SIMULATION OF THE INFLUENCE OF STEEL FIBERS ON THE SHEAR BEHAVIOR OF REINFORCED CONCRETE BEAMS

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ABSTRACT

An analytical model is presented for predicting complete shear load-deformation response of reinforced concrete sections taking into account the steel fibers reinforced concrete. This model is based on the equilibrium, compatibility equations and stress-strain relationships which are formulated in term of average stresses and average strains. This model is able to analyze sections having unusual forms or reinforcing details, loaded in combined bending, axial load and shear. In the case of the steel fiber reinforced concrete, a stress-strain relation proposed by Kachi and al (2002) is adopted. Predictions of the model are compared with several calculated reinforced beams and are shown to estimate the effect of the steel fibbers on the shear stiffness of the sections in the case of the elastic linear domain, after concrete cracking and after reinforcement yielding.

Keywords: Steel fibbers, shear stiffness, equilibrium, compatibility, shear stains.

INTRODUCTION

From that time, many researchers have developed truss model able to determinate the global behavior of reinforced concrete beams subjected to shear loading. Several of such models are based on the Modified Compression Field Theory (MCFT) Belarbi and al (1994), Vechio and Collins (1988), Kachi and al (2006, 2014).

In the present paper, the model developed by Kachi and al (2006), using equilibrium and compatibility equations, for determine the global behavior of reinforced concrete beams, subjected to shear, binding moment and axial load, is used for analyze the global behavior of the steel fiber reinforced concrete beams, in order to simulate the influence of the steel fibers on the reinforced concrete beams behavior subjected to shear loading.

This method is based on the resolving a complex equation system formed by equilibrium equations linking the principal stresses to the longitudinal and transversal stresses, and the compatibility equations linking principal strains to the longitudinal and transversal strains and the stress strain behaviour of materials in compression and traction. In the case of steel fiber reinforced concrete we have adopted a stress-strain low developed by Bouafia and al (2002) for the tensile behaviour and the Sargin low for concrete on the compression with adapted parameters proposed by Bouafia and al (2002) for the steel fiber reinforced concrete.

Some sections are performed. The fiber percentage and fiber length are varied in order to simulate the influence of each parameter on the shear modulus and global behaviour until fracture of the steel fiber reinforced concrete.
Nomenclature

\( f_{c0} \) : Compressive strength of unconfined concrete.
\( E_{ct} \) : Fiber reinforced concrete modulus.
\( \Delta \delta_{u} \) : Increments of axial strain.
\( \Delta \delta_{w} \) : Increments of curvature.
\( \varepsilon_{c0} \) : Concrete strain corresponding to the peak stress.
\( \Delta \gamma_{moy} \) : Increments of the mean distortion.
\( \varepsilon_{f} \) : Normal strains.
\( \varepsilon_{r} \) : Fiber reinforced concrete failure strain.
\( \Delta M \) : Increments of the binding moment.
\( \sigma_{c1} \) : Principal tensile stress.
\( \sigma_{c2} \) : Principal compressive stress.
\( \sigma_{1} \) : Principal tensile strain.
\( \sigma_{2} \) : Principal compressive strain.
\( \Delta \tau_{1} \) : Increments of the shear stress.
\( \varepsilon_{u} \) : Fiber reinforced concrete ultimate strain.
\( \varepsilon_{ft} \) : Fiber reinforced concrete cracking strain.
\( \varepsilon_{c} \) : Nrmale strains.
\( \varepsilon_{f} \) : Concrete compressive strength.
\( E_{co} \) : Initial concrete Young modulus.
\( \varepsilon_{1} \) : Principal tensile strain.
\( \varepsilon_{2} \) : Principal compressive strain.
\( E_{ai} \) : Steel modulus.
\( \rho_{x} \) : Volumetric ration of ties on x direction.
\( \rho_{y} \) : Volumetric ration of ties on y direction.
\( \rho \) : Volumetric ratio of transversal reinforcement.
\( \rho_{a} \) : Ratio of transversal reinforcement.
\( b_{i} \) : Base width of the layer I.
\( h_{i} \) : High of the layer i.
\( \theta \) : Inclination of the compressive principal stress.
\( \delta_{u} \) : Increments of the mean distortion.
\( \delta_{w} \) : Increments of curvature.
\( \Delta \gamma_{moy} \) : Increments of the mean distortion.
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\( \varepsilon_{2} \) : Principal compressive strain.
\( E_{ai} \) : Steel modulus.
\( \rho \) : Volumetric ratio of transversal reinforcement.
\( \rho_{x} \) : Volumetric ratio of ties on x direction.

