VARIABLE STIFFNESS DESIGN OF
ARBITRARY CROSS-SECTION CYLINDERS FOR
BUCKLING WITH STRENGTH CONSTRAINT

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Summary. In this paper, variable stiffness design of general cross section cylinders using lamination parameters as design variables is investigated. A simplified analysis method based on Saint Venant’s solution is implemented in an optimization routine to improve the buckling behavior with strength constraints. The designs are validated using fully nonlinear analysis and are compared against previously reported designs.

1 INTRODUCTION
Circumferential tailoring of circular cylinders for maximum buckling load under bending using constant curvature fibre paths while applying a strength constraint has been investigated by Blom et al. [1]. Analysis and optimisation of the buckling load and failure investigation of elliptical cross section cylinders under axial compression has been studied by Sun et al. [2] using fibre angles as design variables. In this paper, lamination parameters are used as design variables which are the largest possible design space. In addition, they limit the number of design variables and it is shown that the feasible region is convex in lamination parameter space. Buckling improvement of a variable stiffness general cross-section cylinder under strength constraints is investigated and compared with constant stiffness design.

2 ANALYSIS
Using the stationary and minimum potential energy criteria, equilibrium and buckling eigenvalue formulations are derived. The strain-displacement relations for the general thin shells based on the Love's first approximation are applied in total potential energy evaluation. For long cylindrical shells the stress and strain fields can be approximated by St. Venant's solutions. Voigt [3] showed that for extension, bending, and torsion these fields are independent of the axial coordinate and for flexure, they can change at most linearly in the cylinder axial direction. In this paper the cylinder is studied under a combination of internal pressure and corresponding extension, bending, and torsion. Thus the state of stress and strain depends only on the cross section distribution of properties.
3 APPROXIMATION AND OPTIMISATION FORMULATION

Approximation is a widely used technique within structural optimisation. The main goal is to construct separable approximations of objective function and constraints. For the inverse of buckling factor and failure index a convex and conservative approximation is proposed which is a hybrid formulation of linear and reciprocal approximations. Buckling is a multi-modal phenomenon and also there are multi responses for the failure index in a structure. Therefore, the problem of buckling or strength optimization is expressed as minimization of the critical constraint,

\[ \min(\max(g_i)) \]

where \( g_i \) (for \( i = 1, 2, ..., n \)) is the value of inverse of buckling load for mode number \( i \) or failure index value of the node number \( i \). This optimisation problem can be re-expressed using the bound formulation, and subsequently solved using the dual method. The optimisation problem is subject to the constraints on the lamination parameters.

4 PRELIMINARY RESULTS

Preliminary results for a circular cylinder under bending and an elliptical cylinder under compression are shown in Table 1 for constant stiffness and variable stiffness designs.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>CS (%)</th>
<th>VS (%)</th>
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<tbody>
<tr>
<td>Circular cylinder under bending [2]</td>
<td>5.5</td>
<td>21</td>
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</table>

Table 1: Improvement of buckling load for constant stiffness (CS) and variable stiffness (VS) designs with strength constraints over quasi-isotropic (QI) design

5 CONCLUSIONS

Numerical results show relatively large improvements for variable stiffness over constant stiffness designs.

REFERENCES