TRANSIENT ELASTICITY MODELING OF WAVE-SHAPE FORMATION UNDER EXPLOSION WELDING

Szachogluchowicz Ireneusz1(*), Sniezek Lucjan1, Volodymyr Hutsaylyuk1, Heorhiy Sulym2, Iaroslav Pasternak3, Ihor Turchyn4

1Military University of Technology, Warsaw, Poland; 2Bialystok Technical University, Białystok, Poland
3Lutsk National Technical University, Lutsk, Ukraine; 4Ivan Franko National University of Lviv, Lviv, Ukraine

(*)Email: ireneusz.szachogluchowicz@wat.edu.pl

ABSTRACT

Transient elastodynamics problem is considered for a half-plane under the loading, which moves along its boundary with a subsonic velocity. The transient solution of the problem is obtained by means of Laguerre integral transform with respect to the time variable and Fourier integral transform with respect to the spatial coordinate. Based on the numerical analysis the transient wave-shape formation process is studied at the surface of the half-plane under the loading velocities close to the Rayleigh wave velocity of the substrate material.

Keywords: Explosion, elastodynamics, wave-shape formation, Rayleigh wave.

INTRODUCTION

Due to its unique features, the explosion welding is one of the most efficient and in some cases the only possible way of production of hi-tech bimaterials and multilayered composite materials. The explosion welding is a complex physical phenomenon, which is related to several fundamental branches of material science, mechanics, molecular and gas dynamics etc. It is accompanied with several important effects: wave-shape formation process, shaped charge effect, and surface bonding.

One of the characteristic features of explosion welding under a high speed oblique collision is the abovementioned formation of the wave-shaped material interface at the contact zone. The work (Snieżek, 2015) considered the stationary dynamic solution of the problem to explain this phenomenon based on the elastodynamics of the specimen (substrate). The resonance effects were indicated, when the detonation velocity approaches the Rayleigh wave velocity of the substrate material. Due to this phenomenon, the use of the stationary solution of the problem for estimation of the wave-shape formation process under detonation velocities close to the resonance ones is considerably difficult. In addition, the stationary solution of the dynamic problems is theoretically justified only in the case of steady motion, which is achieved after certain time at a constant loading. Meanwhile, under the conditions of the explosion welding, there is not enough time to stabilize the process. Also it is interesting the behavior of the substrate at the time range just right after the detonation, because in this time lapse a significant amount of joined material is formed. In this regard, the paper considers the transient problem of elastodynamics.

Classical method for the solution of transient elastodynamics problems is the Laplace transformation approach for time variable. It is successfully applied to different problems for semi-finite and finite solids. The main problem in its application is to derive the inversion of
the transform, i.e. original time dependences. This is especially important, when besides the Laplace transform another transform is applied for a spatial variable. In the case of a half-space, special approach was developed for their mutual inversion, which is now called Cagniard – de Hoop method (Achenbach, 1973). However, in the case of a moving loading and transition to the moving reference frame the representation of the problem’s solution in the transformed domain is complicated, which leads to significant complexity of Cagniard – de Hoop method application.

Due to these facts this paper proposes to apply the Laguerre polynomials approach to elastodynamics problems, which application recently allowed obtaining several important initial-boundary value problems of mathematical physics (Turchyn, 2013).

PROBLEM FORMULATION

The term “explosion welding” generally means the phenomenon of rigid bondage of metal solids surfaces, which collide under a certain angle and while at least one of them is accelerated to the velocity of 1800...3000 m/s with the products of explosive substance detonation. Herewith it should be mentioned that explosion itself or rather the expansion energy of detonation products acts only the auxiliary role in this process providing accelerated motion of solids one against another and their further collision. Physical nature of the sources of such acceleration can be different, i.e. electromagnetic field (in the magnetic pulse welding), energy of powder charge in the gun barrel, explosion energy of electric conductor under the electric current etc. However, in all the cases the matter of the processes, which occur under the high-speed collision of solids, is the same (Lysak, 2003).

