NUMERICAL MODELLING OF THE DAMAGE-TO-FRACTURE TRANSITION IN A DUCTILE MATERIAL

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ABSTRACT

The present work aims at the numerical treatment of the crack propagation throughout structures made of engineering metals and alloys whose failure results from ductile damage. The challenge consists in reproducing numerically within a unified numerical methodology the successive steps of diffuse damage, strain localisation and crack propagation leading to the ultimate ruin of the structure. This requires the definition of physics motivated criteria and the introduction of specific kinematics with the passage from one step to the other as compatible as possible. In this purpose, we are studying here the ability of the cohesive zone model (CZM) approach to ensure the connection between diffuse damage dealt with in the context of the standard finite element method (FEM) and crack propagation treated in the context of the extended finite element method (X-FEM).

Keywords: Ductile fracture, strain localisation, cohesive zone model.

INTRODUCTION

During accidental events, such as automotive crashworthiness, ships collision or bird strike/ingestion in aeronautics, constitutive materials are subjected to severe loadings which may lead to failure. Predicting numerically the response of the structure during the whole deformation process, i.e. including the phase of crack propagation if any, and the current residual strength of the overloaded structure is of major interest for preserving the main functions and the integrity of the sensitive areas.

When the constitutive materials considered are ductile, the degradation process typically involves three successive steps, namely diffuse micro-voiding induced damage, strain localisation and crack propagation.

As a result of micro-voiding induced damage, the material resistance is exposed to continuous degradation and subsequent softening. There exist several constitutive models able to reproduce the effects of ductile damage on the elastic and plastic properties of metals, see e.g. Gurson (1977), Lemaître (1985), Rousselier (1987) and also Longère (2012). However, results obtained using such continuum models within the standard finite element method (FEM) are subjected to mesh dependence in the ductile damage-induced softening phase and fail to reproduce the material separation during the propagation of discrete macro-cracks. Non-local regularisation techniques can be applied to attenuate the aforementioned pathology, but accurate results require a very fine mesh. Concerning the cracking phase, complementary techniques like the element deletion method (Song 2008, Autenrieth 2009) and the inter-
element crack method (Xu 1994, Song 2008) may be used to describe the crack propagation but they also suffer from mesh dependence. Adaptive remeshing techniques have been shown to better reproduce the crack propagation, see e.g. Bouchard (2000), but require an immense effort in terms of computation time.

Embedding the crack within the finite element and enriching correspondingly the kinematic formulation of the latter has been proposed within the extended finite element method (X-FEM), see e.g. Belytschko (1999) and Moës (1999), to overcome the aforementioned limitations. Works devoted to show the performance of the X-FEM to reproduce the failure of elastic-(quasi-)brittle structures are numerous in literature, see Moës (1999) and Dumstorff (2007), whereas works dealing with ductile, strongly nonlinear structures are still remarkably scarce, see the papers of Crété (2014) and Broumand (2015). This method of embedded finite elements moreover allows for using a coarser mesh and is accordingly more suitable for large structures.

Now, combining a diffuse damage model employed within the continuum mechanics framework using the FEM and a crack propagation model dealt with in the fracture mechanics framework using the X-FEM is generally not sufficient, especially when ductile materials are involved. In the latter case, the drop in the resistance of the structure is indeed too brutal. A key ingredient of a more realistic ductile fracture model is then the proper treatment of the critical transition phase between diffuse damage and crack propagation, namely the strain localisation. The challenge thus consists in taking the strain localisation stage into account within a unified approach connecting the FEM and the X-FEM, as shown in Fig. 1.

One possible approach to make the connection between FEM (diffuse damage) and X-FEM (crack propagation) is the cohesive zone model (CZM). In the sequel, the potentiality of that
‘connection’ model is discussed from both the physical and computational viewpoints, when applied to strongly nonlinear materials and applied in the X-FEM framework.

