Comparing an Extended Kalman Filter with a Hopfield Neural Network for on-line damage detection in Euler-Bernoulli beams

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Abstract In this paper two identification methods are used to detect online damage in an Euler-Bernoulli beam. The damage is simulated by a change in some of the model parameters and is detected by an Extended Kalman Filter and by a Hopfield Neural Network. For tuning the identification methods different scenarios of damage were considered. Both methods track the changes in the beam parameters and the results obtained are very satisfactory.

1 Introduction

With the development of engineering structures (e.g. aircrafts, bridges and off-shore platforms) a greater attention with cracks that endanger the whole structure is necessary. The detection of this damage is an important point to avoid the total crashing of the structure and mainly to repair the cracks in the their early growth state to minimize the costs. In order to detect beam damage, identification methods have been used to identify the beam parameters [6] and monitor their variation. Indeed recent studies show that the beam damage can be represented by a change in the parameters involved in the EB model [6]. Hence, detecting damage is the same of detecting a parameter change.

In this paper two identification methods are presented to identify the beam parameters of a damped Euler-Bernoulli (EB) model representing the time evolution of the transverse displacement along a vibrating beam. Due to its simplicity this model is often used in structural and mechanical engineering [5]. The first method is an Extended Kalman Filter (EKF) [9] and the second one is a Hopfield Neural Network (HNN) [1]. Both identification methods rest on the choice of an initial tuning that influences the quality of the parameter estimates. Here these methods are evaluated for different tunings and the best one is chosen taking into account the mean error between the real parameters and their estimates.

The EKF and the HNN are used to track parameter changes based on simulated vibration data in different scenarios. The tuning chosen for each identification method is applied to three other damage scenarios in order to validate the methods proposed here.

2 Computational model for the Euler-Bernoulli beams

This section describes the computational model for the Euler-Bernoulli model for the deflection of a beam. The method of separation of variables is applied in order to write this model as a state-space system that will serve as basis for the identification procedure.

2.1 Problem formulation

$$\mu \frac{\partial^2}{\partial t^2} w(t,x) + c \frac{\partial}{\partial t} w(t,x) + EI \frac{\partial^4}{\partial x^4} w(t,x) = q(t,x) \tag{1}$$

where *q(t, x)* is the transverse excitation at location *x* and at instant *t*, *m* is the mass per unit length, *c* is the damping coefficient, *E* is the elastic modulus and *I* is the second moment of area. Here these parameters are considered to be constant along the beam, and the excitation *q(t, x)* is taken to be zero (free vibration). Moreover the beam is taken to be simply-supported at both ends, which is translated by the following boundary conditions,

$$w(t,x)|_{x=0,L} = 0 \text{ and } \frac{\partial^2}{\partial x^2} w(t,x)\bigg|_{x=0,L} = 0$$

2.2 First order model

In this subsection the method of separation of variables is applied to the Euler-Bernoulli model to rewrite the model as a first order state-space model. The separation of variables consists in considering that the solution, *w(t, x),* of (1) is given by, $w(t,x) = f(t)g(x)$. Here *g(x)* is taken as $g(x) = \sin\left(\frac{\pi}{L}x\right)$ and the function *f(t)* is the solution of,

$$f''(t) + \frac{c}{\mu}f'(t) + \frac{EI}{\mu}f(t) = 0, \tag{2}$$

with the following initial conditions, $f(0) = \gamma_0$ and $f'(0) = \gamma_1$ .This is corresponds to considering $w(0,x) = \gamma_0 sin\left(\frac{\pi}{L}x\right)$ and $\frac{\partial}{\partial t}w(0,x) = \gamma_1 sin\left(\frac{\pi}{L}x\right)$.

Taking a spatial discretization interval of Δx, a vector $\overline{W}$ is constructed with the values of g(x) at the discretization points, i.e.,

$$\overline{W} = \begin{bmatrix} g(0) \\ g(\Delta x) \\ \vdots \\ g(L) \end{bmatrix} \epsilon \mathbb{R}^{\frac{L}{\Delta x}+1}. \tag{3}$$

This yields $W(t) = f(t)\overline{W}$ as corresponding spatial discretization of *w(t,x)*. The time evolution of *W(t)* is then described by the ODE:

$$f''(t)\overline{W} + \frac{c}{\mu}f'(t)\overline{W} + \frac{EI}{\mu}f(t)\overline{W} = 0, \tag{4}$$