GENERAL HYPOTHESIS

The transverse section of the beam is decomposed into horizontal layers. The deformation of the section follows Bernoulli’s principle. The normal deformation of a section is given by:

\[ \varepsilon_{x}(y) = \delta_{u} + \delta_{w} \cdot y \]  

CONSTITUTIVE LAWS OF MATERIALS

Concrete constitutive low

For compressive concrete low we use the Sargin stress-strain relationship given in Annex 1 of BPEL rules (1999), which involves strength \( f_{c0} \) and the corresponding strain \( \varepsilon_{b0} \). In this case, the stresses are given by:

\[ \sigma = f_{c0} \frac{k_{b} \bar{\varepsilon} - (k_{b} - 1) \bar{\varepsilon}^{2}}{1 + (k_{b} - 2) \bar{\varepsilon} - k_{b} \bar{\varepsilon}^{2}} \]  

Where \( \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_{b0}} \) and \( k_{b} = \frac{E_{co} \varepsilon_{b0}}{f_{c0}} \)  

For a fiber reinforced concrete, it generally takes \( k'_{b} = k_{b} - 0.9 \) Bouafia et al (2002).

This relationship is transformed for describing the concrete behavior in the principal direction \( d_{2} \). The maximal concrete strain is changed according to the proposed by Vecchio and Collins (1988) as a function of the principal tensile strains \( \varepsilon_{1} \), but merely the \( f_{c2} / f_{c} \) report:

\[ \frac{f_{c2}}{f_{c}} = \frac{1}{0.8 - 0.34 \frac{\varepsilon_{1}}{\varepsilon_{c0}}}, \quad \text{With} \quad 0.7 \leq \frac{f_{c2}}{f_{c}} \leq 1 \]  

The concrete tension stress–strain relationship in the principal direction \( d_{1} \) assumed is the Belarbi and Hsu relationship [4]:

\[ \begin{align*} 
\sigma &= E_{c0} \cdot \varepsilon, & \text{Where} & \quad |\varepsilon| < \varepsilon_{ct} \\
\sigma_{t} &= f_{t} \left( \frac{\varepsilon_{b}}{\varepsilon_{t}} \right)^{0.4}, & \text{Where} & \quad |\varepsilon_{ct}| < \varepsilon < |\varepsilon_{rt}| \\
\sigma_{t} &= 0, & \text{Where} & \quad |\varepsilon| > \varepsilon_{rt} 
\end{align*} \]  

-36-
**Steel fiber reinforced Concrete tensile constitutive low**

In the case of steel fiber reinforced concrete we have adopted a stress-strain low developed by Bouafia and al (2002).

\[
\sigma = \begin{cases} 
E_{ct} \varepsilon & \text{if } 0 \leq \varepsilon \leq \varepsilon_{U} \\
\sigma_{uc} \left[ \sigma_{uc} \cdot f_{lt} \left( \frac{\varepsilon_{lt}}{\varepsilon_{lt} - \varepsilon_{u}} \right)^6 \right] & \text{if } \varepsilon_{lt} \leq \varepsilon \leq \varepsilon_{u} \\
\sigma_{uc} \left[ 1 - \left( \frac{\varepsilon_{lt}}{\varepsilon_{lt} - \varepsilon_{u}} \right)^6 \right] & \text{if } \varepsilon_{u} \leq \varepsilon \leq \varepsilon_{r}
\end{cases}
\]

\text{(5)}

**Reinforcement constitutive law**

The behavior of reinforcing steel is characterized by types allowed by the 1999 rules BAEL relationships:

**EQUILIBRIUM OF THE SECTION**

The normal deformation of a section is given by \( \varepsilon_n(y) = \delta u + \delta w.y \) and its transverse deformation (or distortion) is defined by \( \gamma_{moy} \). The contribution of this deformation is taken into account by a non linear approach.

