ABSTRACT
Based on the relations of elastodynamics this paper models the typical wave initiation and wave-shaped structure of the substrate surface due to the explosion welding. Herewith, the substrate is modeled as a half-plane, and the contact interaction of specimens is reduced to the local traction loading, which moves with a constant subsonic velocity at the surface of the half-plane. The solution of this problem in the moving reference frame is obtained based on the Fourier integral transform. Based on the numerical studies it is proved that when the detonation velocity tends to the Rayleigh wave velocity in the substrate material the resonance phenomena are observed.

Keywords: Explosion, elastodynamics, wave-shape formation, Rayleigh wave.

INTRODUCTION
The development and progress of many industries are directly related to new technologies, in particular, new materials, which possess high technical properties and show good performance along with easy processing, production and low cost. Structural composite materials and technologies for their processing and production are among most prospective and science intensive ones, which concern current studies of materials science and have high potential for their future applications in engineering practice (Hutsaylyuk, 2013).

Due to its properties the explosion welding is one of most efficient, and in some cases the only possible way of production of bimaterial and multilayered composite materials (Sniezek, 2013, 2014). The explosion welding is a complex physical phenomenon, which is related to several fundamental branches of material science, mechanics, molecular and gas dynamics etc.

The high-speed collision of the bodies during the explosion welding is accompanied with several important effects: the phenomenon of wave initiation, Munroe effect and surface cohesion (Lysak, 2003). The problem is related to different sides of metal-physics, but in the first place it is the mechanics one. During the cohesion of the materials in a solid phase, which is observed in the high-speed oblique collision of two bodies, at the contact zone the high-intensity narrowly-localized plastic strains are induced mainly in the wave-shape, which are observed at the bimaterial interface (Abrahamson, 1961). Such cohesion of solids occurs without melting of metals, diffusion processes, and the obtained composite materials possess high strength, even in the welding of commonly incompatible metals (Lysak, 2003).
As already mentioned, a characteristic property of the welding under high-speed oblique collision is a wave-shaped surface at the contact zone. There are many works, which explains this phenomenon utilizing the hydrodynamics analogy (Cowan, 1963, 1971; Wang, 2012) molecular dynamics approaches (Godunov, 2013) etc. Nevertheless, this paper provides the explanation of this phenomenon based on the viewpoint of elastodynamics (Achenbach, 1973) of the specimen, which operates as a substrate, neglecting the temperature and sub-surface effect, related with the shaped charge formation.

PROBLEM STATEMENT

The explosion welding stands for the phenomenon of good bondage of surfaces of metal bodies, which collide at a certain angle, wherein at least one of them accelerates to the speed of about 1800...3000 m/s with the products of detonation of explosive charge. Herewith, one should note that the explosion itself, or rather the energy of detonation products expansion in the abovementioned process plays the second role providing the accelerated relative movement of the bodies and their consequent collision. The physical nature of the sources of such acceleration can be different, in instance, electromagnetic field (during the magnetic impulse welding), the energy of powder charge in the barrel of a weapon, the energy of explosion of electric conductor under high current etc. But in all cases the processes during the high-speed collision of solids are the same (Lysak, 2003).

Since this paper considers only the mechanical processes acting in the bottom plate (substrate), the contact interaction of the latter with a specimen is modeled with locally applied tractions \( p(x, t) \), which move with the subsonic velocity at the substrate surface, and the substrate itself is modeled with an elastic half-plane (Fig. 1). Thus, the problem is reduced to determination of the displacement field in the half-plane, and in particular, the deformed shape of its surface \( y = 0 \) ahead of the loading source and beneath it.

In the mathematical sense, the problem is reduced to determination of the elastodynamics solution for the volume expansion function \( \theta(x, y) \) and normal component \( u_x(x, y) \) of the elastic displacement vector from the following system of partial differential equations (Sulym, 2013):

\[
\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{1}{c_1^2} \frac{\partial^2 \theta}{\partial t^2} ; \quad \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} = \frac{1}{c_2^2} \frac{\partial^2 u_x}{\partial t^2} + (1 - \kappa^2) \frac{\partial \theta}{\partial y} ;
\]

\[
\theta(x, y, 0) = \dot{\theta}(x, y, 0) = 0 ; \quad u_x(x, y, 0) = \ddot{u}_x(x, y, 0) = 0 ; \quad \lim_{y \to \pm \infty} \theta(x, y, t) = \lim_{y \to \pm \infty} u_y(x, y, t) = 0 ;
\]

