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COMBINING NEURAL NETWORKS FOR ONLINE DAMAGE DETECTION IN EULER-BERNOULLI BEAMS

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ABSTRACT

The aim of this contribution is to present a method for online damage detection in Euler-Bernoulli beams. This method detects damage by tracking changes in the beam parameters from vibration data collected at different beam points. The method is based on a combination of Hopfield neural networks. At each time instant, each network produces first an estimate of the parameters at a certain beam point and the estimates of neighboring points are then combined to produce a final estimate at each point. The preliminary results obtained encourage the incorporation of this method in an automatic system for online damage detection.

Keywords: Euler-Bernoulli beam, Damage detection, Hopfield neural network.

INTRODUCTION

With the development of engineering structures (e.g. aircrafts, bridges and off-shore platforms) a greater attention to cracks that endanger the whole structure is necessary. The detection of this kind of damage is an important point to avoid the total crashing of the structure and mainly to allow repairing the cracks in their early growth state to minimize the costs. Here, as in [1], it is assumed that damage corresponds to a change in the model parameters, and that it can be detected by tracking parameter variations. More concretely, the basic idea is that model parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping and stiffness). Therefore, changes in these physical properties will cause changes in the model parameters. Thus, parameter identification and monitoring constitutes a good method for early damage detection. Online damage detection is a subject that has received considerable attention. Traditional techniques include the recursive least-squares and the Kalman filter [2]. A more recent technique, based on the Hopfield neural network originally proposed in [3], has already shown a superior performance in practice [4].

This paper presents a contribution for an automatic system for online damage detection, namely a method for tracking changes in beam parameters. This method is based on a combination of Hopfield neural networks [3]. Each network produces a time-evolving estimate of the beam parameters at a certain beam point. The combination of the networks enables them to share information concerning different beam points. The model used here to simulate the beam vibration is the damped Euler-Bernoulli model.

The remainder of the paper is organized as follows. First, the Euler-Bernoulli model is presented. Then, the new method here proposed, based on a combination of neural networks, is described. Finally, some preliminary results are shown and the conclusions are drawn.

DAMPED EULER-BERNOULLI BEAMS

This section describes the Euler-Bernoulli model for the deflection of a damped beam. The method of separation of variables is applied in order to write this model as a first order model that will serve as basis for the identification procedure.

Model Description

The Euler-Bernoulli model is used to simulate the time evolution of the transverse displacement, $w(t,x)$, along a vibrating beam with length L and simply-supported at both ends. The relationship between the transverse displacement and the external force, $q(t,x)$, is described by the following fourth order partial differential equation:

$$\mu\frac{\partial^2}{\partial t^2}w(t,x)+c\frac{\partial}{\partial t}w(t,x)+EI\frac{\partial^4}{\partial x^4}w(t,x)=q(t,x), \tag{1}$$

where μ, c, E and I are the beam parameters, which are related with physical properties of the structure. The parameters are considered to be constant along the beam. The external force is set to zero (free vibration).

The boundary conditions assumed in this paper are translated by:

$$w(t,x)\Big|_{x=0,L}=0 \qquad \frac{\partial^2}{\partial x^2}w(t,x)\Big|_{x=0,L}=0. \tag{2}$$

First order model

In order to use the proposed identification method for online damage detection, the Euler-Bernoulli model needs to be rewritten as a first order model in order to highlight linearity in the parameters.

The separation of variables consists in considering the solution of (1)-(2), $w(t,x)$, as a linear combination of elementary solutions, $w_n(t,x)$, which are separable in the time and space domains, *i.e.*,

$$w_n(t,x)=f_n(t)g_n(x). \tag{3}$$

Hence,

$$w(t,x)=\sum_{n=1}^{\infty}c_n f_n(t)g_n(x)\approx\sum_{n=1}^{N}c_n f_n(t)g_n(x), \tag{4}$$

where the $c_n's$ are arbitrary constants [5], each function $f_n(t)$ is the solution of an equation $f_n''(t)+\theta_2 f_n'(t)+\frac{\theta_1}{\beta^4}f_n(t)=0$, with $\beta=\frac{n\pi}{L}$ and initial conditions $f_n(0)=\gamma_0^n$ and $f_n'(0)=\gamma_1^n$, while $g_n(x)=A\,sin(\beta x)$ with $A\in\mathbb{R}$. This corresponds to consider initial displacements and velocities given by $w(0,x)=\sum_{n=1}^{N}\gamma_0^n\,sin\left(\frac{n\pi}{L}x\right)$ and $\frac{\partial}{\partial t}w(0,x)=\sum_{n=1}^{N}\gamma_1^n\,sin\left(\frac{n\pi}{L}x\right)$, respectively. Here, the elementary solution

