A 2D neural network approach for beam damage detection

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***Abstract*—The aim of this contribution is to propose a 2D Hopfield Neural Network for parameter identification and consequent damage detection in 2D processes. This method is applied for online damage detection in Euler-Bernoulli beams subjected to external forces. Damage is associated with significant change in the parameter values of the model. The 2D Hopfield Neural Network presented here is an extension of a 1D Hopfield Neural Network recently proposed in the literature. At each time instant, the network produces an estimate of the parameters at a certain beam point based on the previous estimates in the point itself and in its neighbours. The network tracks the change in the beam parameters and the results obtained are very satisfactory.**

I. INTRODUCTION

The detection of damage in engineering structures is crucial to avoid the crashing of the whole structure and mainly to repair the cracks in their early growth stage. Damage detection can be associated to an identification problem. Indeed, since damage corresponds to a change in the physical properties of the structure and consequently to a change in the corresponding model parameters, it can be detected by tracking parameter variation through appropriate estimation techniques, like the recursive least-squares and the Kalman filter [1]. Recently, a 1D Hopfield Neural Network (HNN) was proposed in [2] and used to detect damage in 1D processes with very good results.

Here a 2D Hopfield Neural Network for parameter identification is proposed for 2D processes, that can be regarded as an extension of that 1D HNN. This 2D network is used to estimate the parameters of a vibrating beam. At each time instant t, the network produces an estimate $\hat{\theta}(t,x)$ of the parameter values at a certain beam point x taking into account the estimates in the previous time instant. The input for the method consists of the vibration data collected at different beam points.

To simulate the vibration data the damped Euler-Bernoulli model was used. This model relates the beam's deflection with the applied load, and was chosen due to its simplicity [3].

This paper is organized as follows. Section II presents the 2D Hopfield Neural Network proposed in this paper. Section III describes the Euler-Bernoulli model use to simulate the beam data and the results obtained by the application of the identification method; the conclusions are drawn in Section IV.

II. A 2D HOPFIELD NEURAL NETWORK

The 1D Hopfield Neural Network (HNN) introduced in [2] for online parameter estimation in one-dimensional processes is a dynamic network that provides time evolving estimates which are guaranteed to converge to the real values of the parameters under some mild assumptions. Having in mind the problem of damage detection in beams, in this section an extension of this 1D HNN is proposed for parameter estimation in 2D processes in the time/linear-space domain. For this purpose the HNN obtained in [2] is first presented.

This network is designed for 1D processes where the parameter dependence is linear, *i.e.*, for which the dynamical equations can be rewritten as:

$$y(t) \;=\; A(t)\,\theta \;\;, \tag{1}$$

where $y(t)$ and $A(t)$ are a vector and matrix, respectively, computed from the system signals and their derivatives, and θ is the vector of the model parameters to be estimated online. The dynamics of the HNN is then given by:

$$\frac{d}{dt}\hat{\theta}(t) = \frac{1}{\gamma\beta}\, D_\gamma\left(\hat{\theta}(t)\right)\, A(t)^T\, \left(y(t) - A(t)\hat{\theta}(t)\right) \;\;, \tag{2}$$

where γ and β are design parameters to be tuned so as to improve the network performance. $D_\gamma\left(\hat{\theta}(t)\right)$ is the following matrix that depends on γ and $\hat{\theta}(t)$,

$$D_\gamma\left(\hat{\theta}(t)\right) = diag\left(\gamma^2 - \hat{\theta}_i(t)^2\right) \;\;. \tag{3}$$

For this HNN the following result holds.

Theorem 1. *[2] The equilibrium point $\hat{\theta}^* = \theta$ of the HNN (2) at $t = t_0$ is globally uniformly asymptotically stable if,*

for all nondegenerate interval $I \subset [t_0, +\infty[, \bigcap_{t\in I} ker\left(A(t)\right) = 0.$

The differential equation (2) can be discretized using the finite difference method, yielding:

$$\sigma_1\,\hat{\theta}(k) \;=\; \hat{\theta}(k) + \frac{1}{\gamma\beta} D_\gamma\left(\hat{\theta}(k)\right)\, A(k)^T\, \left(y(k) - A(k)\hat{\theta}(k)\right) , \tag{4}$$

where $\hat{\theta}(k)$ is redefined as $\hat{\theta}(k\,\Delta t)$, $k = 0, 1, \ldots$, β is redefined as $\frac{\beta}{\Delta t}$, and σ_1 is the time shift operator, defined by $\sigma_1\hat{\theta}(k) = \hat{\theta}(k+1)$.