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where \( c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}} \) and \( c_2 = \sqrt{\frac{\mu}{\rho}} \) are the longitudinal and transversal waves propagation velocities in the substrate material; \( \lambda \) and \( \mu \) are Lamé’s elastic constants; \( \kappa = c_1 / c_2 \); \( p_x(x, t) \) and \( p_y(x, t) \) are the components of traction vector at the surface \( y = 0 \), which are defined as,

\[
p_x(x, t) = \text{sign}(\phi) \sqrt{p(x, t)} \cos(\phi); \quad p_y(x, t) = \text{sign}(\phi) \sqrt{p(x, t)} \sin(\phi),
\]

and \( \phi \) is an angle, at which upper and lower specimens collide during the explosion welding (Lysak, 2003). It should be mentioned that for successful bondage of two metals with explosion welding there exists certain dependence between the angle \( \phi \) and detonation velocity \( V_D \), which is experimentally determined for different pairs of metals that are welded.

Principal mechanical effects, which determine the quality of welding, occur directly in the loading zone, thus for their accurate definition consider new coordinate system, which moves together with the loading

\[
x_1 = x - V_D t, \quad y = y.
\]

In the moving reference frame (6) governing equations (1) are expressed as follows:

\[
\left( 1 - M_1^2 \right) \frac{\partial^2 \theta}{\partial x_1^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{1}{c_1^2} \frac{\partial^2 \theta}{\partial t^2} + \frac{2V_D}{c_1^2} \frac{\partial \theta}{\partial t} + \frac{V_D^2}{c_1^2} \frac{\partial^2 \theta}{\partial x_1^2};
\]

\[
\left( 1 - M_2^2 \right) \frac{\partial^2 u_j}{\partial x_1^2} + \frac{\partial^2 u_j}{\partial y^2} = \frac{1}{c_2^2} \frac{\partial^2 u_j}{\partial t^2} + \frac{2V_D}{c_2^2} \frac{\partial u_j}{\partial t} + \frac{V_D^2}{c_2^2} \frac{\partial^2 u_j}{\partial x_1^2} + \frac{1}{c_2^2} \frac{\partial^2 \theta}{\partial y}. \tag{8}
\]

Consider only stationary case (Gorskov, 2004), for which one can neglect local time derivatives of the functions sought. Thus, in the dimensionless variables \( \alpha = x_1 / L, \gamma = y / L \), where \( L \) is a characteristic dimension (for example, the length of the segment at which the loading is applied at certain time), the boundary value problem is reduced to

\[
\left( 1 - M_1^2 \right) \frac{\partial^2 \theta}{\partial \alpha^2} + \frac{\partial^2 \theta}{\partial \gamma^2} = 0; \quad \left( 1 - M_2^2 \right) \frac{\partial^2 v}{\partial \alpha^2} + \frac{\partial^2 v}{\partial \gamma^2} = \left( 1 - \frac{c_2^2}{c_2^2} \right) \frac{\partial \theta}{\partial \gamma};
\]

\[
\lim_{y \to \pm \infty} \theta(\alpha, \gamma) = \lim_{y \to \pm \infty} v(\alpha, \gamma) = 0;
\]

\[
\sigma_{\gamma} = -p_x(\alpha), \quad \sigma_{\alpha\gamma} = p_y(\alpha), \quad \gamma = 0,
\]

where \( v(x, y) = u_j(x, y) / L \) is a dimensionless component of displacement vector and \( M_j = V_D / c_j \) are Mach numbers.
It should be noted, that the sign of terms \( 1 - M_i^2 \) determines the type of equations (9), and thus, their solution strategy. Therefore, this paper limits their solution to the case of subsonic detonation velocity \( M_i^2 < M_j^2 < 1 \).

**DERIVATION OF THE PROBLEM’S SOLUTION**

Fourier integral transform (Sneddon, 1951) for the variable \( \alpha \) can be applied to the solution of the problem (9)–(11). Consequently one obtains the boundary value problem for a system of ordinary differential equations

\[
\frac{d^2 \bar{\theta}}{d\gamma^2} - \xi^2 (1 - M_i^2) \bar{\theta} = 0; \quad \frac{d^2 \bar{\nu}}{d\gamma^2} - \xi^2 (1 - M_j^2) \bar{\nu} = \left(1 - \frac{c_1^2}{c_2^2}\right) \frac{d\bar{\theta}}{d\gamma}
\]

\[\text{(12)}\]

\[
\lim_{\gamma \to +\infty} \bar{\theta}(\xi, \gamma) = \lim_{\gamma \to +\infty} \bar{\nu}(\xi, \gamma) = 0;
\]

\[\text{(13)}\]

\[
\bar{\sigma}_{\gamma\gamma} = \bar{p}_\gamma(\xi), \quad \bar{\sigma}_{\alpha\gamma} = \bar{p}_\alpha(\xi), \quad \gamma = 0.
\]

Here \( \left\{ \bar{\theta}(\xi, \gamma) \right\} = \int_{-\infty}^{\infty} \left\{ \theta(\alpha, \gamma) \right\} e^{i\alpha \xi} d\alpha \) are the Fourier transforms (Sneddon, 1951).

