \prod_{j\in s}(1-F(\zeta_{\alpha_j}(x)))\Big) \tag{48}$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L, \forall \alpha_i$.

Directed Competitiveness

- $(m+1)^{th}$ price auction - Known opp. models

Theorem 8 *Consider the same setting as that of theorem 6, with the difference that the models α_i of all bidders are now a vector of coefficients α_{ij}. Then, strategy $g_{\alpha_i}(v_i)$ constitutes a Bayes-Nash equilibrium, where $\zeta_\alpha(x) = g_\alpha^{-1}(x)$ is the solution of the system of differential equations:*

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} \zeta'_{\alpha_j}(x) F'_{\alpha_j}(\zeta_{\alpha_j}(x))\Big)\big((1-\alpha_{ii})(\zeta_{\alpha_i}(x)-x)+\alpha_{ij}(\zeta_{\alpha_j}(x)-x)\big)\cdot$$

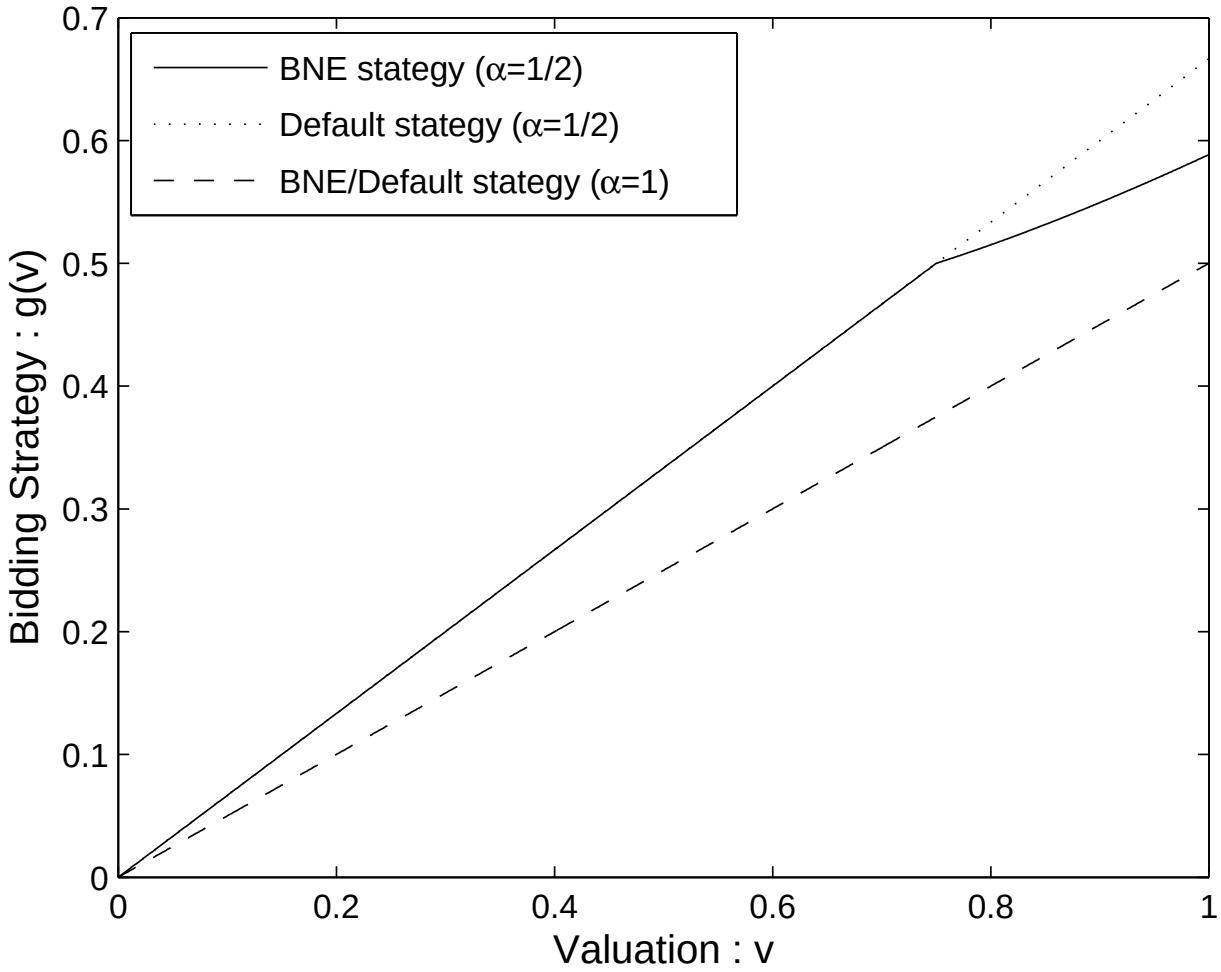
$$\sum_{s\in P_{m-1,N}^{-\{i,j\}}} \Big(\prod_{\substack{\mu\notin s \\ \mu\neq i,j}} F(\zeta_{\alpha_\mu}(x)) \prod_{\mu\in s}\big(1-F(\zeta_{\alpha_\mu}(x))\big)\Big) =$$

