

## IMPERFECTION SENSITIVITY ANALYSIS OF COMPOSITE SHELLS USING A FINITE ELEMENT VERSION OF KOITER'S METHOD

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### ABSTRACT

In the present contribution, a finite element formulation of Koiter's initial post-buckling theory is applied in the buckling analysis of composite shells. Initial post-buckling theory provides direct information about the imperfection sensitivity of structures which are prone to buckling. Initial post-buckling analysis of composite cylindrical and conical shells under axial compressive loading and external pressure are carried out. The accuracy of the post-buckling coefficient resulting from the finite element implementation is shown by a comparison with semi-analytical results, and the feasibility to assess the imperfection sensitivity of composite shells under different loads and for different geometries is illustrated.

Thin-walled composite shells are main structural components in various branching of engineering. Buckling is the key design criterion for these thin-walled structures. Koiter's theory is a valuable tool to assess the imperfection sensitivity of shell structures, but in most of the general purpose finite element codes currently used the method is not available. Finite element implementation of Koiter's perturbation approach for static post-buckling analysis was done by several researchers, with emphasis on beam-type and plate-type structures. Implementations for shells are relatively rare. Peek and Kheyrkahan [1], extending earlier work, considered the effect of pre-buckling nonlinearity in their initial post-buckling analysis of general shells. Furthermore, in previous studies mostly isotropic material was considered. In the work of Bilotta et al. [2] composite panels were analyzed using Koiter's approach. In recent work of the present authors [3], extending an existing implementation in the finite element code DIANA, the potential and capability of the approach and its finite element

implementation have been shown by application to the case of a composite cylindrical shell under axial compressive loading. In the present contribution, the generality of the approach is further illustrated by application to composite cylindrical shells under axial compression and under external pressure, and to composite conical shells under axial compression.

In Koiter's approach a perturbation expansion for the load–displacement relation around the bifurcation buckling point is used, with a corresponding expansion of the initial post-buckling displacement field in terms of buckling modes and corresponding second order modes (Fig. 1). Often a limited number of buckling modes and corresponding second order modes can be used to describe the initial post-buckling behavior. The number of equations in the resulting reduced set of nonlinear algebraic equations is the same as the number of buckling modes chosen in the perturbation expansion, and the perturbation approach can be regarded as the basis of a nonlinear reduced order model for the nonlinear buckling analysis including imperfections.

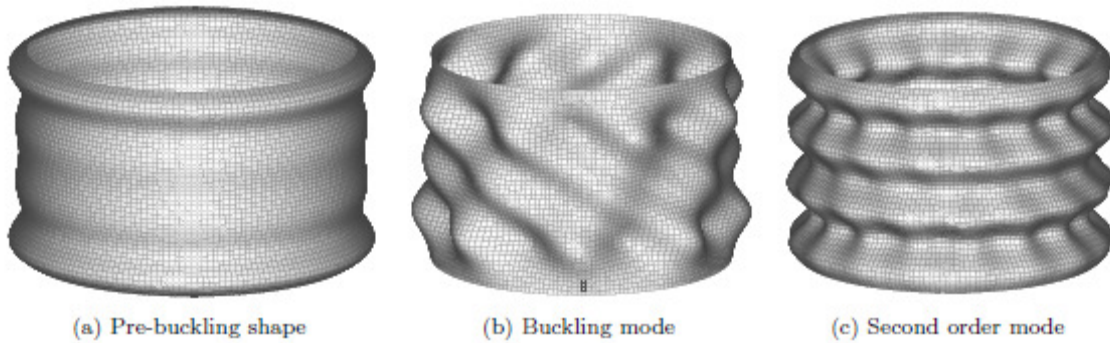


Figure 1: Deformations in perturbation approach for composite cylindrical shell under axial compression

The post-buckling coefficients obtained will be compared with results obtained with semi-analytical tools [4]. The accuracy of the results from the finite element implementation is shown, and the feasibility to assess the imperfection sensitivity of shells under different loads and for different geometries with the present procedure is illustrated.

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