The solution of equations (12), which satisfies Sommerfeld radiation conditions (13), is as follows

\[
\bar{\theta}(\xi, \gamma) = A(\xi) e^{-\beta_1 |\gamma|}; \quad \bar{\nu}(\xi, \gamma) = B(\xi) e^{-\beta_1 |\gamma|} + \frac{\beta_1}{|\xi| M_i^2} A(\xi) e^{-\beta_1 |\gamma|},
\]

where \( \beta_1^2 = 1 - M_j^2 \).

For determination of the component \( u(\alpha, \gamma) \) of the displacement vector Fourier integral transform is applied to the equation for volume expansion \( \theta = \frac{\partial u}{\partial \alpha} + \frac{\partial v}{\partial \gamma} \) as

\[
\bar{\theta} = -i \xi \bar{u} + \frac{\partial \bar{v}}{\partial \gamma}.
\]

Herewith, accounting for the solutions (13), one obtains

\[
\bar{u} = \left( \frac{i}{\xi} \bar{\theta} - \frac{\partial \bar{v}}{\partial \gamma} \right) = \left( \frac{1}{\xi} M_i^2 A(\xi) e^{-\beta_1 |\gamma|} + \beta_1 |\xi| B(\xi) e^{-\beta_1 |\gamma|} \right).
\]

The Fourier transform of the stress tensor components are determined through the Hooke’s law with equations (15) and (16) as
\[
\frac{\sigma_{\gamma \gamma}}{\mu} = \left( \frac{c_1}{c_2} - 2 \right) \bar{D} + 2 \frac{\sigma_{\gamma \gamma}}{D_{\gamma \gamma}} = -\frac{1 + \beta_2^2}{M_i} A(\xi) e^{-\beta_1 |\gamma|} - 2 \beta_2 |\xi| B(\xi) e^{-\beta_1 |\gamma|};
\]
(17)

\[
\frac{\sigma_{\alpha \alpha}}{\mu} = \frac{\sigma_{\alpha \alpha}}{D_{\alpha \alpha}} - i \xi \bar{V} = -i \text{sgn}(\xi) \frac{2}{M_i} \beta_1 A(\xi) e^{-\beta_1 |\gamma|} - i \xi (\beta_2^2 + 1) B(\xi) e^{-\beta_1 |\gamma|},
\]
(18)

where \( \text{sgn}(\xi) = \begin{cases} 1, & \xi > 0 \\ -1, & \xi < 0 \end{cases} \).

Accounting for the boundary conditions (14), one obtains the system of linear algebraic equations

\[
\begin{aligned}
&\frac{1 + \beta_2^2}{M_i} A(\xi) + 2 \beta_2 |\xi| B(\xi) = \frac{\bar{p}_x(\xi)}{\mu}; \\
&\text{sgn}(\xi) \frac{2}{M_i} \beta_1 A(\xi) + \xi (\beta_2^2 + 1) B(\xi) = i \frac{\bar{p}_\alpha(\xi)}{\mu},
\end{aligned}
\]
(19)

which solution results in the following expressions for the unknowns \( A(\xi) \) and \( B(\xi) \):

\[
\begin{aligned}
A(\xi) &= \frac{(1 + \beta_2^2) \bar{p}_x(\xi) - 2i \beta_1 \text{sgn}(\xi) \frac{\bar{p}_\alpha(\xi)}{\mu}}{(1 + \beta_2^2)^2 - 4 \beta_1 \beta_2} M_i^2; \\
B(\xi) &= \frac{i \text{sgn}(\xi)(1 + \beta_2^2) \frac{\bar{p}_x(\xi)}{\mu} - 2 \beta_1 \frac{\bar{p}_\alpha(\xi)}{\mu}}{|\xi| \left( (1 + \beta_2^2)^2 - 4 \beta_1 \beta_2 \right)}. 
\end{aligned}
\]
(20)