Since this paper considers only the mechanical processes, which occur in the bottom plate (substrate) at the zone ahead of the loading segment, its contact interaction with the moving specimen can be modeled with the moving local loading \( p(x,t) \), which moves with the subsonic velocity at the surface of the substrate. The latter is modeled with an elastic half-plane (Fig. 1). The problem is in the derivation of the displacement field in the half-plane, and particularly the shape of the surface \( y = 0 \) ahead of the loading source and just under it.

Mathematically the problem can be reduced to the solution of the following partial differential equations of transient elastodynamics (Sulym, 2013):

\[
(\lambda + \mu) \text{grad div } U + \mu \Delta U = \rho \partial_t^2 U \\
U(x, y, 0) = \partial_t U(x, y, 0) = 0 \\
\lim_{y \to +\infty} U(x, y, t) = 0 \\
\sigma_{yy} = -p_y(x, t), \quad \sigma_{xy} = p_x(x, t), \quad y = 0
\]
where $\mathbf{U}(x,y,t)$ is a displacement vector; $\lambda$ and $\mu$ are elastic (Lame) constants; $\sigma_{xy}$ and $\sigma_{yx}$ are stress tensor components; $p_x(x,t)$ and $p_y(x,t)$ are the components of traction vector at the surface $y=0$, which are defined within equations
\begin{align}
p_x(x,t) &= |p(x,t)|\sin(\phi); \quad p_y(x,t) = |p(x,t)|\cos(\phi),
\end{align}
and $\phi$ is an angle at which the upper sample collides the substrate (Lysak, 2003). It should be mentioned that for a successful bondage of two metals under explosion welding certain dependence exists between an angle $\phi$ and the detonation velocity $V_D$, which is different for different material pairs and is determined experimentally.

Accounting for the potentials $\varphi(x,y,t)$ and $\psi(x,y,t)$ vector equation of motion (1) can be decomposed into two scalar wave equations
\begin{align}
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} &= 1\frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1\frac{\partial^2 \psi}{\partial t^2},
\end{align}
and the components $u_x(x,y,t)$ and $u_y(x,y,t)$ of the displacement vector are related to the potentials $\varphi(x,y,t)$ and $\psi(x,y,t)$ within the equations
\begin{align}
\frac{\partial u_x}{\partial x} &= -\frac{\partial \varphi}{\partial y}; \quad \frac{\partial u_y}{\partial y} = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial x}.
\end{align}

Main mechanical effects, which determine the quality of welding, occur just in the loading zone. Therefore, for their correct modeling consider the moving reference frame, which moves with the loading as
\begin{align}
x_i = x - V_D t; \quad y_i = y.
\end{align}

In the moving reference frame (8) equations (6) reduce to
\begin{align}
\frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \varphi}{\partial y_1^2} &= 1\frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial y_1^2} = V_D^2 \frac{\partial^2 \varphi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2};
\end{align}
\begin{align}
\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial y_1^2} &= 1\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial y_1^2} = V_D^2 \frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2}.
\end{align}

Introducing the dimensionless variables $\alpha = x_i / L, \gamma = y_i / L, \tau = c_1 t / L$, where $L$ is a characteristic linear dimension (i.e. the length of the loading segment), the initial-boundary value problem of elastodynamics reduces to
\begin{align}
\left(1 - M_1^2\right)\frac{\partial^2 \varphi}{\partial \alpha^2} + \frac{\partial^2 \varphi}{\partial \gamma^2} &= 2M_1\frac{\partial^2 \varphi}{\partial \alpha \tau} + \frac{\partial^2 \varphi}{\partial \tau^2};
\end{align}
\begin{align}
\left(1 - M_1^2\right)\frac{\partial^2 \psi}{\partial \alpha^2} + \frac{\partial^2 \psi}{\partial \gamma^2} &= 2M_1\kappa\frac{\partial^2 \psi}{\partial \alpha \tau} + \kappa^2 \frac{\partial^2 \psi}{\partial \tau^2};
\end{align}
\begin{align}
\varphi(\alpha,\gamma,0) = \partial_\gamma \varphi(\alpha,\gamma,0) = \psi(\alpha,\gamma,0) = \partial_\gamma \psi(\alpha,\gamma,0) = 0
\end{align}
\begin{align}
\lim_{\gamma \to \infty} \varphi(\alpha,\gamma,\tau) = \lim_{\gamma \to \infty} \psi(\alpha,\gamma,\tau) = 0;
\end{align}
\begin{align}
\sigma_{\alpha\gamma} = -p_\gamma(\alpha,\tau), \quad \sigma_{\alpha\alpha} = p_\alpha(\alpha,\tau), \quad \gamma = 0,
\end{align}

