Analytical Solution for Electro-magneto-thermo-elastic Behaviors of a Functionally Graded Piezoelectric Composite Cylinder

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Abstract
Functionally graded piezoelectric material (FGPM) is a kind of piezoelectric material with material composition and properties varying continuously along certain directions. FGPM is the composite material intentionally designed so that they possess desirable properties for some specific applications. The advantage of this new kind of materials can improve the reliability of life span of piezoelectric devices. Recently, there has been growing interest in materials deliberately fabricated so that their electric, magnetic, thermal and mechanical properties vary continuously in space on the macroscopic scale.

In this paper, an analytic solution to the axisymmetric problem of a radially polarized, FGPM hollow cylinder is developed. All material constants are assumed to the same power-law dependence through the wall thickness of the FGPM hollow cylinder. The stresses in a three-layered elasto-piezoelectric composite cylinder (the inner and the outer layer are metal and the middle layer is FGPM layer) in the state of axisymmetric plane strain are obtained. Piezoelectric layers that radially polarized are considered as an orthotropic material. For the axisymmetric plane strain case, the non-zero components of displacement and electric potential are only functions of radius. The heat conduction equation in the steady-state condition for the one-dimension problem in polar coordinates and the thermal boundary conditions for the FGPM hollow cylinder is considered. The stress displacement relation for metal and piezoelectric, the equation of equilibrium of every layer and charge equation of electrostatics (in the absence of free charge density) is considered and then the equations are obtained. In this stage, there is a coupled system of second-order ordinary differential equation that by applying boundary conditions, the problem is reduced to solving a system of linear algebraic equations.

The boundary conditions are uniform potential differences between piezoelectric layer and uniform internal and external pressure applying on internal and external surface of cylinder. In addition, two continuity conditions between metal layer and piezoelectric layer are applied.
Finally, three equations for radial stress, hoop stress and potential field in three-layered elasto-FGPM are obtained. The four unknown constants in the equations for piezoelectric layer and the two constants for outer metal layer as well as the two constants for inner metal layer can be found by applying the boundary conditions and the continuity conditions. Finally, the effects of different boundary conditions as well as piezoelectric materials on stress and strain of FGPM composite cylinder are studied.

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