A METHOD FOR HEAT TRANSFER CALCULATION IN FOUR STROKE SPARK IGNITION INTERNAL COMBUSTION ENGINES

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ABSTRACT
This work presents a method for calculation of heat transfer between the combustion chamber walls and the gases in the cylinder for the entire cycle of four-stroke spark ignition internal combustion engines. This method is used in a computer program to model the thermodynamic cycle of four-stroke spark ignition internal combustion engines. The method considers only heat transfer by convection between the combustion chamber walls and the gases in the cylinder. It considers the combustion chamber walls divided in five zones with different surface temperatures and an instantaneous average temperature of the gases. The results of the program are presented and the importance of heat transfer on engine performance is discussed.

Keywords: heat transfer, four-stroke, spark ignition, internal combustion engine.

INTRODUCTION
Many methods have been proposed for the heat transfer calculation in four-stroke spark ignition internal combustion engines. The most simple use time-averaged correlations for the heat flux such and the most sophisticated use instantaneous local calculations of the heat flux such as the method presented by (Esfahanian et al., 2006). The method presented here is used in a zero dimensional thermodynamic model of a four-stroke spark ignition engine cycle (Carvalheira, 2016). The method considers a correlation for instantaneous spatial averaged convective heat transfer coefficient, it considers instantaneous average velocities inside the cylinders according to the engine cycle phase, it considers the combustion chamber walls divided in five zones with different surface temperatures and an instantaneous average temperature of the gases in the cylinder. This method is a modification of the method presented by (Annand, 1963) and intends to reduce some of its limitations.

HEAT TRANSFER
The heat transfer rate from the cylinder wall to the gases in the cylinder, \( \dot{Q} \), is given by Eq. (1) where \( h_c \) is the convection heat transfer coefficient from the cylinder wall to the gases, \( A_{wi} \) is the area of surface \( i \) in the cylinder wall, \( T_g \) is the average temperature of the gas inside the cylinder and \( T_{wi} \) is the temperature of the surface \( i \) in the cylinder wall. The subscript \( i \) is an integer to identify a cylinder wall surface and changes from 1 to 5. 1 is for the cylinder head, 2 is for the intake valves, 3 is for the exhaust valves, 4 is for the lateral wall of the cylinder and 5 is for the piston crown.
The convective heat transfer coefficient is calculated from an equation of the type of Eq. (2) where $\text{Nu}$ is the Nusselt number, $\text{Re}$ is the Reynolds number, $\text{Pr}$ is the Prandtl, and $c$, $m$ and $n$ are constants, as recommended by (Heywood, 1988).

$$\text{Nu} = c \text{ Re}^m \text{Pr}^n$$

The equation used to calculate the convective heat transfer coefficient was derived from the Eq. (3) proposed by (Annand, 1963) which is an equation of the type of Eq. (2). In Eq. (3) $B$ is the cylinder bore, $k$ is the thermal conductivity of the gases in the cylinder, $\rho$ is the density of the gases in the cylinder, $\bar{S}_p$ is the average piston speed, $\mu$ is the viscosity of the gases in the cylinder and $a$ and $b$ are constants. The values used typically for the constants in Annand’s law are $0.35 \leq a \leq 0.80$ and $b = 0.70$ where $a$ depends on the intensity of charge motion and engine design and increases with increasing intensity of charge motion (Heywood, 1988).

$$\left(\frac{h_c B}{k}\right) = a \left(\frac{\rho \bar{S}_p B}{\mu}\right)^b$$

The Prandtl number doesn’t change too much for the gases in the cylinder for the temperature range typical of the gases in the cylinder of an internal combustion engine. Considering the Prandtl number constant Eq. (3) is as an equation of the type of Eq. (2) if we consider that Eq. (4) is valid.

$$a = c \text{ Pr}^n$$

We made two changes to Annand’s correlation. The first change we made to Annand’s correlation was to consider a different velocity for the definition of the Reynolds number in Annand’s correlation. Instead of considering the average piston speed, $\bar{S}_p$, we considered the average velocity of the gas relative to the cylinder walls when both the intake valves and the exhaust valves are closed and the piston displaces from top dead center (TDC) to bottom dead center (BDC) or from BDC to TDC, $\bar{v}_g$. When the intake valves and exhaust valves are both closed there is no outflow or inflow to the cylinder and the average velocity of the gas in the cylinder relative to the cylinder wall is given by Eq. (5). Solving Eq. (5) for $\bar{S}_p$ we get Eq. (6). Substituting the value of $\bar{S}_p$ given by Eq. (6) in Annand’s correlation, Eq. (3), we got the Annand’s correlation given by Eq. (7) now expressed in terms of $\bar{v}_g$.

$$\bar{v}_g = \frac{1}{2} \bar{S}_p$$

$$\bar{S}_p = 2 \bar{v}_g$$

$$\left(\frac{h_c B}{k}\right) = a \left(\frac{2\rho \bar{v}_g B}{\mu}\right)^b$$

$$\bar{v}_g = \frac{1}{2} \bar{S}_p$$

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The second change we made to Annand’s correlation was to substitute \( v_g \) by the modulus of the instantaneous average velocity of the gas in the cylinder relative to the cylinder walls, \( v_g \). By doing this we can calculate an instantaneous heat transfer coefficient between the gases in the cylinder and the combustion chamber walls not only dependent on the instantaneous values of the gas density, viscosity and thermal conductivity but also on the instantaneous average velocity of the gas in the cylinder relative to the cylinder walls. By doing that Annand’s correlation takes the form of Eq. (8).