Here parametrized 2D processes in the time/linear-space are considered, for which the parameter dependence is linear. Like in the 1D case, this means that the system dynamic equations can be rewritten as

$$y(t,x) \ = \ A(t,x)\,\theta \ \ , \tag{5}$$

where the vector $y(t,x)$ and the matrix $A(t,x)$ depend on the system signals and their partial derivatives, and θ is a vector of fixed parameters.

In order to estimate the parameters θ, the following 2D version of the discrete 1D HNN (4) is proposed:

$$\sigma_1\,\hat{\theta}(k,l) = \hat{\theta}(k,l) + \sum_{j=-N^*}^{N^*} \alpha_j(l)\,\sigma_2^j\,\left(F(\hat{\theta}(k,l)), \overline{A}(k,l), \overline{y}(k,l)\right) \tag{6}$$

where, (as before) σ_1 stands for the time-shift, σ_2 is the space-shift defined by $\sigma_2^j\,h(k,l) = h(k,l+j)$, the weights $\alpha_j(l)$ are design coefficients,

$$F\left(\hat{\theta}(k,l), \overline{A}(k,l), \overline{y}(k,l)\right) = \frac{1}{\gamma\beta} D_\gamma(\hat{\theta}(k,l))\overline{A}^T(k,l) \times \times \left(\overline{y}(k,l) - \overline{A}(k,l)\,\hat{\theta}(k,l)\right)$$

$$D_\gamma(\hat{\theta}(k,l)) = diag\left(\gamma^2 - \hat{\theta}_i(k,l)^2\right),$$

and γ and β are again suitable design coefficients. Moreover $\overline{A}(k,l)$ and $\overline{y}(k,l)$ are defined as the discretizations of $A(t,x)$ and $y(t,x)$, respectively, with time lag Δt and spatial lag Δx; more concretely:

$$\begin{aligned} \overline{A}(k,l) &= A(k\,\Delta t, l\,\Delta x) \\ \overline{y}(k,l) &= y(k\,\Delta t, l\,\Delta x) \\ k = 0,1,\dots \ \ &\text{and} \ \ l = 0,1,\dots \end{aligned}$$

The updating structure of this nonlinear difference equation is shown in Fig. 1.

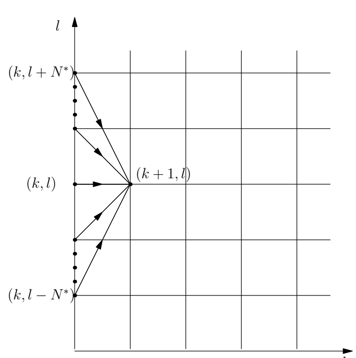

Fig. 1. Updating structure for Equation (6).

The theoretical analysis of this 2D discrete HNN will be performed elsewhere. In the next section its use for parameter estimation and damage detection in vibrating beams is illustrated.

III. AN APPLICATION: BEAM DAMAGE DETECTION

As mentioned before, damage can be associated with significant change in the parameter values of a process. In this section, the 2D discrete HNN (6) is used in order to detect damage in beams modelled by Euler-Bernoulli equations.

A. Theoretical model

As is well-know, the Euler-Bernoulli model describes the vibration of a beam with mass per unit length μ, damping coefficient c and stiffness EI by means of a PDE of the form:

$$\mu\,\frac{\partial^2}{\partial t^2}\,w(t,x) \ + \ c\,\frac{\partial}{\partial t}w(t,x) \ + \ EI\,\frac{\partial^4}{\partial x^4}w(t,x) \ = \ q(t,x) \ \ , \tag{7}$$

where $w(t,x)$ is the transversal displacement and $q(t,x)$ is the external force at instant t and at location x. For simulation purposes the external force $q(t,x)$ is considered to be a harmonic excitation with amplitude F and frequency ω applied at the middle point of the beam. Moreover, it is assumed that the beam has length L and that both ends are clamped, which corresponds to the boundary conditions:

$$\begin{aligned} w(t,x)\big|_{x=0,L} &= 0 \\ \frac{\partial}{\partial x}w(t,x)\bigg|_{x=0,L} &= 0. \end{aligned} \tag{8}$$

Dividing both sides of equation (7) by μ yields

$$\frac{\partial^2}{\partial t^2}\,w(t,x) \ + \ \theta_1\frac{\partial}{\partial t}w(t,x) \ + \ \theta_2\frac{\partial^4}{\partial x^4}w(t,x) \ = \theta_3\,q(t,x) \ \ , \tag{9}$$

with

$$\theta_1 = \tfrac{c}{\mu} \quad \theta_2 = \tfrac{EI}{\mu} \quad \theta_3 = \tfrac{1}{\mu}$$

Here we shall take θ_1, θ_2 and θ_3 as the parameters to be identified.