-19-
where $M_j = \frac{V_j}{c_j}$ is the Mach number; $\kappa = c_1 / c_2$.

Previously in (Snieżek, 2015) the stationary case was considered, for which the local time derivative of the sought quantities had been neglected. This work considers the transient case. It should be mentioned, that the sign of terms $\left(1 - M_j^2\right)$ influences the type of equations (11), (12), and thus their solution strategy. This paper considers only subsonic detonation velocities ($M_1^2 < M_2^2 < 1$).

The solution of Eqs. (11), (12) is sought in a function class, which satisfies the condition

$$\int_0^\infty \exp(-\tau) \left[ \phi(\alpha, \gamma, \tau) \right]^2 d\tau < \infty.$$  

Then the following series can be associated with these functions

$$\left\{ \phi(\alpha, \gamma, \tau) \right\} = \lambda \sum_{n=0}^{\infty} \left\{ \phi_n(\alpha, \gamma) \right\} L_n(\lambda \tau)$$  

(16)

where

$$\left\{ \phi_n(\alpha, \gamma) \right\} = \int_0^\infty \exp(-\lambda \tau) \left\{ \phi(\alpha, \gamma, \tau) \right\} L_n(\lambda \tau) d\tau.$$  

(17)

Transform (17) is further called the Laguerre integral transform of the functions $\phi(\alpha, \gamma, \tau)$ and $\psi(\alpha, \gamma, \tau)$, and Eq. (16) is the inversion of this transform. The parameter $\lambda > 0$ can be considered as a scale factor, which is helpful in optimization of the series (16) summation numerical procedure.

Multiplying Eqs. (11), (12) by the kernel $\exp(-\lambda \tau)L_n(\lambda \tau)$ of transform (17), integrating obtained relations by parts in the range $[0; +\infty)$, and accounting for the differentiation rules

$$\frac{\partial}{\partial \tau} \left[ \exp(-\lambda \tau)L_n(\lambda \tau) \right] = -\lambda \exp(-\lambda \tau) \sum_{k=0}^{n} L_k(\lambda \tau);$$

$$\frac{\partial^2}{\partial \tau^2} \left[ \exp(-\lambda \tau)L_n(\lambda \tau) \right] = -\lambda^2 \exp(-\lambda \tau) \sum_{k=0}^{n} (n-k+1) L_k(\lambda \tau)$$

and zero initial conditions (13), one obtains the following equation sequences

$$\left(1 - M_1^2\right) \frac{\partial^2 \phi_n}{\partial \alpha^2} + \frac{\partial^2 \phi_n}{\partial \gamma^2} = 2M_1 \lambda \frac{\partial}{\partial \alpha} \sum_{k=0}^{n} \phi_k + \lambda^2 \sum_{k=0}^{n} (n-k+1) \phi_k, \ n = 0,1,2,\ldots;$$  

(18)

$$\left(1 - M_2^2\right) \frac{\partial^2 \psi_n}{\partial \alpha^2} + \frac{\partial^2 \psi_n}{\partial \gamma^2} = 2M_2 \lambda \kappa \frac{\partial}{\partial \alpha} \sum_{k=0}^{n} \psi_k + \kappa^2 \lambda^2 \sum_{k=0}^{n} (n-k+1) \psi_k, \ n = 0,1,2,\ldots.$$  