$$g_1(x)=\sin\left(\frac{\pi}{L}x\right) \tag{5}$$

obtained by taking $A = 1$ and $n = 1$ (first harmonic) will be considered. This corresponds to the solution $w(t,x) = f_1(t)\, sin\left(\frac{\pi}{L}x\right)$ with initial conditions

$$w(0,x) = f_1(0)\sin\left(\frac{\pi}{L}x\right) = \gamma_0^1 \sin\left(\frac{\pi}{L}x\right)$$
$$w'(0,x) = f'_1(0)\sin\left(\frac{\pi}{L}x\right) = \gamma_1^1 \sin\left(\frac{\pi}{L}x\right). \quad (6)$$

The values of γ_0^1 and γ_1^1 are arbitrary and, in case of practical experiments, can be adequately chosen according to the desired experimental setup. Taking a spatial discretization step of Δx, a vector $\overline{W}$ is constructed with the values of $g_1(x)$ at the discretization points, *i.e.*,

$$\overline{W} = \begin{bmatrix} g_1(0) \\ g_1(\Delta x) \\ \vdots \\ g_1(L) \end{bmatrix} \in \mathbb{R}^m \quad (7)$$

where $m = \frac{L}{\Delta x} + 1$. This yields $W(t) = f_1(t)\overline{W}$ as the corresponding spatial discretization of $w_1(t,x)$. The time evolution of $W(t)$ is then described by the following ODE:

$$\mu\, W''(t) + c\, W'(t) + EI\, W(t) = 0. \quad (8)$$

Defining $F_1(t) = W(t)$ and $F_2(t) = W'(t)$, the following first order model is obtained:

$$\begin{cases} F'(t) = \phi(\theta)F(t) \\ W(t) = GF(t) \end{cases}, \quad (9)$$

with $F = [F_1(t) \quad F_2(t)]^T$ and $G = [I \quad \underline{0}_{(m\times m)}]$, where I is the identity matrix with dimension m. The matrix ϕ depends on the parameter vector $\theta = [\theta_1 \ \ \theta_2]^T$, with $\theta_1 = \frac{EI}{\mu}$ and $\theta_2 = \frac{c}{\mu}$, and is given by

$$\phi(\theta) = \begin{bmatrix} \underline{0}_{(m\times m)} & I \\ -\theta_1 I & -\theta_2 I \end{bmatrix}. \quad (10)$$

COMBINATION OF HOPFIELD NEURAL NETWORKS

As mentioned before, damage can be detected by a variation in the model parameters. Identification methods are crucial to detect damage as they monitor the time evolution of the model parameters. This section presents an identification method based on a combination of several Hopfield neural networks.

Hopfield neural network

The Hopfield neural network (HNN) considered in the combination presented here was proposed in [3]. It can be used to produce an estimate of the beam parameters at each time instant. In order to apply this method, model (9)-(10) needs to be written in the form $y(t,x) = A(t,x)\theta$, where $y(t,x)$ and $A(t,x)$ are a certain vector and a certain matrix computed from the system data, respectively, and θ is the vector of the model parameters to be identified. This yields:

$$\underbrace{\begin{bmatrix} F'_1(t) - F_2(t) \\ F'_2(t) \end{bmatrix}}_{y(t,x)} = \underbrace{\begin{bmatrix} \underline{0}_{(m\times 1)} & \underline{0}_{(m\times 1)} \\ -F_1(t) & -F_2(t) \end{bmatrix}}_{A(t,x)} \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}}_{\theta}. \tag{11}$$

The HNN produces a time-evolving estimate of the beam parameters at a certain beam point through the following discrete time model [3]:

$$\hat{\theta}(t+1,x) = \hat{\theta}(t,x) + \frac{\Delta t}{c\beta} D_c\left(\hat{\theta}(t,x)\right) A^T(t,x) \left(y(t,x) - A(t,x)\hat{\theta}(t,x)\right), \tag{12}$$

where Δt is the time step, c, β are parameters that can be tuned to adjust the network performance, and $D_c(\hat{\theta}(t,x))$ is the following matrix:

$$D_c\left(\hat{\theta}(t,x)\right) = \begin{bmatrix} c^2 - \hat{\theta}_1(t,x)^2 & 0 \\ 0 & c^2 - \hat{\theta}_2(t,x)^2 \end{bmatrix}. \tag{13}$$