B. Simulation model

In order to simulate the vibration data, equation (7) is reduced to a first order model using Galerkin's method [4].

The idea of this method is to expand the transversal displacement function, $w(t,x)$, in an orthonormal basis for the spatial component with time-dependent coefficients and then to use the PDE itself to generate an ODE for those time-dependent coefficients. In this way the function $w(t,x)$ is separated in the time and space domains as,

$$w(t,x) \ = \ \sum_{i=0}^{\infty} f_i(t)g_i(x) \ \ , \tag{10}$$

and approximated as $w(t,x) = \sum_{i=1}^{M} f_i(t) g_i(x)$, where the functions $g_i(x)$ correspond to natural vibration forms for a beam with clamped ends, *i.e.*, each $g_i(x)$ is the solution of a 1D boundary problem:

$$\begin{array}{l} \frac{d^4}{dx^4} g(x) - \zeta_i^4\, g(x) = 0 \\ g(0) = g(L) = 0 \\ \frac{dg}{dx}(0) = \frac{dg}{dx}(L) = 0, \end{array} \tag{11}$$

where ζ_i is a solution of the equation:

$$cos(\zeta\, L) cosh(\zeta\, L) = 1\ , \tag{12}$$

being given by:

$$g_i(x) = cosh(\zeta_i\, x) - cos(\zeta_i\, x) - \alpha_i\, (sinh(\zeta_i\, x) - sin(\zeta_i\, x))\ , \tag{13}$$

with

$$\alpha_i = \frac{cos(\zeta_i\, L) cosh(\zeta_i\, L)}{sinh(\zeta_i\, L) sin(\zeta_i\, L)}. \tag{14}$$

Here, for simplicity of the exposition, we shall assume that $M = 1$, *i.e.*, only one natural vibration mode is considered, and in this way the beam deflection is given by

$$w(t,x) \ = \ f(t) g(x) \tag{15}$$

where $g(x) = g_1(x)$ is given by (13) for a suitable value $\zeta_1 = \zeta^*$.

This corresponds to have initial beam displacements and velocities given by

$$\begin{array}{rcl} w(0,x) & = & f(0) g(x) \\ \frac{\partial}{\partial t} w(0,x) & = & \frac{d}{dt} f(0) g(x). \end{array}$$

Substituting (15) in (9), and considering the external force as: $q(t,x) \ = \ F\,\delta \left(x - \frac{L}{2}\right)\, \cos(\omega\, t)$, the model equation and the boundary conditions become:

$$\begin{array}{rcl} f''(t)\, g(x) & + & \theta_1\, f'(t)\, g(x) \, + \, \theta_2\, f(t)\, \frac{d^4\, g(x)}{dx^4} \\ & = & \theta_3\, F \delta \left(x - \frac{L}{2}\right) cos(\omega\, t) \end{array} \tag{16}$$

where $f'(t) \ = \ \frac{d}{dt} f(t)$.

Multiplying both sides of equation (16) by $g(x)$ and integrating with respect to x in the interval $[0, L]$ leads to the ODE:

$$f''(t) \, + \, \theta_1 f'(t) \, + \, \theta_2 \eta f(t) = \theta_3 K F cos(\omega t), \tag{17}$$

where $G = \int_0^L (g(x))^2\, dx$, $K = \frac{g(L/2)}{G}$ and $\eta = \zeta^{*4}$.

Multiplying both sides of (17) by $g(x)$ yields:

$$f''(t) g(x) + \theta_1\, f'(t) g(x) + \theta_2\, f(t) \eta g(x) \, = \, \theta_3\, K\, F\, cos(\omega t) g(x), \tag{18}$$

or equivalently, taking to account that $w(t,x) = f(t) g(x)$,

$$\frac{\partial^2 w}{\partial t^2}(t,x) + \theta_1\, \frac{\partial w}{\partial t}(t,x) + \theta_2\, \eta w(t,x) = \theta_3 K F cos(\omega t) g(x). \tag{19}$$

