PROCEDURE FOR HYGROSCOPIC ANALYSIS OF A COLD CHAMBER ENVELOPE

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ABSTRACT
The diffusion of water vapour in the atmospheric air through the elements of the envelope of a cold room, caused by the pressure gradient between the external environment and interior, is inevitable in most situations, it should be properly studied in the design phase, otherwise it can be a serious problem in the camera's use phase. In fact, if the conditions in the interior of an envelope element are such as to enable the vapour freezing of the migrant water, the increase in volume from the formation of ice causes the deformation of this element with very serious consequences, which can go up to its partial or total destruction. In this scenario, readily note the importance of vapour barriers associated with a properly designed insulation, tend to reduce not only the amount of water diffused, but also prevent the achievement of the conditions for freeze inside the engaging elements. The purpose of this work is to formulate the procedure for design of vapour barriers connected with the design of optimized thermal insulation, and then apply it to a cold chamber located in Portugal.

Keywords: Ice formation, Fick’s Law, vapour barriers, cold chamber.

INTRODUCTION
The IIR (International Institute of Refrigeration, 2015) estimates that there are operating throughout the world, about 3000 million refrigeration systems, which represents about 17% of the electricity used. So, besides the improvement in all equipment’s of these systems in order to achieve better efficiencies, there has to be special attention in the envelope of the cold rooms. Therein occurs the heat and water vapor transfer between the inside and the outside environment.

The mechanism underlying strength to the vapor diffusion through the successive layers of materials forming the envelope element is the gradient of partial pressures of the vapor between the outside and inside air, while the temperature gradient causes the energy transfer. The interdependence of temperature and pressure studied in thermodynamics (Afonso, 2013) allow to state that the regions of higher temperatures are also of higher pressures. Therefore, given the interdependence between pressure and temperature, mass and heat transfer processes are intrinsically related and must be analyzed simultaneously.

The way to avoid / minimize the vapor condensation inside the structure of the envelope, is the placement of vapor barriers in the warmer side of the insulation. There are two types vapor barrier for their ability to prevent vapor migration: total limiting barriers and partial vapor migration. The first is of metallic or plastic type, whereas the second are essentially of the bituminous type. In this work, only the second type will be analyzed.
The aim of this study is to calculate the vapor mass flowrate and heat transfer through the successive elements of the envelope of a cold chamber located in Portugal. Then, with the results obtained, analyze if there is vapor condensation inside the envelope (with all the inherent risks) and finally chose the appropriate vapor barrier.

THEORETICAL FORMULATION

The study of moisture transfer can be done from the Navier-Stokes equation, where it follows the Fick's law, which is the basis for mathematical models of thermodiffusion (Incropera, 1990):

\[ \hat{g} = -k_{\text{diff}} \cdot \text{grad}(cp) \]  

From this equation it is possible to deduce the general equation of vapor diffusion through porous materials, known as Fick's law:

\[ \frac{\partial(cp)}{\partial t} = \text{div} [k_{\text{diff}} \cdot \text{grad}(cp)] + I \]  

Considering a typical situation, which is a homogeneous non-hygroscopic wall with flat and parallel internal surfaces and without internal flow production, the concentration can be obtained from the following equation:

\[ c = \frac{m_d}{m_d + m_D} = \frac{\rho_d}{\rho_d + \rho_D} = \frac{\rho_d}{\rho} \]  

By making use of the Clapeyron equation (Çengel, 1989) for steam, it follows that:

\[ p v_d = \frac{p}{\rho_d} = ZRT \]  

From equations 3 and 4:

\[ cp = \frac{1}{R \ ZT} \]  

So, equation 2 can then be rewritten as:

\[ \frac{\partial \left( \frac{p}{ZT} \right)}{\partial t} = \text{div} \left[ k_{\text{diff}} \cdot \text{grad} \left( \frac{p}{ZT} \right) \right] \]  

Assuming steady state with a constant diffusion coefficient, which is acceptable in most cases of vapor diffusion through the elements of the envelope, equation (6) is reduced to:

\[ \nabla^2 \left( \frac{p}{ZT} \right) = 0 \]  

On the other hand, if the flow through one envelope element is unidirectional, which is an acceptable simplification for the current zones of the element, and assuming that the steam has a perfect gas behavior (Z = 1) it is finally obtained:

\[ \frac{\partial^2 \left( \frac{p}{T} \right)}{\partial x^2} = 0 \]  

To better understand the applicability of these equations, consider the extreme case of vapor diffusion through a wall in isothermal regime. If the wall is at "T" temperature, has a thickness "e", and is subject to an inner partial pressure "p_i" and external "p_e", the integration
of equation (8) determines the distribution law the partial pressure of the vapor inside the wall. This law is translated by equation (9) and is illustrated in Figure 1a).

\[ p_i(x) = (p_e - p_i) \frac{x}{e} + p_i \]  

Therefore, in permanent and monodirectional regime, the pressure evolves linearly inside a solid component of the envelope. Note that the saturation pressure of the migrant steam is constant because the temperature does not vary, and has always been considered superior to the partial vapor pressure over the element, which does not always happen.