$$-\sum_{s\in P_{m,N}^{-\{i\}}} \Big(\sum_{l\in s}\alpha_{il}\Big)\Big(\prod_{\substack{j\notin s \\ j\neq i}} F(\zeta_{\alpha_j}(x)) \prod_{j\in s}\big(1-F(\zeta_{\alpha_j}(x))\big)\Big) \qquad (52)$$

with boundary conditions: $g_{\alpha_i}(v_i^L) = v_i^L, \forall \alpha_i.$

Examples

Example 1:
Asymmetric Risk Attitudes



Example 1: Asymmetric Risk Attitudes

- Why does this happen? When?

Lemma 1 *Consider the same case as in theorem 1, with the simplification that all bidders draw their valuations from the same distribution F, meaning that $F_\alpha = F, \forall \alpha$, but still with different risk attitudes (CRRA) $u_\alpha(x) = x^\alpha$. In this case, any bidder i can ignore the fact that his opponents have different risk attitudes $u_{\alpha_j}(), j \neq i$, as his equilibrium strategy is identical to the case when all his opponents have the same risk attitude $u_{\alpha_i}(x) = x^{\alpha_i}$ as him.*

Example 2: Asymmetric Spite 2nd price Auction

- Two cases: either self interested (*α=0*) with probability *p*, or competitive using coefficient *α* with probability *(1-p)*
- Equilibrium:
  - Bid truthfully if *α=0*
  - Bid: $$g_{\alpha}(v) = \beta v + 1 - \beta$$

