NON-LINEAR DYNAMIC RESPONSE OF ACTIVE HYBRID COMPOSITE BEAM EMBEDDED WITH SMA WIRES

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Summary. This research considered the dynamic response of active prestrained SMA hybrid composite beam, by taking into account the phase transformation effects, for every point along the beam. A new approach procedure developed based on the transient finite element along with an iterative incremental method, to solve the nonlinear equations of motion.

1 INTRODUCTION

In recent years, considerable attention is devoted to the use of SMA wires to form adaptive composite structures. Unlike the traditional materials or even most of the engineering material systems, SMAs can undergo a reversible phase transformation between a martensite and an austenite phases under thermo-mechanical loading. Due to this transformation, SMAs have some unique properties, such as, superelasticity, high damping capability and recovery effects which lead to their wide use in the mechanical and aerospace engineering components.

Due to having non-linear hysteresis behaviours which is resulted from the phase transformation, it is difficult to present a comprehensive and effective mathematical model. Hence, in the most cases, the suggested models have been proposed based on numerous simplifications. In this study we investigate non linear dynamic response of a continuous active SMA-embedded composite beam. SMA wires are prestrained at the low temperature and are embedded in the composite medium layers. SMA with heating leads to the generation of high recovery stresses within the composite structures. What makes this paper different from the previous works is that, at every time, the phase transformation effects are taken in to account for all points of the beam.

2 FORMULATIONS AND RESULTS

In the present study, the one-dimensional constitutive equation of the SMA proposed by Brinson is employed. Using Euler–Bernoulli beam theory and Hamilton’s principle, the following equations of motion are obtained for the SMA hybrid composite beam:

\[
\frac{\partial}{\partial x} \left( A_{11}(\xi) \frac{\partial u_0(\xi)}{\partial x} - B_{11}(\xi) \frac{\partial^2 w_0(\xi)}{\partial x^2} + N^s(\xi) \right) = l_0 \frac{\partial w_0(\xi)}{\partial x} - l_1 \frac{\partial^2 w_0(\xi)}{\partial x^2} + \mathcal{N}^s(\xi) + q(x, t) \]

(1)

\[
\frac{\partial^2}{\partial x^2} \left( B_{11}(\xi) \frac{\partial u_0(\xi)}{\partial x} - D_{11}(\xi) \frac{\partial^2 w_0(\xi)}{\partial x^2} + M^s(\xi) \right) + \frac{\partial}{\partial x} \left( N^s(\xi) + q(x, t) \right)
\]

(2)
According to equations of motion, it can be seen that its coefficients are dependent on the martensite volume fraction and therefore, these coefficients are dependent on time and location of any point along the beam. Meanwhile, with respect to the martensite volume fraction dependency on the stress and subsequently the displacement values, these coefficients are unknown. Therefore, in this study not only the material properties are variable with respect to time and location, but they are also unknown. Therefore, the equations of motion and the phase transformation’s kinetic equations are coupled together, which makes the problem more complicated.

\[
\mathbf{I}_0 \dot{\mathbf{w}}(\xi) + \mathbf{I}_1 \frac{\partial \mathbf{w}(\xi)}{\partial x} - \mathbf{I}_2 \frac{\partial^2 \mathbf{w}(\xi)}{\partial x^2} = \mathbf{f}(\xi)
\]

The validation is omitted here due to space limitations. In this work, the SMA hybrid composite beam is subjected to a step impulse load at the center of the beam, having the intensity \( F=75 \text{ kN} \) and the time duration \( t=0.005 \text{ s} \) for SMA wires with 0.5% prestrained at the temperature \( T=50 \text{ °C} \). The results from the present model are shown in Fig. 1. In this figure the beam response is considered for three cases; the beam with SMA wires without pseudoelastic effect (NPE), beam with SMA wires with pseudoelastic effect (WPE) and beam with prestrained SMA wire considering both of the shape memory and pseudoelastic effects together. Fig. 1(a) shows the stress-strain response at the middle of the beam taking into account the pseudoelastic behavior of SMA wires. As can be seen, the stress-strain curve shows the hysteresis loops. As can be seen in these figures, under NPE conditions, the beam oscillates with constant vibration amplitudes, while considering the pseudoelastic effect, the decay of vibration response of the beam can be observed. It can be also observed that when SMA wires are prestrained, the reduction of response amplitude is more remarkable. The main reason of this phenomenon is the generation of stress recovery in the SMA wires.

3 CONCLUSIONS

In this study, a new approach procedure developed for analysis the active SMA hybrid composite beam. To access the performance of the procedure developed, dynamic response of SMA hybrid composite beam subjected to impulse load is analyzed. Results showed the efficiency of the proposed model and the relevant solution algorithm.