Size Dependent Buckling Analysis of Functionally Graded Timoshenko Micro Beams Based on Modified Couple Stress Theory

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Key words: buckling, size dependency, modified couple stress theory, shear deformation, GDQ method.

Summary. Buckling analysis of functionally graded micro beams based on modified couple stress theory is presented. To this end, first order shear deformation beam theory is developed based on modified couple stress theory for the FG micro beam. Governing equations and boundary conditions are derived using principle of minimum potential energy. Afterwards, generalized differential quadrature (GDQ) method is applied to solve the obtained differential equations. GDQ method can deal with different types of boundary conditions; therefore, beams with various boundary conditions are considered. Some numerical results are presented to study the effects of material length scale parameter, beam thickness, Poisson ratio and power index of material distribution on size dependent buckling load.

1 INTRODUCTION

FG specimens have special advantages due to their constituent properties and have wide applications in biomedicine, optics, electronics etc. Many micro scaled beams and plates are used in different applications such as in micro electro mechanical systems (MEMS) and micro
sensors or actuators and it has been observed through experimental results that their behavior are quite size dependent in such scales [1, 2]. Classical continuum theory is not capable of capturing this size dependency; therefore, many different higher order continuum theories were introduced to improve the results obtained for micro systems. These theories, mainly, try to improve the model by introducing length scale parameters to capture the so-called size effects. One of the higher order continuum theories is couple stress theory which was presented by Toupin [3]. Yang et al. [4] modified the couple stress theory to introduce an applicable theory which can capture size dependencies considering only one additional constant other than Lame' constants. Asghari et al. [5, 6] investigated static and dynamic behavior of micro FG beams based on the same theory considering Euler-Bernoulli and Timoshenko beam theories. Their results proved that micro scale influences are not negligible and should be considered when a thorough analysis of any micro system is needed.

In this paper, modified couple stress theory is used to perform a buckling analysis on FG micro beams based on first order shear deformation (Timoshenko) beam theories. The beam is assumed to be functionally graded in the thickness direction; while Poisson ratio is assumed to be constant, but it's not neglected. The principal of minimum potential energy is applied to obtain the governing equations and boundary conditions of the FG beam. To analyze different boundary conditions, generalized differential quadrature (GDQ) method is used to solve the governing equations numerically. The GDQ method is a very fast converging method and can be applied to such systems quite easily. To analyze effect of shear deformation on critical buckling load, the results obtained by three different beam theories are compared. Also effect of FGM parameter of material distribution and Poisson ratio on the critical buckling load is investigated.

It is observed that buckling loads predicted by modified couple stress theory deviates significantly from classical ones, especially for thin beams. It is shown that size dependency of FG micro beams differs from isotropic homogeneous micro beams as it is a function of power index of material distribution. In addition, the general trend of buckling load with respect to Poisson ratio predicted by the present model differs from classical one.