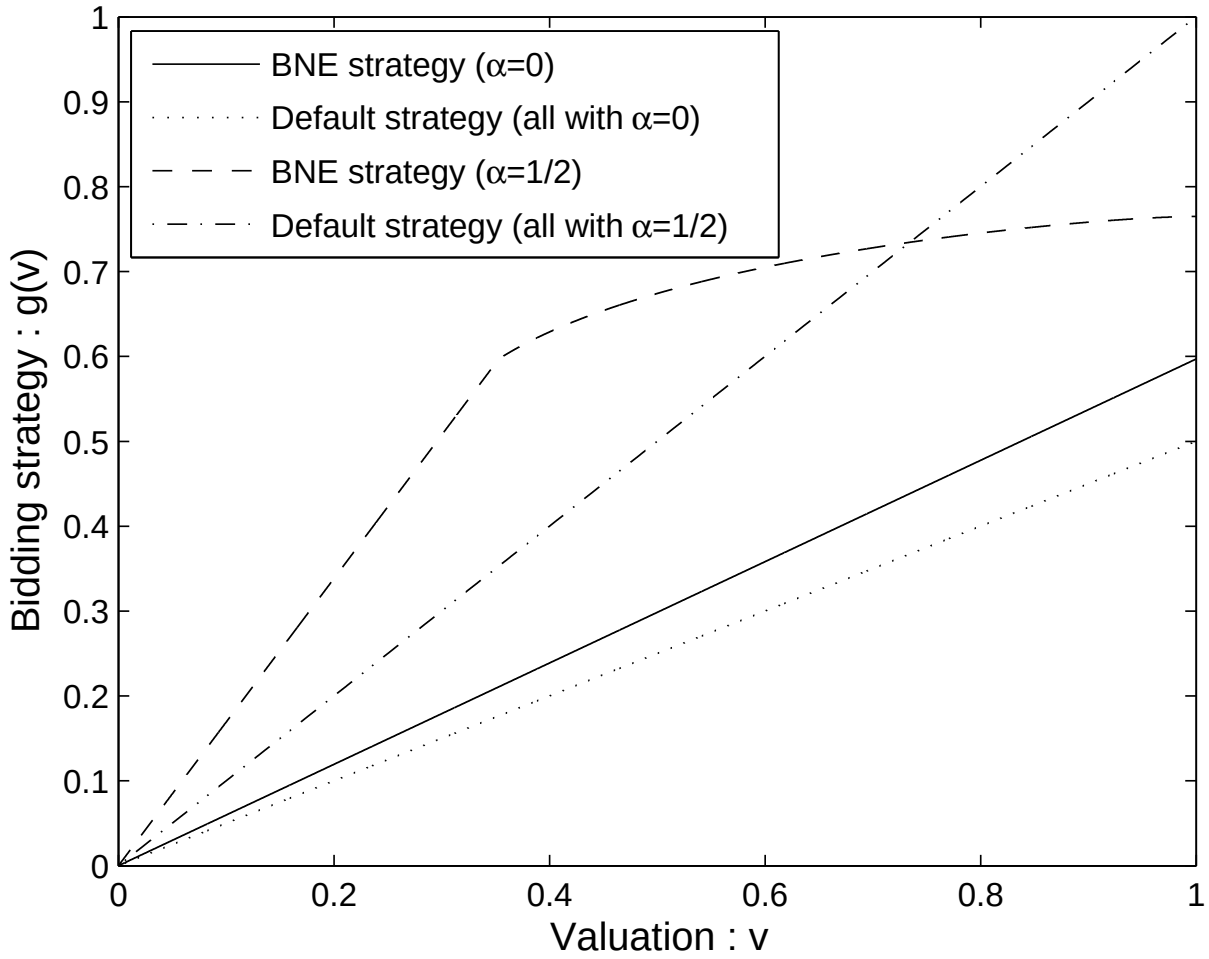
where: $$\beta = \frac{1 - \alpha - 2p + \sqrt{1 - 2\alpha + \alpha^2 + 4\alpha p}}{2(1-p)}$$

Example 3: Using the Methodology

- m^{th} price auction
- *N=3* bidders
- *M=2* items for sale
- Two models each with probability 0.5:
  - *α=0*
  - *α=0.5*

- The system is actually unstable probably due to the boundary conditions, but the solution is:

Example 3: Using the Methodology



Summary

- Presented equilibrium analysis of cases with asymmetric bidders:
  - bidders can have different utility functions and valuation distributions
  - bidders can have different competitiveness
- Showed how to solve the systems of differential equations that characterize the equilibria in this case (and in other auction problems)

Other Related Work

The Big Picture

Why Should We Care?

- This type of system of differential equations seems to appear in other types of problems

- Examples:

To follow in the next slides

Problem Setting for Multi-unit Auctions

- **N** bidders, each wanting 1 item
- Valuations are private information which is i.i.d. drawn from know distribution **F(u)**
- **m** identical items for sale in 1 auction
- Each bidder maximizes own utility:
  - Risk neutral agents, i.e. profit equals utility
  - Utility: U_i = (own profit)
- No Reserve value, and infinite budget
- Uniform pricing rule for winners:
  - Auction closes immediately (1 round of bids)
  - m^{th} price, or $(m+1)^{th}$ price

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Problem Setting for Multi-unit Auctions

- **N** bidders, each wanting L (multiple) items
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- **m** identical items for sale in m (multiple) auctions
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