From this analysis, the Fick's law for porous materials can be written as follows:

\[ \dot{g} = -\Pi \frac{dp}{dx} \]  

Figure 1b) illustrates this law, which is valid for vapor diffusion conditions stated above.

\[ \text{Fig. 1 - a) Isothermal vapor diffusion in steady state; b) Linear variation of pressure in the interior of a solid element.} \]

However, it is not real the equality between the inside and outside temperatures of the envelope, because it is subjected to temperature gradients. In this case, the saturation pressure varies from point to point through the thickness of the element. So:

- If the line of partial pressures \((p_e - p_i)\) does not intercept the saturation pressure curve (as shown in Figure 2a), there will be no condensation in the interior of the element.
- If, however, the two curves intersect (Figure 2b), condensation will take place.

Making use of Fick's law (Eckert, 1992) it can be determined the amount of water that deposits inside an envelope element. Given that the condensate flow is equal to the difference between the input and output flows on the surfaces \(x_1\) and \(x_2\) (see figure 2b), it is obtained:

\[ \dot{g} = \Pi \left[ \left( \frac{dp}{dx} \right)_{x=x_1} - \left( \frac{dp}{dx} \right)_{x=x_2} \right] \]  

Generally, the envelope elements of refrigeration chambers are not homogeneous, but rather composed of layers of materials that can be considered homogeneous, each with its conductivity and permeability. Thus, if \(e_1, e_2, ..., e_n\) and \(\Pi_1, \Pi_2, ..., \Pi_n\) are respectively the thicknesses and permeability’s of the different layers, the diffused steam flow in steady state is obtained by integrating equation 10 and will be equal to:

\[ \dot{g} = \Pi_1 \frac{p_e - p_2}{e_1} = \Pi_2 \frac{p_2 - p_3}{e_2} = ... = \Pi_n \frac{p_n - p_{n-1}}{e_n} = \sum_{i=1}^{n} \left( \Pi_i \frac{\Delta p}{e_i} \right) \]  

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Defining vapor diffusion resistance of a given material "i", the relationship between the thickness and permeability is:

\[ R_{v,i} = \frac{e_i}{\Pi_i} \]  

(13)

So, the total resistance of the element envelope to vapor diffusion can then be defined as:

\[ R_{v,eq} = \sum_{i=1}^{n} R_{v,i} = \sum_{i=1}^{n} \frac{e_i}{\Pi_i} \]  

(14)

and vapor flow per unit area is determined by:

\[ \dot{g} = \frac{\Delta p}{R_{v,eq}} = \frac{p_e - p_i}{R_{v,eq}} \]  

(15)

![Fig. 2 - Vapor diffusion without / with condensation in an element subject to a temperature gradient.](image)

As can be seen from figure 2, the saturation pressure of the water vapour varies inside each element of the envelope as a function of the temperature in it. Thus, priority to any calculation, it is necessary to know in each section of the element what is the prevailing temperature. This can be done through the heat transfer analysis of the wall, which is composed of several elements, as shown in Fig.3.

![Fig. 3 - Heat transfer in a composite wall.](image)

The heat transfer through the wall is given by (Çengel, 1998, 2001):

\[ \dot{q} = \frac{T_{Proj, Ext} - T_{Proj, Int}}{R_{t, Ext-Int}} \]  

(16)

where the total thermic resistance between the outside and inside environment is:
The temperature in each section (x) is:

\[ T_x = T_{\text{proj, Ext}} - q \cdot R_{t, \text{Ext} - x} \]  \hspace{1cm} (18)

where:

\[ R_{t, \text{Ext} - x} = \frac{1}{\alpha_{\text{Ext}}} + \sum_{i=1}^{x} \left( \frac{e}{\lambda_{i}} \right) \]  \hspace{1cm} (19)

The knowledge of the values of the temperatures at the interfaces allows the determination the partial pressure of vapor, which is used in the study of vapor diffusion through the element, Fig. 4.