The deformation increase vector of the section is given by (Eq.6):

\[
\Delta \tilde{\delta} = \begin{bmatrix} \Delta \delta u \\ \Delta \delta w \\ \Delta \gamma_{moy} \end{bmatrix}
\]

\text{(6)}

The equilibrium equation of the section in the intrinsic system is given by:

\[
\Delta \tilde{F}_s + \Delta \tilde{P}_s = K_s \Delta \tilde{\delta}
\]

\text{(7)}

This equation is solved by an iterative method. Its solution may be written as (Eq.8):

\[
\Delta \tilde{\delta} = K_s^{-1} \left( \Delta \tilde{F}_s + \Delta \tilde{P}_s \right)
\]

\text{(8)}

The expression of the stiffness matrix \([K_s]\) of the section is then written as (Eq.9):
In (Eq. 9), \( \Delta N \), the axial load increase; \( \Delta M \), the bending moment increase; and \( \Delta V \), the shear force increase.

In the expression adopted for \([K_s]\) the shearing modulus \( G = f(E) \) is not assumed to be constant as in linear elasticity but is taken as a function of the shear variation. Indeed, the coupling terms between \( \Delta N \) and \( \Delta \delta w \) and those between \( \Delta M \) and \( \Delta \delta u \) cannot be ignored in flexural analysis. But we assume that the coupling terms between \( \Delta N \) and \( \Delta \gamma_{\text{moy}} \) (or \( \Delta V \) and \( \Delta \gamma_{\text{moy}} \)), and between \( \Delta M \) and \( \Delta \gamma_{\text{moy}} \) (or \( \Delta V \) and \( \Delta \delta w \)), are negligible. The couplings mentioned in the introduction between the strength due to \( V \) and those due to \( (N,M) \) may be taken into account by the dependence of \( \Delta \gamma_{\text{moy}} \) on stresses and strains due to \( N \) and \( M \). The flexural terms \((\Delta N/\Delta \delta u, \Delta N/\Delta \delta w, \Delta M/\Delta \delta u \) and \( \Delta M/\Delta \delta w \)) are evaluated by classical methods. For the evaluation of the shear terms \((\Delta V/\Delta \gamma_{\text{moy}})\), a method is proposed. In this way, the problem is reduced to the evaluation of the middle distortion \( \gamma_{\text{moy}} \) of transverse sections submitted to combined bending moment, normal load and shear. In this field, using the virtual work theorem, we use the equality of the external shear work to the internal shear work evaluated in each layer:

\[
\Delta W_e = \sum_i \Delta W_i
\]

In this expression, \( \Delta W_e \) is the external shear work, and \( \Delta W_i \) is the work of internal shear evaluated in each layer, the expression of which may be written as:

\[
\Delta W_e = \Delta V \cdot \Delta \gamma_{\text{moy}} \quad \text{and} \quad \Delta W_i = b_i \cdot h_i \cdot \Delta \tau_i \cdot \Delta \gamma_i
\]

Finally the mean distortion of the section is given by (Eq. 10):

\[
\Delta \gamma_{\text{moy}} = \sum_i \frac{\Delta \tau_i \cdot b_i \cdot h_i \cdot \Delta \gamma_i}{\Delta V}
\]