Discretizing the spatial function $g(x)$ with interval Δx, the following vector $\overline{W}$ is constructed:

$$\overline{W} = \left[\begin{array}{c} g(0) \\ g(\Delta x) \\ \vdots \\ g(L) \end{array} \right] \in \mathbb{R}^m\ , \tag{20}$$

Multiplying both sides of the equation (17) by $\overline{W}$, a lumped version of $w(t,x)$ is obtained as $W(t) = f(t)\, \overline{W}$. The time evolution of $W(t)$ is given by:

$$\begin{array}{rcl} W''(t) \, + \, \theta_1\, W'(t) & + & \theta_2\, \eta\, W(t) \\ & = & \theta_3\, K\, F\, cos(\omega t) \overline{W}\ \ . \end{array} \tag{21}$$

Taking $Q(t) = F\, cos(\omega\, t)$, $X_1(t) = W(t)$ and $X_2(t) = W'(t)$ the following state-space model is obtained:

$$\left\{ \begin{array}{rcl} X'(t) & = & \Phi(\theta)\, X(t) + B(\theta)\, Q(t) \\ W(t) & = & C\, X(t) \end{array} \right. \tag{22}$$

with

$$\begin{array}{rcl} \Phi(\theta) & = & \left[\begin{array}{cc} \underline{0}_{(m\times m)} & I \\ -\eta \theta_2\, I & -\theta_1\, I \end{array} \right]\ , \\ B(\theta) & = & \left[\begin{array}{c} \underline{0}_{(m\times 1)} \\ \\ \theta_3 K \overline{W} \end{array} \right] \quad \text{and} \\ C & = & \left[\begin{array}{cc} I & \underline{0}_{(m\times m)} \end{array} \right] \end{array} \tag{23}$$

where I is the identity matrix of size m and $X(t) = [X_1(t)\ \ X_2(t)]^T = \left[(W(t))^T\ \ (W'(t))^T\right]^T$. The initial conditions are $X(0) = [W(0)\ \ W'(0)]^T = \left[f(0)\overline{W}\ \ f'(0)\overline{W}\right]^T$.

Now, in order to apply the proposed identification method, (19) is rewritten in order to express the parameter dependence yields:

$$\underbrace{\frac{\partial^2 w}{\partial t^2}(t,x)}_{y(t,x)} = \underbrace{\left[\begin{array}{ccc} -\frac{\partial w}{\partial t}(t,x) & -\eta w(t,x) & -K F cos(\omega t) g(x) \end{array} \right]}_{A(t,x)} \underbrace{\left[\begin{array}{c} \theta_1 \\ \theta_2 \\ \theta_3 \end{array} \right]}_{\theta}$$

For a time discretization lag of Δt and a spatial discretization lag $\Delta x = \frac{L}{m-1}$, corresponding to the division of the interval $[0, L]$ into $m-1$ intervals to equal length, one obtains:

$$\overline{y}(k,l) = \overline{A}(k,l)\, \theta$$

where

$$\begin{array}{rcl} \overline{y}(k,l) & = & y(k\Delta t, l\Delta x) \ \ \text{and} \\ \overline{A}(k,l) & = & A(k\Delta t, l\Delta x). \end{array}$$

Finally, it is not difficult to check that $\overline{y}(k,l)$ and $\overline{A}(k,l)$ can be obtained as

$$\begin{array}{rcl} \overline{y}(k,l) & = & \left(X_2'(k\Delta t)\right)_{l+1} \\ \overline{A}(k,l) & = & \left[-\left(X_2(k\Delta t)\right)_{l+1} \ - \eta\, \left(X_1(k\Delta t)\right)_{l+1} \ - K F cos(\omega k \Delta t) \left(\overline{W}\right)_{l+1} \right] \end{array}$$

where $(V)_{l+1}$ denotes the $(l+1)$-*th* component of the vector V.

C. Simulation setting and results

In this subsection the 2D HNN (6) is applied for damage detection in a beam with the following characteristics [5]: $L = 406\times10^{-3}\,m$, $E = 71.72\times10^{9} N/m^2$, $I = 1.33\times10^{-11}\,m^4$, $\mu = 0.112\,kg/m$ and $c = 1\,N\,s/m^2$ and this way $\theta_1 = \frac{c}{\mu} = 8.9286\,s^{-1}$, $\theta_2 = \frac{EI}{\mu} = 8.5168\,m^4/s^2$ and $\theta_3 = \frac{1}{\mu} = 8.9286\,m/kg$. The beam is subjected to a harmonic excitation with amplitude $F = 1\,N$ and frequency $\omega = 396.54\,rad/s$.