(19)

Applying the Fourier integral transform for the variable $\alpha$ to equations (18), (19), one obtains the sequences of ordinary differential equations for the transformed functions

$$\frac{\partial^2 \tilde{\phi}_n}{\partial \gamma^2} - \left(1 - M_1^2\right) \xi^2 \tilde{\phi}_n = 2i \xi \lambda M_1 \sum_{k=0}^{n} \tilde{\phi}_k + \lambda^2 \sum_{k=0}^{n} (n-k+1) \tilde{\phi}_k, \ n = 0,1,2,\ldots$$  

(20)
\[
\frac{\partial^2 \psi_n}{\partial \gamma^2} - (1 - M_i^2) \xi^2 \psi_n = 2i\xi M_i \lambda \kappa \sum_{k=0}^{n} \bar{\psi}_k + \kappa^2 \lambda^2 \sum_{k=0}^{n} (n-k+1) \bar{\psi}_k, \quad n = 0, 1, 2, \ldots
\] (21)

where \( \{ \bar{\psi}_n (\xi, \gamma) \} \) are the Fourier transforms (Sneddon, 1951).

Transferring the terms in Eq. (20) with the index \( n \) into the left hand side, one obtains
\[
\frac{\partial^2 \bar{\phi}_n}{\partial \gamma^2} - \left( (1 - M_i^2) \xi^2 + 2i\xi \lambda + \lambda^2 \right) \bar{\phi}_n = 2i\xi M_i \lambda \sum_{k=0}^{n-1} \bar{\phi}_k + \lambda^2 \sum_{k=0}^{n-1} (n-k+1) \bar{\phi}_k, \quad n = 0, 1, 2, \ldots
\] (22)

The obtained sequence has the triangular structure, since its right hand side is a linear combination of the solutions obtained for previous values of \( n \). This allows presenting its solution in the form of the algebraic convolution
\[
\bar{\phi}_n (\xi, \gamma) = \sum_{j=0}^{n} A_{n-j}(\xi) G_j^{(1)}(\xi, \gamma),
\] (23)

where \( A_j, \quad (j = 0, 1, 2, \ldots) \) is a sequence of arbitrary constants (which are the function of Fourier transform parameter \( \xi \)), and \( G_j^{(1)}(\xi, \gamma) \) are fundamental solutions of the ODE sequence (18), which can be rewritten regarding these functions as
\[
\frac{\partial^2 G_j^{(1)}}{\partial \gamma^2} - \omega_i^2 G_j^{(1)} = \sum_{k=0}^{j-1} (2i\xi M_i \lambda + (n-k+1) \lambda^2) G_k^{(1)}, \quad j = 0, 1, 2, \ldots
\] (24)

where
\[
\omega_i = \sqrt{(1 - M_i^2) \xi^2 + 2i\xi \lambda + \lambda^2} = \frac{1}{\sqrt{2}} \left( \sqrt{\left((1 - M_i^2) \xi^2 + \lambda^2\right)^2 + 4\xi^2 \lambda^2} + \left(1 - M_i^2\right) \xi^2 + \lambda^2 + ight.
\]
\[+ i \sqrt{\left((1 - M_i^2) \xi^2 + \lambda^2\right)^2 + 4\xi^2 \lambda^2} - \left(1 - M_i^2\right) \xi^2 - \lambda^2 \right) \] (25)

The structure of the sequence (24) allows applying to it the standard procedure of the method of undefined multipliers. Accounting for the fact that for \( M_i^2 < 1, \Im \xi = 0, \Im \lambda = 0 \) the term \( (1 - M_i^2) \xi^2 + 2i\xi \lambda + \lambda^2 \) has positive real part, the general solution of Eq. (20) for \( j = 0 \) writes as,
\[
\frac{\partial^2 G_0^{(1)}}{\partial \gamma^2} - \omega_i^2 G_0^{(1)} = 0,
\] (26)

which according to the condition (14) can be presented as
\[
G_0^{(1)}(\xi, \gamma) = a_{0,0}^{(1)}(\xi) \exp(-\gamma \omega_i),
\] (27)