Where \( \Delta \tau_i \) is the shear stress increase evaluated at the layer \( i \). It is determined by analysing for each section \( S_1 \) of the beam loaded by an axial load \( N_1 \), bending moment \( M_1 \) and shear \( V_1 \), a second section \( S_2 \) of the beam, loaded by \(( N_2 = N_1, M_2 = M_1 - s V_1, V_2 = V_1)\), located at a small distance \( s \) from the first. Both sections are analyzed for the same shear stress distribution, satisfying sectional equilibrium in each case. The value of the shear stress in the layer \( i \) is obtained by solving the free-body equilibrium of the layer \( i \) between the two sections. The local distortion increase \( \Delta \gamma_i \) is determined at each layer by solving a complex system of equations, namely equilibrium equations, compatibility equations and constitutive laws of the materials. It is worth to note that one uses an iterative technique to find the angle of inclination of the diagonal compression based on stress and strain Mohr’s circles properties. A more detailed description of the solution procedure for the determination of these parameters may be found in Kachi et al. (2006).

**Equilibrium equations**

The three equilibrium equations of the truss model show that the stresses in the concrete satisfy Mohr’s stress circle. Assuming that the steel bars can resist only axial stresses, and then the superposition of the concrete stress and the steel stress are given by:
\[
\begin{align*}
\sigma_x &= \sigma_{c1} \sin^2 \theta_c + \sigma_{c2} \cos^2 \theta_c + \rho_x f_{ex} \quad (11) \\
\sigma_y &= \sigma_{c1} \cos^2 \theta_c + \sigma_{c2} \sin^2 \theta_c + \rho_y f_{ey} \quad (12) \\
\tau &= (\sigma_{c1} - \sigma_{c2}) \sin \theta_c \cos \theta_c \quad (13)
\end{align*}
\]

We assume the coincidence of the principle direction of the stresses and the principle direction of the strains \( \theta_c = \theta_c. \)

**Compatibility equations**

Having assumed that the reinforcement is anchored to the concrete, any change in the concrete strain will be accompanied by an equal change in steel strain. If the three strains components \( \varepsilon_1, \varepsilon_2 \) and \( \gamma_{xy} \) are known, then the strain in any direction can be found from the Mohr’s circle geometry as:

\[
\begin{align*}
\varepsilon_x &= \varepsilon_1 \sin^2 \theta + \varepsilon_2 \cos^2 \theta \quad (14) \\
\varepsilon_y &= \varepsilon_1 \cos^2 \theta + \varepsilon_2 \sin^2 \theta \quad (15) \\
\gamma_{xy} &= 2 (\varepsilon_1 - \varepsilon_2) \sin \theta \cos \theta \quad (16)
\end{align*}
\]

The direction D2 is also the cracks, at least at first cracking. When the stresses grow, this direction may deviate from the initial cracks, which is physically admissible due to shear stresses transmitted by engagement lip cracks. This also corresponds to the concept of variable inclination of Eurocode 2 (1992), section 4.3.2.4.4 rods Shear - Method rods of variable inclination.

**RESULTS AND CONCLUSIONS**

The results from the calculated sections of two reinforced concrete beams are shown in fig 2 and fig 3. Geometrical and mechanical characteristics can be found in Vechio and al (1986). In the case of steel fiber reinforced concrete, we have taken account variably percentage and length of the steel fibers. The shear load-shear strains curve has three different regions with different shear stiffness. The first region is elastic linear and they are no influence of the steel fibers. The second region is between concrete cracking end reinforcement yielding, in this region, they are equally no influence of the steel fibers. The third region is located after reinforcement yielding; in this region we can see that the steel fibers increase the shear strength of the section and its shear stiffness.

![Shear force vs Shear strains](image1)

![Shear force vs Shear strains](image2)

Fig. 2 - Increase of the shear load with increase of the shear strains
This study shows that there is no influence of the steel fibers on the shear stiffness before yielding of the reinforcement. After reinforcement yielding we can see then they is differences on the shear behavior, Steel fibers increase in this case shear stiffness and shear strength. Further this model should be performed in order to analyze other mechanical properties interaction, such bending and axial load and shear interaction.

REFERENCES