After a brief overview in Sect. 2 of the CZM/X-FEM approach adopted in the present work, propositions are made to describe the transition between FEM (diffuse damage) and CZM/X-FEM (strain localisation) in Sect. 3; then between CZM/X-FEM (strain localisation) and X-FEM (crack propagation) in Sect. 4. Concluding remarks are finally given.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$t_n$, $t_m$</td>
<td>Normal and tangential components of the current (local) traction vector</td>
</tr>
<tr>
<td>$n$, $m$</td>
<td>Normal/ tangential vector of the cohesive surface</td>
</tr>
<tr>
<td>$t_n^0$, $t_m^0$</td>
<td>Normal and tangential components of the initial (local) traction vector</td>
</tr>
<tr>
<td>$T_n$, $T_m$</td>
<td>(Local) cohesive stiffness</td>
</tr>
<tr>
<td>$[[u]]_n$, $[[u]]_m$</td>
<td>Normal and tangential components of the (local) cohesive displacement jump</td>
</tr>
<tr>
<td>$x$</td>
<td>Arbitrary point within the finite element</td>
</tr>
<tr>
<td>$u(x)$</td>
<td>Approximated displacement at the coordinate $x$</td>
</tr>
<tr>
<td>$I$, $J$</td>
<td>Set of all nodes of the structure mesh/ of the cut elements</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Degrees of freedom (node $i$) of the standard FEM</td>
</tr>
<tr>
<td>$b_j$</td>
<td>Enriched degrees of freedom (node $j$) of the X-FEM</td>
</tr>
<tr>
<td>$N_i$</td>
<td>Classical FEM shape functions</td>
</tr>
<tr>
<td>$H(x)$</td>
<td>Heaviside function ($+0.5$ if $x&gt;0$; $-0.5$ if $x&lt;0$)</td>
</tr>
<tr>
<td>$H_j$</td>
<td>Heaviside function at node $j$</td>
</tr>
<tr>
<td>$[[u]]$</td>
<td>(Global) cohesive displacement jump</td>
</tr>
<tr>
<td>$\tilde{\sigma}_{ij}$</td>
<td>Averaged stress</td>
</tr>
<tr>
<td>$\sigma_{nn}$</td>
<td>Projected normal stress</td>
</tr>
<tr>
<td>$\tau_{nm}$</td>
<td>Projected shear stress</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Cohesive strength</td>
</tr>
<tr>
<td>${\sigma_{xx}, \sigma_{yy}, \tau_{xy}}$</td>
<td>Components of the stress vector $\sigma$ (Voigt notation)</td>
</tr>
<tr>
<td>$B_j$</td>
<td>Strain-displacement interpolation matrix at node $j$</td>
</tr>
<tr>
<td>$y_A$, $y_B$</td>
<td>$y$-coordinates of intersection of cohesive surface with element edges</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Orientation angle of the cohesive surface</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of the cohesive surface</td>
</tr>
<tr>
<td>$f^{int}$</td>
<td>Internal force vector</td>
</tr>
<tr>
<td>$K_{ui}$, $K_{ub}$, …</td>
<td>Contributions to the global stiffness matrix</td>
</tr>
</tbody>
</table>
OVERVIEW OF THE ‘CONNECTION’ METHOD CZM/X-FEM

The proposed ‘connection’ model is characterized as follows:

- The bulk behaviour is governed by the pre-localisation continuum model.
- The (weak discontinuity) localisation band is condensed to a (strong discontinuity) cohesive surface.
- The strain softening response in the post-localisation phase is described by an additional cohesive law. We choose the linear softening cohesive law (Figure 2)

\[
\begin{pmatrix}
    t_m \\
    t_n
\end{pmatrix}
= \begin{pmatrix}
    t_m^0 \\
    t_n^0
\end{pmatrix}
+ \begin{bmatrix}
    T_m & 0 \\
    0 & T_n
\end{bmatrix}
\begin{pmatrix}
    [u]_m \\
    [u]_n
\end{pmatrix}
\]

(1)

Fig. 2 - Linear softening cohesive law

- The X-FEM is used to embed the cohesive surface into the finite element. Based on the enriched (X-FEM) displacement approximation with shifted basis (Zi 2003)

\[
u(x) = \sum_{i \in I} u_i N_i(x) + \sum_{j \in J} b_j N_j(x)(H(x) - H_j)
\]

(2)

the cohesive jump is derived from the enriched degrees of freedom \( \mathbf{b} \) in (2)

\[
[u](x) = \sum_{j \in J} b_j N_j(x)
\]

(3)

- The cohesive force and stiffness terms contributing to the global system of equations are integrated using two Gauss points per element along the cohesive surface (Fig. 3).

Fig. 3 - Integration of the CZM
FROM FEM TO CZM/X-FEM

The passage from the continuum model FEM to the ‘connection’ localisation model CZM requires the definition of physical criteria for localisation band initiation and a proper treatment of the numerical FEM-to-CZM transition.

a. Criteria for localisation band formation

The detection of localisation band formation is based on three subsequent steps (Fig. 4)

- Localisation band orientation: Bifurcation analysis combined with the evaluation of quantities averaged over a patch (Fig. 4, leftmost) is employed to calculate the potential orientation of localisation, as done in Crété (2014).

- Localisation initiation criterion: The localisation band (cohesive surface) propagates as soon as the stored energy around the crack tip exceeds a critical limit, as done in Crété (2014).

- Incorporation of the CZM: As soon as the initiation criterion is met, a cohesive surface is inserted into the crack-tip element. In order to ensure a smooth transition, the initial tractions of the cohesive law (1) are calculated according to the traction continuity principle. These are derived from a stress measure $\tilde{\sigma}_{ij}$ which is averaged over the oriented crack-tip patch and projected onto the cohesive surface:

$$t^0_n = \sigma_{nn} = n_i^t \tilde{\sigma}_{ij} n_j$$

$$t^0_m = \tau_{nm} = m_i^t \tilde{\sigma}_{ij} n_j$$

b. Computational issues

When the aforementioned transition criteria are applied to numerical failure simulations, a problematic numerical issue is encountered which may be explained as follows. For simplicity, let us analyse a single element with the boundary conditions shown in Fig. 5 (left). The material is assumed to behave elastically. When the vertical component of the stress vector exceeds the cohesive strength $f_c$, the cohesive surface is inserted with a pre-defined orientation with a traction force-displacement jump law of the type shown in Fig. 2. The resulting vertical force-displacement response of the top edge of the element is plotted in Fig. 5 (right).
According to Fig. 5, at the activation of the cohesive surface, the traction continuity is not satisfied leading to an unexpected drop in force before continuing on the strain softening path. The magnitude of the drop in force is strongly dependent on the traction force-displacement jump law and the cohesive surface orientation. This pathologic behaviour is analysed mathematically in the sequel, based on the ideas of Jirasek (2000) and Wu (2011). Let us consider now the example in Fig. 6.