Defining $X_1(t) = f(t)\overline{W}$ and $X_2(t) = f'(t)\overline{W}$ the following first order model is obtained:

$$\begin{cases} X'(t) = F(\theta)X(t), \\ W(t) = CX(t) \end{cases} \tag{5}$$

with $F(\theta) = \begin{bmatrix} 0 & I \\ -\theta_1 I & -\theta_2 I \end{bmatrix}$, where I is the identity matrix with dimension $\frac{L}{\Delta x} + 1$ and $X(t) = \left[f(t)\overline{W} \quad f'(t)\overline{W}\right]^T$. Moreover $\theta = [\theta_1 \quad \theta_2]^T$ with $\theta_1 = \frac{EI}{\mu}$ and $\theta_2 = \frac{c}{\mu}$ is the parameter vector. The initial conditions for this model $X(0) = \left[\gamma_0\overline{W} \quad \gamma_1\overline{W}\right]^T$ with $\gamma_0 = f(0)$ and $\gamma_1 = f'(0)$.

3 Identification methods

Damage in a beam can be detected by a variation in its parameters. Therefore, in order to detect damage it is necessary to identify these parameters and to monitor their evolution in time. In this section two identification methods are described: in the first subsection the Extended Kalman Filter algorithm [9] is explained, and the second subsection presents the Hopfield Neural Networks [1] to be used in the sequel.

3.1 Extended Kalman Filter

In order to identify the beam parameters θ_1 and θ_2, these parameters are incorporated as states whose evolution is constant along time. This originates a nonlinear state-space model with enlarged state to which an Extended Kalman Filter (EKF) is applied. The EKF allows an individualized tuning of the covariance matrix of the process and measurement noise. Here the covariance matrix of the process is taken to be diagonal, since it is assumed that the enlarged states, and in particular the parameters θ_1 and θ_2, are independent. Moreover, the continuous-time representation (8) is discretized by the zero-order hold method [3] using a sampling interval Δt yielding

$$\begin{cases} X(k+1) = \Phi(\theta)X(k), \\ W(k) = CX(k) \end{cases} \tag{6}$$

where $\Phi(\theta) = e^{F(\theta)\Delta t}$ is the sampled system matrix.

The enlarged state vector is given by $\overline{X}(k) = \begin{bmatrix} X(k) \\ \theta(k) \end{bmatrix}$.

Consequently the enlarged state-space model stands as follows:

$$\begin{cases} \overline{X}(k+1) = \underbrace{\begin{bmatrix} \Phi(\theta) & \underline{0}_{(2n\times 2)} \\ \underline{0}_{(2\times 2n)} & I_{(2\times 2)} \end{bmatrix}}_{h\left(k,\overline{X}(k)\right)} X(k) + v(k) \\ W(k) = \underbrace{\begin{bmatrix} C & \underline{0}_{(n\times 2)} \end{bmatrix}}_{h\left(k,\overline{X}(k)\right)} X(k) + e(k) \end{cases} \tag{7}$$

Here $v(k)$ and $e(k)$ are mutually independent Gaussian white noise sequences with zero mean and covariances R_1 and R_2, respectively, tuned by an empirical analysis.

3.2 Hopfield Neural Networks

As an alternative identification method, a Hopfield Neural Network (HNN) was applied to the beam vibration data. Similarly what happens in EKF, the HNN produces a time-evolving estimate parameters. The system equations are reduced to the form, $y(t) = A(t)\theta$, where $y(t)$ and $A(t)$ are a certain vector and a certain matrix, respectively, computed from the system data, and θ is the vector of model parameters to be estimated online.

The HNN to be used here as an online estimator was proposed in [1] and is given by the following dynamics,

$$\frac{d\hat{\theta}}{dt}(t) = \frac{1}{c\beta} D_c\left(\hat{\theta}(t)\right) A(t)^T\left(y(t) - A(t)\hat{\theta}(t)\right) \tag{8}$$

where $c, \beta > 0$ and $\forall\, i \in 1, \dots, N, t \geq t_0, s_i(t) \in]-s, s[$ and $D_c(\hat{\theta}(t))$ is a positive definite and invertible matrix defined by,

$$D_c\left(\hat{\theta}(t)\right) = \begin{bmatrix} c^2 - \theta_1(t)^2 & 0 \\ 0 & c^2 - \theta_2(t)^2 \end{bmatrix} \tag{9}$$

In order to apply this HNN, the model (5) is rewritten in the form $y(t) = A(t)\theta$ as,

$$\begin{bmatrix} X_1'(t) - X_2(t) \\ X_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -X_1(t) & -X_2(t) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{10}$$

where $X_1'(t)$ and $X_1''(t)(= X_2'(t))$ are determined numerically from $X_1(t)$.

