Orthogonal projection technique for resolution enhancement of
the Fourier transform fringe analysis method

Paulo J. Tavares a,*, Mário A. Vaz b

a Prudente & Tavares, Technical Consulting, Edifício Península, E# 305, Praça do Bom Sucesso 127, 4150-146 Porto, Portugal
b Mechanical Engineering Department, Engineering Faculty, Porto University, Portugal

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Abstract

The spatial resolution of the phase map in the Fourier transform fringe analysis method is determined by the size of the filter’s window in frequency domain. This article reports a straightforward technique to improve the method’s resolution by a factor of nearly two. The technique requires capturing a second image with a fringe pattern orthogonal to the first one, therefore using the information from both patterns to eliminate the central component in frequency space. The resulting spectrum supports double sized filter windows for removal of the carrier frequency without leaking into adjacent orders. The overall spatial resolution of the method is thus increased. In the following, the Fourier fringe analysis method is briefly reviewed, the new technique is described and analyzed and the experimental results are shown and discussed.

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1. Introduction

The Fourier transform fringe analysis method is one of the most successful techniques for fringe pattern analysis. First introduced by Takeda [1] in 1982, the technique was adapted to two-dimensions by Macy [2], although that was essentially a one-dimensional extension of the original work by Takeda. Bone [3], provided a full 2D version in 1986, along with a set of very interesting observations regarding the inherent errors of the technique and some suggestions about how these could be overcome. Several other works have been published throughout the years, as this method remains one of the simplest and more effective methods for shape measurement. Another important reason is its dependence on one frame only, contrary to the phase shift methods that need three frames at the very least. This makes it the only method capable of dealing with dynamic situations where the target object moves and taking three frames becomes prohibitive.

The technique can be simply described as follows: a fringe pattern with a known spatial frequency is projected onto the object under test and a two-dimensional Fourier transform of the resulting image is calculated. The spectrum that results from this fringe projection has two sharp peaks, centered at the carrier fringe frequency. In order to remove this carrier and thus obtain an expression of the phase variation, proportional to the object profile at each image point, a certain amount of frequencies surrounding the carrier are filtered with a suitable window. This window is then moved to the center of the spectrum, and an inverse Fourier transform of those selected frequencies is calculated. This inverse transform holds the phase information which can finally be calculated by the logarithm of a simple inverse tangent calculation.

The method is simple and effective, yet plagued with many error sources. One of the most important is the error that results from the fact that the frequencies spread out to infinity in frequency space, and a too wide filtering window
is bound to include frequencies that belong to adjacent orders. This is a very stringent constraint, in that the filter size is very limited and a number of high frequencies responsible for the image detail can thus be lost. What we propose herein, is a simple technique to eliminate the zero-order in frequency space and widen the filter window size to approximately twice the maximum admissible size without including adjacent orders. The resolution enhancement so obtained is near to double the original’s method.

2. Analysis

A sinusoidal fringe pattern projected onto an object, as shown in Fig. 1, is adequately described by

\[ g_1(x,y) = a(x,y) + b(x,y) \cos[2\pi f_0 x + \phi(x,y)] \]  

where \( a(x,y) \) and \( b(x,y) \) represent non-uniform distributions of reflectivity on the surface of the object, \( f_0 \) is the fundamental frequency of the observed grating image and \( \phi(x,y) \) is the phase modulation resulting from the object height distribution.

For convenience of analysis, this fringe pattern can be rewritten as

\[ g_1(x,y) = a(x,y) + c(x,y) \exp[2\pi j f_0 x] + c'(x,y) \exp[-2\pi j f_0 x] \]  

with the understanding that,

\[ c(x,y) = \frac{b(x,y) \exp[j \phi(x,y)]}{2} \]

and * denotes complex conjugation.