The stress vector at the moment of activation of the cohesive zone model takes the form:

\[
\sigma(x, y) = \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
0 \\
f_t \\
0
\end{bmatrix}
\]  
(6)

The y-component of the nodal force at node 3 is computed from the internal force vector

\[
F_{3y} = A_e \int_{\Omega} (H(x, y) - H_3) \cdot B_3^T(x, y) \cdot \sigma(x, y) \, d\Omega
\]  
(7)

where \( A_e = 4 \) and
The integral over x and y finally yields

\[
F_{3y} = -\int_{x=-1}^{x=1} \int_{y=-1}^{y=y_A + \frac{y_B - y_A x}{2}} (1 + x) f_t \, dy \, dx
\]

\[
= -f_t \left( \frac{1}{3} (y_B - y_A) + 2y_A + 2 \right)
\]

The smearing procedure leads to the fictitious tractions

\[
t_n = \frac{F_{3y}}{l} = -\frac{1}{2} \cos(\alpha) f_t \left( \frac{1}{3} (y_B - y_A) + 2y_A + 2 \right)
\]

which are only equal to the cohesive strength in case of a horizontal cohesive surface, i.e. if \(y_A = y_B = 0\) and thus also \(\alpha = 0\). In all other cases, i.e. if the cohesive surface is inclined with respect to the horizontal line, the natural traction continuity condition is violated.

In the paper of Wu (2011) a modified X-FE formulation was proposed which naturally satisfies the traction continuity condition. However the approach emanates from the hypothesis of an elastic stress field. In the case of a strongly non-linear material law, that approach is inappropriate. In this concern, more research work is needed to provide an X-FEM for ductile materials which also satisfies the traction continuity.

**FROM CZM/X-FEM TO X-FEM**

The transition from the ‘connection’ model (CZM) to the crack propagation model (X-FEM) requires an appropriate indicator for crack formation and the analysis of the coupling of the two numerical methods.

**a. Criterion for crack formation**

As soon as the cohesive traction is zero, the crack faces are entirely separated and start to form the macro-crack, introducing the transition from the CZM/X-FEM to the X-FEM (Fig. 7). This transition is mainly determined by the traction-separation energy \(G_c\), which can be obtained from experimental observations and energy equivalence considerations.
b. Computational issues

The advantage of using the same enrichment formulation for the CZM (X-FEM) and for the final stage of failure (also X-FEM) allows simply passing from the one phase to the other. As the cohesive traction forces approach zero, the internal cohesive force \( f_{coh}^{int} \) and the cohesive stiffness matrix \( K_{coh} \) become zero, hence naturally resulting in the global (incremental) set of equations of the classical (traction-free) X-FEM (Fig. 8).

![Fig. 8 - Numerical transition CZM – X-FEM regarding to the global set of equations](image)

Also the 64 Gauss point integration scheme is maintained throughout the transition phase as the contributions of the cohesive Gauss points equal to zero at the initiation of the macro-crack (Fig. 9).

![Fig. 9 - Numerical transition CZM – X-FEM regarding to the integration scheme](image)

**CONCLUSION**

This paper is devoted to the numerical treatment of the ductile fracture process in engineering materials undergoing large deformation and further failure. The approach proposed in the present paper aims at coupling a ductile strongly non-linear constitutive model taking into account micro-voiding induced damage with the crack propagation model.
We proposed a linear softening cohesive law (CZM/X-FEM) serving as a ‘connection’ model between the FEM (diffuse damage) and the X-FEM (crack propagation). For the modelling of the initiation of the cohesive zone at the onset of strain localisation, we considered quantities averaged over a patch located at the crack tip. This concept was also adopted to calculate the initial traction forces of the linear cohesive law.

In a subsequent mathematical analysis we revealed a rarely considered numerical deficit of the X-FEM, especially when applied to cohesive zone models. It turned out that the X-FEM is not able to correctly satisfy the traction continuity condition in the case of a inclined cohesive surface. Hence, a numerically erroneous representation of the cohesive law can lead to wrong physical interpretations of the simulation results.

Finally, we discussed the manageable transition from the CZM/X-FEM to the final stage of the ductile fracture process which is also modelled with X-FEM. This transition is naturally provided as soon as the traction forces vanish.

The implementation of the cohesive zone model is in progress satisfying the compatibility with the X-FEM framework.

ACKNOWLEDGMENT
This work was supported by a DGA-MRIS grant.

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