3.3 Off-line tunning

In order to tune the EKF and the HNN the beam parameters are assumed to be known and fixed in space and the performance of each identification method is analysed under four different scenarios, namely:

S1 - The beam parameters are fixed in time;

S2 - E suffers a decrease of 10%, 20% and 30% at time t = 0.5s (which implies a decrease of θ_1) [7];

S3 - c suffers a increase of 50%, 100% and 200% at time t = 0.5s (which implies an increase of θ_2) [8];

S4 - μ suffers a decrease of 10%, 20% and 30% at time t = 0.5s (which implies an increase of θ_1 and θ_2) [4].

These scenarios are tested for 8 different tunings of the EKF and 29 different tunings of the HNN. The performance of each tuning is measured in terms of the mean identification error.

Table 1 - *mean error obtained with the application of the EKF and the HNN in each scenario S1, S2, S3 and S4.*

		S1	S2	S3	S4	S1, S2, S3, S4	S1, S3, S4
EKF	Mean id error	1.7×10^{-2}	2.2×10^{-2}	3.1×10^{-2}	2.4×10^{-2}	2.8×10^{-2}	2.2×10^{-2}
	Tuning	6	8	6	8	8	8
HNN	Mean id error	1.7×10^{-2}	1.6×10^{-2}	7.0×10^{-2}	2.5×10^{-2}	4.1×10^{-2}	2.0×10^{-2}
	Tuning	24	28	14	21	17	24

4 Results

This section presents the results obtained in the analysis of the performance of each identification method taking in account the four scenarios described in subsection 3.3. The Euler-Bernoulli model parameters used to simulate the beam vibration data were obtained in [2] as $E = 2 \times 10^{-11} \mathrm{N/m^2}$, $I = 2.45 \times 10^{-7} \mathrm{m^4}$, $\mu = 23.1$ kg/m and $c = 32$ N s/m^2 and in this way $\theta_1 = 2129.5 \frac{\mathrm{m^4}}{\mathrm{s^2}}$ and $\theta_2 = 1.38\ \mathrm{s^{-1}}$. With the aim of normalizing the spatial discretization between 0 and π, the beam length was considered equal to π with 0.1π as spatial sampling interval.

Table 1 shows the mean error obtained with the application of the tuning in the scenarios S1, S2, S3 and S4. As it is possible to see both identification methods have similar results in scenarios S1 and S4. In scenario S2 the HNN has a good performance whereas the EKF presents a good achievement in scenario S3. The EKF is globally the best identification method whereas the HNN presents good results if S2 is not considered. In this case, the best tuning for the EKF is tuning 8, and the best tuning for the HNN (neglecting S2) is 24.

In order to validate these results the EKF and the HNN were applied with tunings 8 and 24, respectively, to the following three scenarios:

S5 - E suffers a decrease of 15% at time t = 0.3s and at time t = 0.7s (which implies a decrease of θ_1) [7];

S6 - c suffers a increase of 50% at time t = 0.3s and at time t = 0.7s (which implies an increase of θ_2) [8];

S7 - μ suffers a decrease of 15% at time t = 0.3s and at time t = 0.7s (which implies an increase of θ_1 and θ_2) [4];

Figure 1 shows the time evolution for the parameters estimates given by the EKF (blue line) and by the HNN (red line) under the conditions of scenario S5. Both identification methods detect the two variations in parameter θ_1.

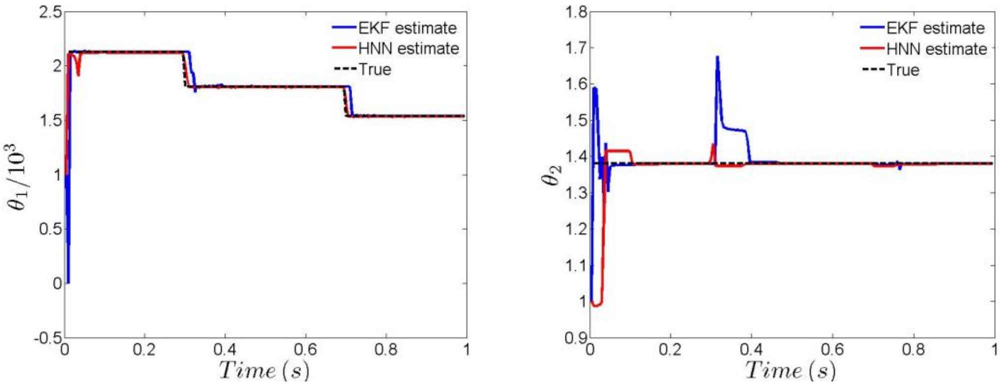

Figure 1 - *Time-evolution of beam parameters under the conditions of S5.*

Figure 2 represents the scenario S6 where both methods detect the beam damage that it is translated by the variation in parameter c but only the EKF converges for the new value of this parameter.

