ANALYSIS OF COMPOSITE LAMINATED AND SANDWICH PLATES USING AN UNSYMMETRIC RADIAL BASIS FUNCTIONS COLLOCATION METHOD: A NOVEL ALGORITHM FOR CHOOSING THE SHAPE PARAMETER

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Summary. Many radial basis function (RBF) methods contain a free shape parameter that plays an important role for the numerical accuracy of the meshless method. In most papers the authors end up choosing this shape parameter by trial and error or some other ad-hoc means. In this paper, a novel algorithm, based on a convergence analysis, to select the shape parameter is presented. The efficacy of this algorithm is assessed by static analyses of laminated composite and sandwich plates.

ABSTRACT

Meshless Methods based on the strong form of a problem are truly Meshless because they do not require any computational grid. In recent years, the theory of radial basis functions (RBFs) has undergone intensive research and enjoyed considerable success as a technique for interpolating multivariable data and functions. In 1982, Franke [1] published a review article evaluating virtually all the interpolation methods for scattered data sets available at that time. Among the methods tested, RBFs outperformed all the other methods regarding accuracy, stability, efficiency, memory requirement and simplicity of implementation [2].

A radial basis function, \( g(\|x - x_i\|) \) is a continuous spline which depends upon a separation distance of a subset of data centers scattered in the problem domain. In the open literature it is possible to find different RBFs but the multiquadratic (MQ) functions converge exponentially and always produce a minimal error [2]. However, despite MQ’s excellent performance, they depend upon a free parameter, \( c \), called shape parameter.

The choice of the value of \( c \) can greatly affect the accuracy of the approximation as the MQ coefficient matrix could become ill-conditioned according to the value of the shape parameter. Although the choice of \( c \) is a key problem, no mathematical theory has been developed yet to determine an optimal value. In literature there are several proposed methods for choosing a shape parameter, all somehow related with the number of points in the grid and the distance between these points. Among these methods, the most important are those of
Fasshauer [3], Franke [1] and Hardy [4]. However, according to Rippa [5], the shape parameter should depend on many other factors, such as the distribution of grid points, the interpolation function \( g \) but also the computer precision and the condition number of the coefficient matrix. In [6] it is possible to find some important details on Rippa’s algorithm. Because of the irregularity of the cost function which has to be minimized in order to find the optimal value of the shape parameter and the arbitrariness of the interval inside which finding the optimal value, the process could be very expensive.

The algorithm proposed in this paper estimates the optimal shape parameter through a convergence analysis varying the number of nodes and the value of the shape parameter in a range defined by the user. The variable (control variable) taken into consideration is the displacement. Thus, varying the number of nodes, for a specific value of \( c \), it is possible to explore the behavior of the solution. Due to the convergence behavior of the collocation method, the solution tends to the asymptotic value when increasing the number of nodes; while increasing the value of \( c \) and for the same number of nodes, the solution is closer to the asymptotic value. This is not always true, in fact as long as the value of \( c \) is lower than a value called “critical value”, the solution remains stable in the interval of number of nodes \( N \) investigated; if the value is greater than the critical one, the solution becomes unstable. According to this algorithm, the optimal value of the shape parameter and the critical one are the same, in fact we define the optimal value as that which guarantees the fastest convergence and that ensures the stability of the solution. In Figure 1 an example of the convergence analysis for a specific problem is showed.

Several numerical tests are performed and the optimal value of the shape parameter is estimated by the present algorithm. The results obtained are compared with solution presented in literature in order to validate the accuracy and reliability of the proposed algorithm.

The Radial Basis Function method coupled with the present algorithm is applied to laminated composite and sandwich plates, both simply supported and fully clamped, subjected to uniform or sinusoidal pressure. The results obtained are in a very good agreement with analytical solutions.
Figure 1 - Example of convergence analysis.

REFERENCES