Combination of several Hopfield neural networks

The motivation for combining several Hopfield neural networks is to enable them to share information concerning different beam points. Assuming that the beam is spatially discretized in k equally spaced points $x_1, x_2, \dots, x_k$, k networks are considered. The corresponding k networks are linearly combined as follows:

$$\begin{bmatrix} \hat{\theta}(t+1,x_1) \\ \hat{\theta}(t+1,x_2) \\ \hat{\theta}(t+1,x_3) \\ \vdots \\ \hat{\theta}(t+1,x_k) \end{bmatrix} = M \begin{bmatrix} H(t,x_1) \\ H(t,x_2) \\ H(t,x_3) \\ \vdots \\ H(t,x_k) \end{bmatrix}, \tag{14}$$

with

$$H(t,x) = \hat{\theta}(t,x) + \frac{\Delta t}{c\beta} D_c\left(\hat{\theta}(t,x)\right) A^T(t,x) \left(y(t,x) - A(t,x)\hat{\theta}(t,x)\right) \tag{15}$$

and the weight matrix, M, given by:

$$M = \begin{bmatrix} \frac{k-1}{k} & \frac{k-(k-1)}{k} & 0 & \dots & \dots & \dots & 0 \\ \frac{k-(k-1)}{2k} & \frac{k-1}{k} & \frac{k-(k-1)}{2k} & 0 & \dots & \dots & 0 \\ 0 & \frac{k-(k-1)}{2k} & \frac{k-1}{k} & \frac{k-(k-1)}{2k} & 0 & \dots & 0 \\ \vdots & & & & & & \\ \vdots & & & & & & \\ \vdots & & & & & & \\ 0 & \dots & \dots & \dots & 0 & \frac{k-(k-1)}{k} & \frac{k-1}{k} \end{bmatrix}. \tag{16}$$

RESULTS

This section presents simulation results obtained by applying the proposed identification method based on a combination of several Hopfield neural networks. The Euler-Bernoulli model is simulated with the parameters values as in [6] as: $E = 2 \times 10^{11}\ N/m^2$, $I = 2.45 \times 10^{-7}\ m^4$, $\mu = 23.1\ kg/m$, $c = 32\ N/m^2$ and $L = \pi\ m$. This results in $\theta_1 = 2129.5\ m^4/s^2$ and $\theta_2 = 1.38\ s^{-1}$. The beam was spatially discretized, and three equally spaced points $x_1,\ x_2, x_3$ were considered. The corresponding three networks were linearly combined as follows:

$$\begin{bmatrix} \hat{\theta}(t+1, x_1) \\ \hat{\theta}(t+1, x_2) \\ \hat{\theta}(t+1, x_3) \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 2/3 & 1/6 \\ 0 & 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} H(t, x_1) \\ H(t, x_2) \\ H(t, x_3) \end{bmatrix}. \tag{5}$$

To analyze the performance of the identification method, the following relative error was computed:

$$ER(t) = max\{ER_1^{\theta_1}(t) \quad ER_2^{\theta_1}(t) \quad ER_3^{\theta_1}(t) \quad ER_1^{\theta_2}(t) \quad ER_2^{\theta_2}(t) \quad ER_3^{\theta_2}(t)\}, \tag{6}$$

where $ER_i^{\theta_j}(t)$ refers to the estimate of the *j-th* parameter produced by the *i-th* network at time *t*:

$$ER_i^{\theta_j}(t) = 100 \times \left| \frac{\theta_{ji} - \hat{\theta}_{ji}(t)}{\theta_{ji}} \right| \qquad i=1,2,3 \text{ and } j=1,2. \tag{7}$$

In a first stage no damage was considered and θ_1, θ_2 were kept constant. Fig. 1 presents the evolution of the $\hat{\theta}_1$ obtained by the combination of HNNs.