Accounting for (27), Eq. (24) for \( j = 1 \) is as follows
\[
\frac{\partial^2 G_1^{(1)}}{\partial \gamma^2} - \omega_i^2 G_1^{(1)} = \left(2i\xi M_i \lambda + 2\lambda^2\right) a_{0,0}^{(1)}(\xi) \exp(-\gamma \omega_i).
\] (28)
The right hand side of Eq. (28) satisfies the homogeneous equation, therefore its general solution is presented as

\[ G_i^{(1)}(\xi, \gamma) = a_{i,1}^{(1)}(\xi) \exp(-\gamma \omega_1) + a_{i,0}^{(1)}(\xi) \exp(-\gamma \omega_1). \]  

Substituting (27) into (26) one obtains the relation between \( a_{i,0}(\xi) \) and \( a_{0,0}(\xi) \)

\[ a_{i,0}(\xi) = -\frac{(2i \xi M, \lambda + 2 \lambda^2)}{2 \omega_1} a_{0,0}^{(1)}(\xi) \]  

Generalizing this procedure for arbitrary \( j \) the fundamental solution of the sequence (24) are presented as

\[ G_j^{(1)} = \exp(-\gamma \omega_1) \sum_{k=0}^j a_j^{(1)}(\xi) \gamma^k. \]  

Their direct substitution into Eq. (24) and further change in the summation order leads to the recurrent relations

\[ a_j^{(1)} = \frac{1}{2 \omega_1} \left[ a_{j+1}^{(1)} - \sum_{p=k}^{j-1} (2i \xi M, \lambda + (j - p + 1) \lambda^2) a_p^{(1)} \right], \]  

where \( a_j^{(1)} = 0 \) for \( k > j \), and all \( a_j^{(1)}, j = 0, 1, 2, \ldots \) stay arbitrary.

Applying the same considerations to Eq. (20) one obtains its general solution as,

\[ \psi_n(\xi, \gamma) = \sum_{j=0}^n B_{n-j}(\xi) G_j^{(2)}(\xi, \gamma), \]  

where \( B_j, (j = 0, 1, 2, \ldots) \) is the sequence of arbitrary constants, and \( G_j^{(2)}(\xi, \gamma) \) are fundamental solutions of the sequence (21), which analogously to (31) are presented as

\[ G_j^{(2)} = \exp(-\gamma \omega_2) \sum_{k=0}^j a_j^{(2)}(\xi) \gamma^k. \]  

for

\[ \omega_2 = \sqrt{\left(1 - M_z^2\right) \xi^2 + 2i \lambda \xi \kappa + \lambda^2 \kappa^2} = \frac{1}{\sqrt{2}} \left( \sqrt{\left((1 - M_z^2) \xi^2 + \lambda^2 \kappa^2\right)^2 + 4 \xi^2 \lambda^2 \kappa^2 + \left(1 - M_z^2\right) \xi^2 + \lambda^2 \kappa^2} + i \sqrt{\left((1 - M_z^2) \xi^2 + \lambda^2 \kappa^2\right)^2 + 4 \xi^2 \lambda^2 \kappa^2 - \left(1 - M_z^2\right) \xi^2 - \lambda^2 \kappa^2} \right). \]  

The factors \( a_j^{(2)}(\xi) \) are determined from the recurrent relations

\[ a_j^{(2)} = \frac{1}{2 \omega_2} \left[ a_{j+1}^{(2)} - \sum_{p=k}^{j-1} (2i \xi M, \lambda \kappa + (j - p + 1) \lambda^2 \kappa^2) a_p^{(2)} \right] \]  

for arbitrary \( a_{j, j}, j = 0, 1, 2, \ldots \).