The Fourier transform of the deformed fringe pattern, shown in Fig. 2, is,

\[ G_1(u,v) = A(u,v) + C(u - f_0, v) + C^*(u + f_0, v) \]  

A second fringe pattern projected over the object, with fringes orthogonal to the first fringe pattern, Fig. 3 can be described by,

\[ g_2(x,y) = a(x,y) + c(x,y) \exp[2\pi j f_0 y] + c'(x,y) \exp[-2\pi j f_0 y] \]

and its Fourier transform by,

\[ G_2(u,v) = A(u,v) + C(u, v - f_0) + C^*(u, v + f_0) \]  

The phase, \( \phi(x,y) \), which depends only on the object height at each \( (x,y) \) point and the structural parameters of the crossed optical axes geometry, remains unchanged, as do \( a(x,y) \) and \( b(x,y) \) if the contrast and gain of the two fringe patterns is kept constant and the background intensity variation under control. It is clear from Eqs. (4) and (6) and Figs. 2 and 4 that the DC or zero-order component is identical in both spectra.

In order to enforce minimum background intensity variations, and if the problem allows taking more than just one image, several recordings of the deformed fringe pattern can be made and a weighted average calculated, thus eliminating spurious background variations.

An added benefit of the orthogonal projection technique is the total removal of these constraints, for if the two spectra are subtracted, the obtained spectrum is,

\[ G_1(u,v) - G_2(u,v) = C(u - f_0, v) - C(u, v - f_0) + C^*(u + f_0, v) - C^*(u, v + f_0) \]
thus totally eliminating the DC or central component in frequency space. Now the resulting spectrum is digitally manipulated in order to clip the negative going values, i.e., all values below a zero threshold, as seen in Fig. 5, $C(u - f_0, v) + C^*(u + f_0, v)$  

$$C(u - f_0, v) + C^*(u + f_0, v)$$ (8)

Even if there is some variation on the background illumination leading to a different contrast of the second fringe pattern, $b_2(x, y)$, this will be totally eliminated on this clipping operation. Other techniques to improve the method’s resolution, such as the “π phase shifting technique” introduced by Su [4], rely on a constant fringe contrast between two consecutive images, which hardly ever occurs.

The usual FTP filtering and centering operations can now be performed on one of the side lobes, say $C(u - f_0, v)$, by first isolating it with a suitable filter window centered at $f_0$ and then removing the carrier frequency altogether by moving it to the center of the spectrum. The fact there isn’t a DC term anymore, allows using a double sized filter window without leaking into the zero order, thus doubling the range of selected frequencies and improving the method’s resolution by a factor of two (see Fig. 5).

The resulting spectrum is, $G(u, v) = C(u, v)$  

$$G(u, v) = C(u, v)$$ (9)

with an inverse Fourier transform equal to $g(x, y) = c(x, y) \exp[\jmath \phi(x, y)]$

$$g(x, y) = c(x, y) \exp[\jmath \phi(x, y)]$$ (10)

As usual also, the wrapped phase, shown in Fig. 6, is given by, $\phi(x, y) = \tan^{-1} \left( \frac{\text{Im}(c(x, y))}{\text{Re}(c(x, y))} \right)$  

$$\phi(x, y) = \tan^{-1} \left( \frac{\text{Im}(c(x, y))}{\text{Re}(c(x, y))} \right)$$ (11)

3. Results and discussion

The sinusoidal 0.75 fringes/mm pattern used on the previous images was calculated on a PC and projected onto the object using an InFocus LP-70 DLP video projector. The images were captured with an 8 bit ES 1.0 Kodak CCD and a Matrox Genesis frame grabber and stored for processing.
It is clear from the analysis of the previous images that the advantage of the proposed technique is the use of a much larger window for the removal of the carrier frequency. Theoretically, the filter's window size can be made as big as twice the carrier frequency, $f_0$, (cf. Fig. 5) before the two remaining terms overlap.

The wrapped phase map, once obtained, can be demodulated using any unwrapping technique, though we found the Volkov [5] deterministic phase unwrapping method to perform well under most circumstances. Figs. 7 and 8 represent the unwrapped phase map and the three dimensions cloud point obtained with the previous object.

4. Conclusions

The Fourier transform profilometry method for 3D surface measurement has become very popular due to the fact that it is inherently simple, standing out as the only method applicable to dynamic situations because it relies on one image only. One of the method’s chief restrictions is the limited resolution, due to the well known leakage artifact from the application of FFTs and the constraint it imposes on the carrier removal filter size.

The method presented in this note removes this limitation to a great extent, by eliminating the central DC frequency component in frequency space, thus allowing the filter’s size to be twice the carrier frequency.

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