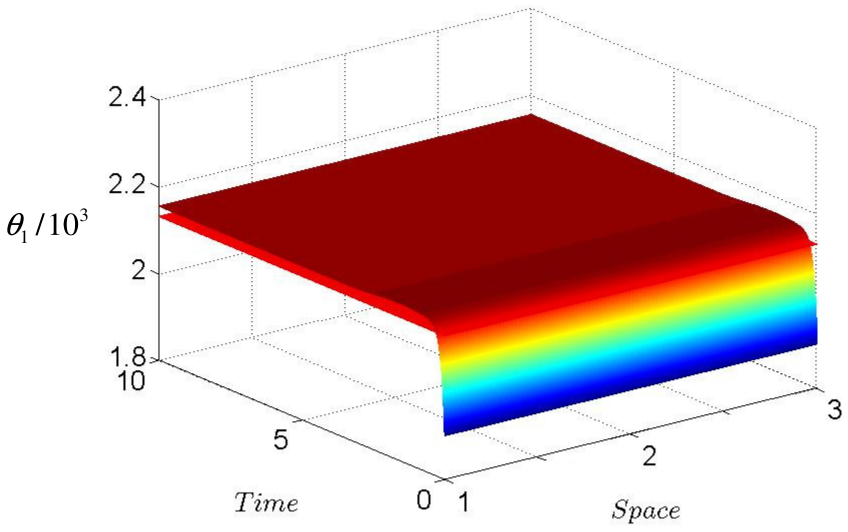

Fig. 1. Time-evolution of the estimate of the beam parameter θ_1 obtained by the combination of HNNs when no damage is considered.

The estimation error obtained in this case is represented in the following figure:

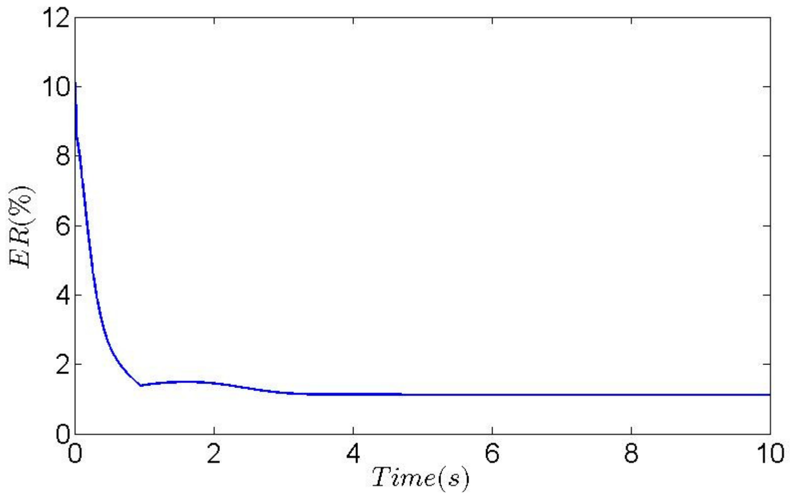

Fig. 2. Time-evolution of the estimation error when no damage is considered.

In a second stage damage was considered and represented by a variation of 50% of θ_1 at time $t = 5\ s$. The change in parameter θ_1 corresponds to a change in Young's modulus parameter, E. The estimation error obtained in this case is illustrated in the following figure:

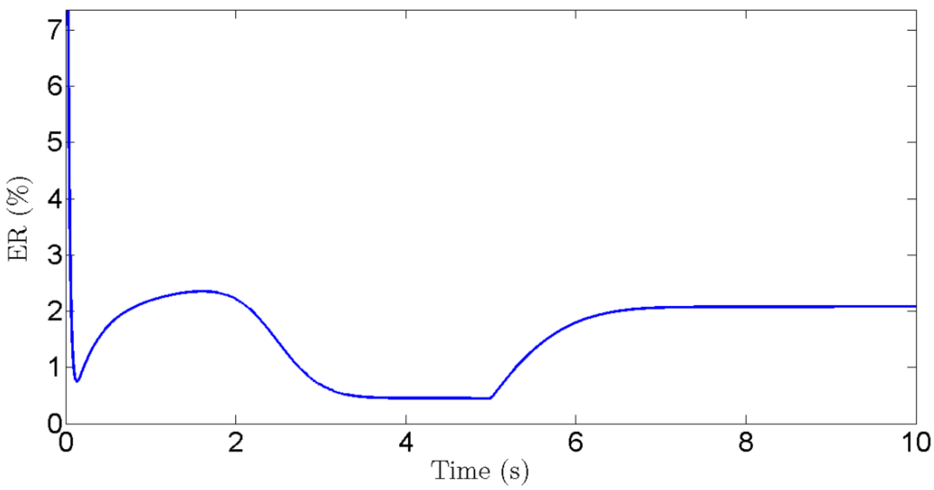

Fig. 3. Time-evolution of the estimation error when a variation of parameter θ_1 occurs.

As can be seen the method converges before $t = 5\ s$ and the error is small. Then a change in the beam parameter is detected.

CONCLUSIONS

This paper presents a combination of Hopfield neural networks for online damage detection in Euler-Bernoulli beams. At each time instant, each network produces first an estimate of the parameters at a certain beam point and the estimates of neighboring points are then combined to produce a final estimate at each point. The preliminary results obtained encourage the use of the proposed method in the case of non-homogeneous beams and its inclusion in an automatic system for online damage detection.

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