Consider the boundary conditions (15) in the terms of potentials \( \varphi \) and \( \psi \). In view of this, one should account for the stress-strain relations in terms of dimensionless variables.
\[
\varepsilon_{aa} = \frac{\partial u_x}{\partial \alpha}; \varepsilon_{\gamma\gamma} = \frac{\partial u_y}{\partial \gamma}; \varepsilon_{a\gamma} = \frac{1}{2} \left( \frac{\partial u_x}{\partial \gamma} + \frac{\partial u_y}{\partial \alpha} \right); \theta = \frac{\partial u_x}{\partial \alpha} + \frac{\partial u_y}{\partial \gamma}; \]

\[
\sigma_{\gamma\gamma} = \lambda \theta + 2 \mu \varepsilon_{\gamma\gamma}; \sigma_{a\gamma} = 2 \mu \varepsilon_{a\gamma}; \sigma_{aa} = \lambda \theta + 2 \mu \varepsilon_{aa}.
\]

Therefore, according to Eq. (7)

\[
\varepsilon_{aa} = \frac{\partial^2 \varphi}{\partial \alpha^2} - \frac{\partial^2 \psi}{\partial \alpha \partial \gamma}; \varepsilon_{\gamma\gamma} = \frac{\partial^2 \varphi}{\partial \gamma^2} + \frac{\partial^2 \psi}{\partial \alpha \partial \gamma}; \varepsilon_{a\gamma} = \frac{1}{2} \left( 2 \frac{\partial^2 \varphi}{\partial \alpha \partial \gamma} - \frac{\partial^2 \psi}{\partial \gamma^2} + \frac{\partial^2 \psi}{\partial \alpha^2} \right); \theta = \Delta \varphi;
\]

\[
\frac{\sigma_{\gamma\gamma}}{\mu} = (\kappa^2 - 2) \Delta \varphi + 2 \left( \frac{\partial^2 \varphi}{\partial \alpha \partial \gamma} + \frac{\partial^2 \psi}{\partial \alpha \partial \gamma} \right); \frac{\sigma_{a\gamma}}{\mu} = 2 \frac{\partial^2 \varphi}{\partial \alpha \partial \gamma} - \frac{\partial^2 \psi}{\partial \gamma^2} + \frac{\partial^2 \psi}{\partial \alpha^2}; \frac{\sigma_{aa}}{\mu} = (\kappa^2 - 2) \Delta \varphi + 2 \left( \frac{\partial^2 \varphi}{\partial \alpha^2} - \frac{\partial^2 \psi}{\partial \alpha \partial \gamma} \right)
\]

Applying the Laguerre and Fourier integral transforms to the boundary conditions (15) and accounting for (37) one obtains

\[
(\kappa^2 - 2) \left( \frac{\partial^2 \tilde{\varphi}_n}{\partial \gamma^2} - \xi^2 \tilde{\varphi}_n \right) + 2 \left( \frac{\partial^2 \tilde{\varphi}_n}{\partial \gamma^2} + i \xi \frac{\partial \tilde{\psi}_n}{\partial \gamma} \right) = \frac{\tilde{P}_{r,n}(\xi)}{\mu}, \gamma = 0
\]

\[
2i \xi \frac{\partial \tilde{\varphi}_n}{\partial \gamma} - \xi^2 \tilde{\psi}_n = \frac{\tilde{P}_{a,n}(\xi)}{\mu}, \gamma = 0
\]