As a result, the Fourier transforms of components of the displacement vector with the account of Eq (5) take the form

\[
\begin{aligned}
\bar{v}(\xi, \gamma) &= \frac{\bar{p}(\xi)}{\mu} \left[ \beta_1 \frac{(1 + \beta_2^2) \cos \varphi - 2i \beta_1 \text{sgn} \sin \varphi e^{-\beta_1 |\gamma|}}{(1 + \beta_2^2)^2 - 4 \beta_1 \beta_2} \frac{\bar{p}_x(\xi)}{|\xi|} \right] + \\
&+ \frac{i \text{sgn}(\xi)(1 + \beta_2^2) \sin \varphi - 2 \beta_1 \cos \varphi e^{-\beta_1 |\gamma|}}{(1 + \beta_2^2)^2 - 4 \beta_1 \beta_2} \frac{\bar{p}_\alpha(\xi)}{|\xi|}; \\
\bar{u}(\xi, \gamma) &= \frac{\bar{p}(\xi)}{\mu} \left[ \frac{(1 + \beta_2^2) \cos \varphi - 2i \beta_1 \text{sgn} \sin \varphi e^{-\beta_1 |\gamma|}}{(1 + \beta_2^2)^2 - 4 \beta_1 \beta_2} \frac{\bar{p}_x(\xi)}{|\xi|} \right] + \\
&+ \frac{i \text{sgn}(\xi)(1 + \beta_2^2) \sin \varphi - 2 \beta_1 \cos \varphi e^{-\beta_1 |\gamma|}}{(1 + \beta_2^2)^2 - 4 \beta_1 \beta_2} \frac{\bar{p}_\alpha(\xi)}{|\xi|}.
\end{aligned}
\]
(21)
For obtaining the inverses of expressions (21), (22) one can utilize the convolution theorem for the Fourier transform and the fact that the inverses of \( p(\xi), \frac{ie^{-|\xi|}}{|\xi|^2} \) and \( e^{-|\xi|/|\xi|} \) for \( \zeta \geq 0 \) are, respectively,

\[
p(\alpha), \frac{1}{\pi} \arctg \left( \frac{\alpha}{\zeta} \right), \quad -\frac{1}{\pi} \left( C + \ln \left( \sqrt{\alpha^2 + \zeta^2} \right) \right). \tag{23}
\]

It is known that vertical displacement in the elasticity problem for a half-plane are determined up to a constant. Therefore, one can assume that \( C = 0 \) in Eq (23). Moreover, one can assume, that the loading distribution at the surface of the half-plane fits the Hertz contact distribution

\[
p(\alpha) = \begin{cases} p^* \sqrt{(1-\alpha^2)}, & |\alpha| \leq 1; \\ 0, & |\alpha| > 1,
\end{cases}
\]

where \( p^* \) is a pressure at the center of a loading segment.

Consequently based on Eqs (21), (22) one obtains

\[
u(\alpha, \gamma) = \frac{p^*}{\mu \left(1 + \beta_2^2\right)^2 - 4 \beta_1 \beta_2} \left\{ \frac{(1 + \beta_2^2) \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta - \frac{2 \beta_2 \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta \right\}; \tag{24}
\]

\[
u(\alpha, \gamma) = \frac{p^*}{\mu \left(1 + \beta_2^2\right)^2 - 4 \beta_1 \beta_2} \left\{ -\frac{\beta_1 (1 + \beta_2^2) \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \ln \left( \sqrt{\beta_2^2 \gamma^2 + (\alpha - \eta)^2} \right) d\eta - \frac{2 \beta_2 \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta \right\}.
\]

\[
u(\alpha, \gamma) = \frac{p^*}{\mu \left(1 + \beta_2^2\right)^2 - 4 \beta_1 \beta_2} \left\{ -\frac{\beta_1 (1 + \beta_2^2) \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \ln \left( \sqrt{\beta_2^2 \gamma^2 + (\alpha - \eta)^2} \right) d\eta - \frac{2 \beta_2 \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta + \frac{2 \beta_1 \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta + \frac{1 + \beta_2^2 \cos \phi}{\pi} \int_{-1}^{1} \sqrt{(1-\eta^2)} \arctg \left( \frac{\alpha - \eta}{\beta_2 \gamma} \right) d\eta \right\}. \tag{25}
\]
At the surface $\gamma = 0$ the components of the displacement vector are as follows

\[
u(\alpha,0) = \frac{p^* / \mu}{\left(1 + \beta_2^2\right)^2 - 4\beta_1\beta_2} \left( (1 + \beta_2^2 - 2\beta_1\beta_2) \cos \varphi \int_{-1}^{1} \sqrt{1 - \eta^2} \ \text{sgn}(\alpha - \eta) \ d\eta + \int_{-1}^{1} \frac{M_2^2 \beta_2 \sin \varphi \int_{-1}^{1} \sqrt{1 - \eta^2} \ln|\alpha - \eta| \ d\eta}{\pi} \right);
\]