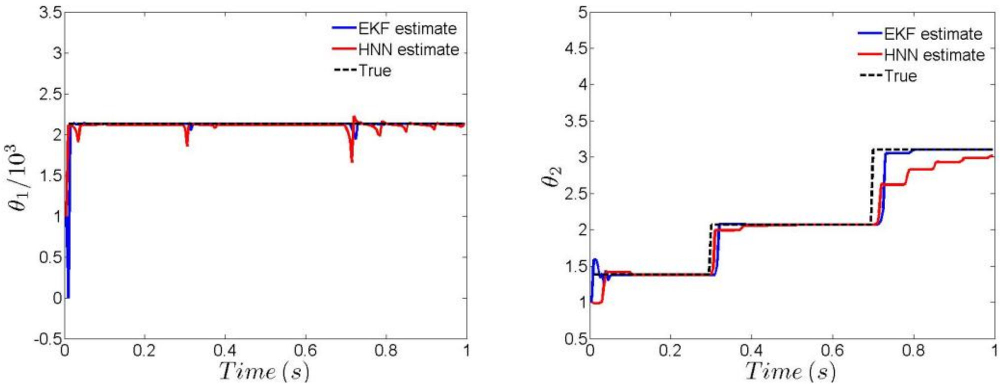

Figure 2 - *Time-evolution of beam parameters under the conditions of S6.*

The variation in parameter μ implies an increase of θ_1 and θ_2 as is shown in Figure 3. Both identification methods detect these changes in parameter values. However the identification of θ_1 is better than the identification of θ_2. For the change in θ_1 the EKF and the HNN have similar results and the HNN method detects almost instantaneously this change. The identification of θ_2 given by the EKF is globally better than the identification given by the EKF.

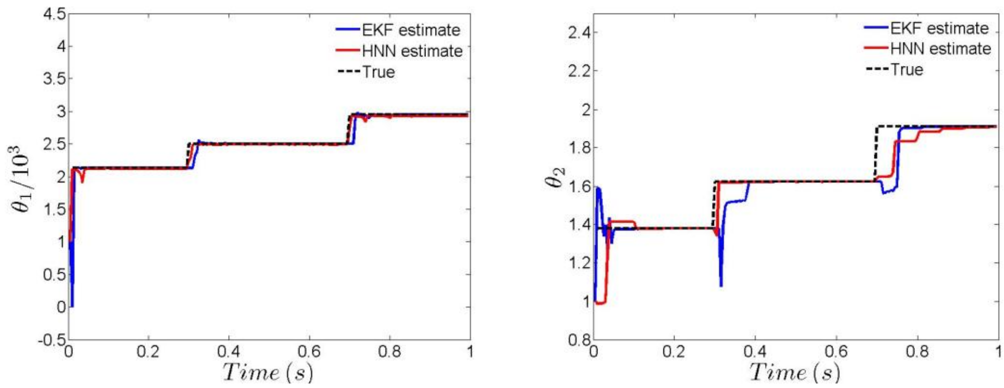

Figure 3 - *Time-evolution of beam parameters under the conditions of S7.*

Table 2 presents the value of the mean error obtained with the application of each identification method in each of the scenarios S5, S6 and S7. The results obtained agree with the results obtained in Figures 1, 2 and 3.

Table 2 - *mean error obtained with the application of the EKF and the HNN in scenarios S5, S6 and S7.*

	S5	S6	S7
EKF	2.5×10^{-2}	4.5×10^{-2}	3.6×10^{-2}
HNN	1.7×10^{-2}	1.2×10^{-2}	3.6×10^{-2}

5 Conclusions

This paper presents a comparison between two identification methods for online damage detection in Euler-Bernoulli beams. The first method is an Extended Kalman Filter and the second one is a Hopfield Neural Network. The damage is represented by a variation in some of the model parameters. Results were obtained under various identification scenarios to find the optimal tuning for each method. In order to validate these tunings both identification methods were applied to three other identification scenarios. The results are very satisfactory for both identification methods and encourage their incorporation in an automatic system for online beam damage detection.

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