The initial displacements and velocities were taken as

$$\begin{aligned} w(0,x) &\equiv 0 \\ \frac{\partial}{\partial t}w(0,x) &\equiv 0, \end{aligned}$$

(corresponding to $f(0) = 0$ and $f'(0) = 0$); further $g(x)$ was taken to be given by (13) for $\zeta^* = 11.65$.

This beam was spatially discretized in seven equally spaced points $x_l = l\,\Delta x$ with $l = 0,\ldots,6$. Moreover in equation (6) N^* is taken equal to 1 and the weights $\alpha_j(l)$ are defined as:

$$\begin{aligned} \alpha_{-1}(0) &= 0 \\ \alpha_0(0) &= 6/7 \\ \alpha_1(0) &= 1/7 \\[1em] \alpha_{-1}(l) &= 1/14 \\ \alpha_0(l) &= 6/7 \qquad l = 1,\ldots,5 \\ \alpha_1(l) &= 1/14 \\[1em] \alpha_{-1}(6) &= 1/7 \\ \alpha_0(6) &= 6/7 \\ \alpha_1(6) &= 0 \\[1em] \alpha_j(l) &= 0 \qquad l < 1 \text{ or } l > 6 \end{aligned}$$

The damage of the beam is represented by a significant change in the model parameters. Here the parameter θ_2 suffers an abrupt change as a consequence of a sudden variation in the parameter E, [6]. This change takes place at time $t = 25\,s$.

To analyse the performance of the 2D HNN the following error measure was considered:

$$ECH(t) = mean\left\{\left\|y(t,x_l) - A(t,x_l)\,\hat{\theta}(t,x_l)\right\|_2\right\} \qquad l = 0,\ldots,6 \tag{24}$$

The obtained error profile is presented in Fig. 2. As can be seen, the estimation error converges to zero and presents a peak when damage occurs. The estimation of the parameter θ_2 for each time instant and for each beam point performed by this method is illustrated in Fig. 3.

These results shown that the 2D HNN was successful in tracking the parameter change and hence was successful in detecting the simulated damage. This encourages further tests in an experimental context. Moreover although this feature has not be exploited in the present work, the methodology presented here seems to be appropriate to treat the case of non-homogeneous beams, where the parameters vary along the beam length.

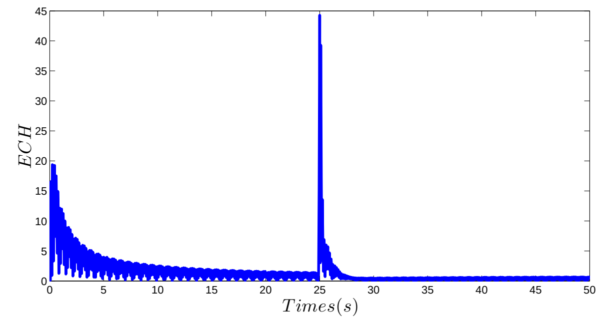

Fig. 2. Time-evolution of the estimation error of the beam parameters obtained by the combination of HNNs.

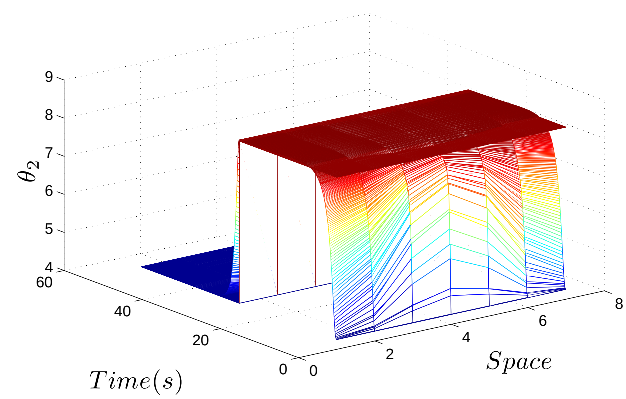

Fig. 3. Time-evolution of the beam parameter estimation obtained by the combination HNNs.

IV. Conclusion

In this paper preliminary results on the use of a 2D Hopfield Neural Network for online damage detection in Euler-Bernoulli beams actuated by external forces were presented. This network provides time evolving parameter estimates along the beam and turns out to be effective. Moreover although this feature has not be exploited in the present work, the methodology presented here seems to be appropriate to treat the case of non-homogeneous beams, where the parameters vary along the beam length. Together with the theoretical analysis of the 2D discrete HNN this is the subject of current investigation.

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