\[
u(\alpha,0) = \frac{p^* / \mu}{\left(1 + \beta_2^2\right)^2 - 4\beta_1\beta_2} \left( \beta M_2^2 \cos \varphi \int_{-1}^{1} \sqrt{1 - \eta^2} \ \text{ln}|\alpha - \eta| d\eta + \int_{-1}^{1} \frac{(1 + \beta_2^2 - 2\beta_1\beta_2) \sin \varphi \int_{-1}^{1} \sqrt{1 - \eta^2} \ \text{sgn}(\alpha - \eta) d\eta}{\pi} \right).
\]

(26)

It can be shown (Gorskov, 2004) that the relation $\left(1 + \beta_2^2\right)^2 - 4\beta_1\beta_2 = 0$ is equivalent to the known equation, which defines the Rayleigh wave velocity; therefore, when the detonation velocity tends to the Rayleigh wave velocity in the substrate material the resonance phenomena are observed.

**NUMERICAL ANALYSIS AND DISCUSSION**

The numerical analysis is held for a substrate made of aluminium alloy, which has the following elastic properties (Totten, 2003): $\lambda = 53.1$ GPa, $\mu = 26.5$ GPa; therefore, $c_1 = 6.3 \cdot 10^4$ m/s and $c_2 = 3.2 \cdot 10^3$ m/s, respectively. The Rayleigh wave velocity in this medium equals $c_R = 2.98 \cdot 10^3$ m/s.

As it was mentioned above, the explosion welding occurs only at certain ratios between the detonation velocity and the angle, at which the oblique collision of the specimens occurs. At the calculations the detonation velocity $V_D$ (m/s) is selected in the range $[2000; 2900]$; the collision angle is in the range $\varphi \in [0.24; 0.35]$ (radian); and $p^* / \mu = 0.01$.

Fig. 2 presents the results of calculation of normal displacement $\nu(\alpha,0)$ at the boundary of the half-plane for $p^* / \mu = 0.01$, $\varphi = 0.3$ and different values of the detonation velocity $V_D$, and Fig. 3 depicts normal displacement of the boundary for $p^* / \mu = 0.01$, $V_D = 2800$ m/s and different values of collision angle $\varphi$. One can see in these figures that the dominant influence on the displacement of the half-plane’s boundary under the constant loading magnitude $p^*$ is caused by the detonation velocity, especially those in the range, which tends to the Rayleigh wave velocity in the substrate material.
The influence of ratio $p^*/\mu$ on the separate components of the displacement vector is reduced to the scale factor. However, this ratio has significant effect on the shape of the half-plane’s surface, which is shown in Fig. 4 for $V_D = 2800\text{ m/s}$ and $\varphi = 0.3$. Here it should be noted that for linearity conservation of the problem this ratio should be much less than one,
however, in a crude approximation the results presented can predict the typical wave-structure of big scale at the surface of the substrate.

Herewith it should be accounted for the fact that the colliding specimen just before its contact with the substrate due to the explosion welding is under high-magnitude alternating bending loading. The latter can initiate plastic bands (zones of softened material) in this specimen, which will “fill” the wave-shaped substrate after the collision. This process will be accompanied and supported with high pressure of gases at the zone just beneath the contact area.

**CONCLUSIONS**

Based on the relations of elastodynamics this paper models the typical wave initiation and wave-shaped structure of the substrate surface due to the explosion welding. Herewith, the substrate is modeled as a half-plane, and the contact interaction of specimens is reduced to the local traction loading, which moves with a constant subsonic velocity at the surface of the half-plane. The solution of this problem in the moving reference frame is obtained based on the Fourier integral transform. Based on the numerical studies held for an aluminium alloy half-plane it is proved that when the detonation velocity tends to the Rayleigh wave velocity in the substrate material the resonance phenomena are observed. At certain collision angles of specimens, detonation velocities, and especially magnitudes of loading, the surface of the half-plane acquires the wave-shape typical for the explosion welding. Regardless the number of assumptions and approximations made in the modeling of explosion welding process, the results obtained explains experimentally observed wave-shape structure of the interface within the framework of elastodynamics.

**ACKNOWLEDGMENTS**

The authors gratefully acknowledge the funding by The National Centre for Research and Development of Poland under the grant No PBS/A5/35/2013

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