Fig. 4 - Vapor diffusion in a composite element

So, the partial pressure of vapor at the interface 'x' \( (p_x) \), is similar to that indicated for the temperature (equation 18) with the expression:

\[ p_x = p_{\text{Ext}} - \dot{g} \cdot R_{v, \text{Ext} - x} \]  \hspace{1cm} (20)

where:

\[ R_{v, \text{Ext} - x} = \sum_{i=1}^{x} R_{v, i} = \sum_{i=1}^{x} \left( \frac{e}{\Pi L_{i}} \right) \]  \hspace{1cm} (21)

Despite the implementation of good vapor barriers, it is virtually impossible to prevent small quantities of water vapor entering the warm side of the insulation (unless, as stated above, with metal or plastic barriers). It is therefore necessary that the water vapor that passes through the vapor barrier can be spread over the insulation and out of the cold side into the cooling chamber, which eventually will condense in the evaporators. Thus, it is important that the coating of the internal elements (inside the chamber) is very permeable because if it is too tight, the greater must be the sealing of the vapor barrier, in order to avoid, in any event, that enters more water vapor to the isolation than what goes into the cooling chamber.

With the calculated partial pressures of the vapor and, as said, if \( p_v(T_x) < p_{v, \text{sat}}(T_x) \), there is no occurrence or non-condensing a given surface. If not, it is necessary to calculate the partial pressures of the element where condensation occurs, and with them the flow of vapor that migrates from the outside, and the flow of vapor which reaches the interior of the cooling chamber. The difference between them is the condensed vapor flow. Consider figure 2, and suppose that the condensation occurs at the interface "x". So, this vapor is in the saturated
state at the surface which means that the pressures are equal: \( p_v(T) = \phi \cdot p_{v, sat}(T) \). In this situation, the vapour flow into the element is:

\[
\dot{g}_{\text{ext}} = \frac{p_1 - p_x}{R_{v, 1-x}}
\]

(22)

The vapor flow that reaches the inner surface of the refrigeration chamber is:

\[
\dot{g}_{\text{Int}} = \frac{p_x - p_{n+1}}{R_{v, x-(n+1)}}
\]

(23)

The difference between these vapor flows (equation 22 and 23) is the vapor that has been condensate:

\[
\dot{g}_{\text{Cond}} = \dot{g}_{\text{Ext}} - \dot{g}_{\text{Int}}
\]

(24)

APPLICATION OF THE METHODOLOGY TO COLD CHAMBER IN PORTUGAL

The above methodology was tested in a cold chamber located in Portugal. Two models were analysed: one without vapour barrier, Figure 5a) and another with vapour barrier, Figure 5b). The wall structure model is diagrammed in the same figure, where it is observed the identification of the construction materials (“1” - “9”) and the interfaces between the layers of materials (“a” - “k”). The thermal and hygroscopic properties of these materials are registered in Table 1.

Models without / with vapor barrier serve the objectives of the analysis. Both models have thermal insulation with a thickness equal to the optimal thickness (Afonso, 2013)

Simulation were done for the three most common types of thermal insulation (ERP rigid polyurethane foam; PLE- expanded polyethylene and black CRT- agglomerated cork) and limiting vapor barriers (bituminous surface, aluminium foil with \( e > 40 \mu m \) and polyethylene with \( e = 100 \mu m \).
Table 1 - Characteristics of building materials of walls (ASHRAE, 1994, 1997).

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductibility $\lambda$ [W/(m.K)(^{-1})]</th>
<th>Thickness e [cm]</th>
<th>Permeability $\Pi$ [g/(m.h.mmHg)(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Cement grout</td>
<td>1.105</td>
<td>1.5</td>
<td>7.00E-04</td>
</tr>
<tr>
<td>2 Brick wall (22 cm)</td>
<td>0.480</td>
<td>22.0</td>
<td>5.00E-03</td>
</tr>
<tr>
<td>3 Primary bituminous</td>
<td>0.230</td>
<td>1.0</td>
<td>2.20E-03</td>
</tr>
<tr>
<td>4 Asphaltic bitumen</td>
<td>0.230</td>
<td>0.5</td>
<td>2.20E-03</td>
</tr>
<tr>
<td>5 Vapor barrier</td>
<td></td>
<td></td>
<td>1.00E-03</td>
</tr>
<tr>
<td>6 Asphaltic bitumen</td>
<td>0.230</td>
<td>0.5</td>
<td>2.20E-03</td>
</tr>
<tr>
<td>7 Thermal insulation</td>
<td></td>
<td>Optimal</td>
<td>5.00E-03</td>
</tr>
<tr>
<td>8 Asphaltic bitumen</td>
<td>0.230</td>
<td>1.5</td>
<td>2.20E-03</td>
</tr>
<tr>
<td>9 Reinforced plastering</td>
<td>1.150</td>
<td>2.0</td>
<td>6.50E-04</td>
</tr>
</tbody>
</table>

RESULTS

The calculations were carried out using the models described in the previous section to demands corresponding to three different temperatures within the chamber at -10, -20 and -30ºC. Regarding the outside temperature, although there were analyzed the hourly temperatures for one day of summer and winter, it was only considered here the maximum temperature in summer, 37ºC, being the critical temperature.