Substituting the general solutions (23) and (33) into Eqs. (38) and (39) it follows

\[
\sum_{j=0}^{n} A_{n-j}(\xi) \left( \kappa^2 \frac{\partial^2 G_j^{(1)}(\xi,0)}{\partial \gamma^2} - (\kappa^2 - 2) \xi^2 G_j^{(1)}(\xi,0) \right) + 2i \xi \sum_{j=0}^{n} B_{n-j}(\xi) \frac{\partial G_j^{(2)}(\xi,0)}{\partial \gamma} = \frac{\tilde{P}_{r,n}(\xi)}{\mu}
\]

\[
2i \xi \sum_{j=0}^{n} A_{n-j}(\xi) \frac{\partial G_j^{(1)}(\xi,0)}{\partial \gamma} - \sum_{j=0}^{n} B_{n-j}(\xi) \left( \frac{\partial^2 G_j^{(2)}(\xi,0)}{\partial \gamma^2} + \xi^2 G_j^{(2)}(\xi,0) \right) = \frac{\tilde{P}_{a,n}(\xi)}{\mu}
\]

Assume that \( a_{0,0}^{(1)} = a_{2,0}^{(1)} = 1, a_{1,0}^{(1)} = a_{1,0}^{(2)} = 0, j = 1, 2, 3, \ldots \) in Eqs. (32) and (35). Then

\[
G_0^{(1)}(\xi,0) = G_0^{(2)}(\xi,0) = 1, G_j^{(1)}(\xi,0) = G_j^{(2)}(\xi,0) = 0, j = 1, 2, 3, \ldots
\]

\[
\frac{\partial G_0^{(1)}(\xi,0)}{\partial \gamma} = -\omega_1, \frac{\partial^2 G_0^{(1)}(\xi,0)}{\partial \gamma^2} = \omega_1^2, \frac{\partial G_0^{(2)}(\xi,0)}{\partial \gamma} = -\omega_2, \frac{\partial^2 G_0^{(2)}(\xi,0)}{\partial \gamma^2} = \omega_2^2.
\]

Accounting for Eq. (42), only the terms with \( A_n(\xi) \) and \( A_n(\xi) \) are left in the left hand sides of Eqs. (40) and (41), and the rest are transferred into the right hand sides. As a result, one obtains the sequence of equations for determination of \( A_n(\xi) \) and \( B_n(\xi) \)

\[
\begin{align*}
A_n(\xi) & \left( \kappa^2 \omega_1^2 - (\kappa^2 - 2) \xi^2 \right) + 2i \xi \omega_2 B_n(\xi) = f_n^{(1)}(\xi) \\
-2i \xi \omega_1 A_n(\xi) - (\omega_2^2 + \xi^2) B_n(\xi) & = f_n^{(2)}(\xi)
\end{align*}
\]

where
\[
\begin{align*}
f_n^{(1)}(\xi) &= \frac{p_{a,n}(\xi)}{\mu} - \sum_{j=0}^{n-1} A_{n-j}(\xi) \left( \kappa^2 \frac{\partial^2 G_j^{(1)}(\xi,0)}{\partial \gamma^2} - (\kappa^2 - 2)\xi^2 G_j^{(1)}(\xi,0) \right) - 2i\xi \sum_{j=0}^{n-1} B_{n-j}(\xi) \frac{\partial G_j^{(2)}(\xi,0)}{\partial \gamma} , \\
f_n^{(2)}(\xi) &= \frac{p_{a,n}(\xi)}{\mu} - \sum_{j=0}^{n-1} A_{n-j}(\xi) \frac{\partial G_j^{(1)}(\xi,0)}{\partial \gamma} + \sum_{j=0}^{n-1} B_{n-j}(\xi) \left( \frac{\partial^2 G_j^{(2)}(\xi,0)}{\partial \gamma^2} + \xi^2 G_j^{(2)}(\xi,0) \right) .
\end{align*}
\]

The recurrent solution of Eq. (43) is obtained as

\[
A_n(\xi) = \frac{f_n^{(1)}(\xi) \left( \alpha_1^2 + \xi^2 \right) + 2i\xi\alpha_2 f_n^{(2)}(\xi)}{\Delta(\xi)} ;
\]

\[
B_n(\xi) = -\frac{2i\xi\alpha_1 f_n^{(1)}(\xi) + f_n^{(2)}(\xi) \left( \kappa_1^2 \alpha_1^2 - (\kappa_1^2 - 2)\xi^2 \right)}{\Delta(\xi)} .
\]

\[
\Delta(\xi) = \left( (2 - M_1^2)\xi^2 + \kappa_1^2(2i\xi\lambda + \lambda^2) \right) \left( (2 - M_2^2)\xi^2 + 2i\xi\lambda + \lambda^2 \right) + 4\xi^2 \sqrt{1-M_1^2}\xi^2 + 2i\xi\lambda + \lambda^2 \sqrt{1-M_2^2}\xi^2 + 2i\xi\lambda + \lambda^2 \lambda^2
\]

Equations (44)–(45) formally complete the solution of the transient problem. The originals of potentials \( \phi(\alpha,\gamma,\tau) \) and \( \psi(\alpha,\gamma,\tau) \) equal to

\[
\begin{align*}
\phi(\alpha,\gamma,\tau) &= \frac{\lambda}{\kappa} \sum_{n=0}^{\infty} L(n,\lambda,\tau) \sum_{j=0}^{n} \int_{-\infty}^{\infty} \exp(-i\xi\alpha) A_n(\xi) G_j^{(1)}(\xi,\gamma) d\xi ; \\
\psi(\alpha,\gamma,\tau) &= \frac{\lambda}{\kappa} \sum_{n=0}^{\infty} L(n,\lambda,\tau) \sum_{j=0}^{n} \int_{-\infty}^{\infty} \exp(-i\xi\alpha) B_n(\xi) G_j^{(2)}(\xi,\gamma) d\xi .
\end{align*}
\]

With the known potentials \( \phi(\alpha,\gamma,\tau) \) and \( \psi(\alpha,\gamma,\tau) \) the displacement vector and stress tensor are determined within Eq. (37) for arbitrary time.

**NUMERICAL RESULTS**

The numerical analysis is held for a substrate made of aluminium alloy, which has the following elastic properties (Totten, 2003): \( \lambda = 53.1 \text{ GPa} \), \( \mu = 26.5 \text{ GPa} \); therefore, \( c_1 = 6.3 \cdot 10^3 \text{ m/s} \) and \( c_2 = 3.2 \cdot 10^3 \text{ m/s} \), respectively. The Rayleigh wave velocity in this medium equals \( c_R = 2.98 \cdot 10^3 \text{ m/s} \).

Figure 2 depicts the calculation results for normal displacement \( u_y(\alpha,0,\tau) \) at the boundary of the half-space for the ratio of detonation velocity to the Rayleigh wave velocity of \( V_D / c_R = 0.9 \); and Figure 3 for \( V_D / c_R = 0.95 \). Here it should be mentioned that for the selected material the value of dimensionless time \( \tau = 1 \) under \( L = 0.1m \) corresponds to real time \( t = 1.6 \cdot 10^{-5} \text{ s} \).
One can see from the results presented that when the detonation velocity tends to the Rayleigh wave velocity, resonance phenomena occur at the boundary of the half-space. The speed of increase of these effects depending on $V_D / c_R$ is depicted in Figure 4, where the time dependence of the ratio of maximal amplitude to the initial one is presented.
CONCLUSIONS
The paper utilizes the Laguerre polynomial approach to obtain exact analytic solution of the transient elastodynamics problem for a stress-strain state of a half-space, which boundary is loaded with moving traction loading applied at a certain angle. Formulated problem is closely related to modeling of the wave-formation process observed at the surface of two samples, which undergo explosion welding. In general, the dynamic plots of wave propagation at the surface of a half-space due to the influence of a moving loading is in good qualitative agreement with the results of corresponding stationary problem obtained previously by the authors, which can be used for quantitative estimation of the interfacial shape of specimens bonded under explosion welding.

ACKNOWLEDGMENTS
The authors gratefully acknowledge the funding by The National Centre for Research and Development of Poland under the grant No PBS/A5/35/2013

REFERENCES