In all simulated models without a vapor barrier it has been found that there is a possibility of condensation (unless a limiting vapor barrier is placed). Fig. 6 shows the temperature decay along the envelope and the evolution of the saturation and partial pressure of the vapor along the wall. These results were obtained when the thermal insulation used was the expanded polyethylene (for others insulations the situation is identical).

As noted, in the interfaces "i-k" the partial pressure of saturated vapor is below the partial pressure, indicating that there is condensation. The vapour flow that is condensed was calculated with the equation 24 and are indicated in Table 2. It follows therefore that the constructive solution using the optimal thickness of insulation is not enough to prevent condensation inside the element, requiring the placement of a limiting vapor barrier to prevent it.
Table 2 - Results obtained with/without vapour barrier.

<table>
<thead>
<tr>
<th>T&lt;sub&gt;int&lt;/sub&gt; [°C]</th>
<th>Thermal insulation</th>
<th>Without vapour barrier</th>
<th>With vapour barrier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>e&lt;sub&gt;opt&lt;/sub&gt; [cm]</td>
<td>Heat flux [W]</td>
</tr>
<tr>
<td>ERP -10</td>
<td>19.9</td>
<td>6.4</td>
<td>26994</td>
</tr>
<tr>
<td></td>
<td>PLE 21.0</td>
<td>7.0</td>
<td>20816</td>
</tr>
<tr>
<td></td>
<td>CRT 22.3</td>
<td>8.7</td>
<td>24267</td>
</tr>
<tr>
<td>ERP -20</td>
<td>25.5</td>
<td>6.2</td>
<td>27667</td>
</tr>
<tr>
<td></td>
<td>PLE 26.9</td>
<td>6.8</td>
<td>20280</td>
</tr>
<tr>
<td></td>
<td>CRT 28.7</td>
<td>8.5</td>
<td>24307</td>
</tr>
<tr>
<td>ERP -30</td>
<td>31.2</td>
<td>6.0</td>
<td>27209</td>
</tr>
<tr>
<td></td>
<td>PLE 33.0</td>
<td>6.6</td>
<td>19056</td>
</tr>
<tr>
<td></td>
<td>CRT 35.3</td>
<td>8.3</td>
<td>23404</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERP -30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that in the analysed situations, it is not possible to avoid condensation using only the variation of the thickness of the thermal insulation because it would be required for unsustainable thickness values (of the order of 3m in thickness).

Knowing this, the main objective of the hygroscopic analysis is sizing of the vapor barrier in order to prevent this occurrence. Following the procedure already described, the results obtained for the three types of barriers can be seen in table 2.

For the temperature inside the chamber equal to -10 °C (for other analysed temperatures the relationship is identical), Fig. 7 shows the magnitude of the thickness of the vapor barriers depending on the type of insulation used. It is clearly seen that the aluminium foil (e > 40 µm)
requires a substantially lower thickness when compared with the others, as would be expected due to its lower permeability.

![Comparison between vapour barriers](image)

**Fig. 7 - Simulation results for the three types of vapor barriers (T_{int} = -10^\circ C).**

It is possible to arrive at the same conclusion by analyzing Fig. 8, which shows the evolution of thicknesses for each of the tested vapor barriers, depending on the type of isolation and temperature within the chamber.

![Vapor Barrier Thickness](image)

**Fig. 8 - Variation of the vapor barrier thickness as a function of the type of insulation and internal temperature.**

In this figure, it is easily observed that the thinner insulation material for the three temperatures, corresponds to black cork agglomerate, whose optimum thickness is, however, superior to the other two insulating materials (see table 2).

**CONCLUSIONS**

The herein performed analysis shows that:

Only with a thermal insulation with minimized thickness is impossible to prevent condensation within the cold room envelope. Only with a non-technically feasible thicknesses would it be possible to avoid it.

Vapor barriers are mandatory, and the procedure for its design was outlined in the previous sections.

The required thickness of the vapor barrier is relatively small, and the most appropriate isolation (maintaining the optimal thickness) is the black cork agglomerate. However, one must note that in order to establish a definitive conclusion, an economic analysis is needed that is not within the scope of this work.
REFERENCES