\[
\left( \frac{h_c B}{k} \right) = a \left( \frac{2 \rho |v_g| B}{\mu} \right)^b
\]  

Eq. (8) has two advantages relative to Eq. (3). The first advantage is that it allows the calculation of the effect of the instantaneous average velocity of the gas in the cylinder relative to the cylinder walls, caused by the instantaneous piston speed, on the instantaneous heat transfer coefficient. The second advantage is that it allows the calculation of the effect of the intake and exhaust flows on the average velocity of the gas relative to the cylinder walls during the intake and exhaust processes and on the instantaneous heat transfer coefficient. This advantage is particularly important for the calculation of the amount of air admitted in the cylinder on each cycle and in consequence in the calculation of the volumetric efficiency of naturally aspirated engines, due to the effect of heating of the charge by the combustion chamber walls during the intake process. Eq. (8) has one important disadvantage relative to Eq. (3). This disadvantage is that the convective heat transfer coefficient is equal to zero when \( v_g = 0 \). This can happen when the piston is on top dead center (TDC) and the intake and exhaust valves are both closed as it happens in the end of the compression stroke. To overcome this disadvantage if the calculated value of \( v_g \) goes below a certain threshold value we make \( v_g \) equal to this threshold value.

By convention the intake mass flow rate is positive if it is into the cylinder and is negative if it is out of the cylinder. If the intake mass flow rate is positive the velocity of the gas inside the cylinder due to the intake gas flow, \( v_{gc,\text{in}} \), is given by Eq. (9) where \( \dot{m}_l \) is the intake mass flow rate, \( \rho \) is the density of the gas inside the cylinder, \( A_l \) is the minimum flow area of one intake valve, \( A_c \) is the area of the cross section of the cylinder normal to the cylinder axis, \( n_{IV} \) is the number of intake valves, \( A_{IV} \) is the area of the intake valve head. \( A_c \) is given by Eq. (10) where \( B \) is the cylinder bore. \( A_{IV} \) is given by Eq. (11) where \( D_{IV} \) is the intake valve head diameter. \( v_{gc,\text{in}} \) is an average velocity of the gas inside the cylinder which is a weighted average of the velocity of the gas in the intake valve throat, in the cylinder below the intake valves and in the cylinder close to the piston crown.

\[
v_{gc,\text{in}} = \frac{1}{4} \left( \frac{\dot{m}_l}{\rho A_l} + 2 \frac{\dot{m}_l}{\rho (A_c - n_{IV} A_{IV})} + \frac{\dot{m}_l}{\rho A_c} \right)
\]  

\[A_c = \frac{\pi}{4} B^2\]  

\[A_{IV} = \frac{\pi}{4} D_{IV}^2\]
If the intake mass flow rate is negative the velocity of the gas inside the cylinder due to the intake gas flow, \( v_{gc,i,\text{out}} \), is given by Eq. (12) where \( \dot{m}_l \) is the intake mass flow rate, \( \rho \) is the density of the gas inside the cylinder and \( A_c \) is the area of the cross section of the cylinder normal to the cylinder axis.

\[
v_{gc,i,\text{out}} = \frac{\dot{m}_l}{\rho A_c}
\]

(12)

By convention the exhaust mass flow rate is positive if it is into the cylinder and is negative if it is out of the cylinder. If the exhaust mass flow rate is positive the velocity of the gas inside the cylinder due to the exhaust gas flow, \( v_{gc,e,\text{in}} \), is given by Eq. (13) where \( \dot{m}_E \) is the exhaust mass flow rate, \( \rho \) is the density of the gas inside the cylinder, \( A_E \) is the minimum flow area of one exhaust valve, \( A_c \) is the area of the cross section of the cylinder normal to the cylinder axis, \( n_{EV} \) is the number of exhaust valves, \( A_{EV} \) is the area of the exhaust valve head. \( A_c \) is given by Eq. (10) where \( B \) is the cylinder bore. \( A_{EV} \) is given by Eq. (14) where \( D_{EV} \) is the exhaust valve head diameter. \( v_{gc,e,\text{in}} \) is an average velocity of the gas inside the cylinder which is a weighted average of the velocity of the gas in the exhaust valve throat, in the cylinder below the exhaust valves and in the cylinder close to the piston crown.

\[
v_{gc,e,\text{in}} = \frac{1}{4} \left( \frac{\dot{m}_E}{\rho A_E} + 2 \frac{\dot{m}_E}{\rho (A_c - n_{EV} A_{EV})} + \frac{\dot{m}_E}{\rho A_c} \right)
\]

(13)

\[
A_{EV} = \frac{\pi}{4} D_{EV}^2
\]

(14)

If the exhaust mass flow rate is negative the velocity of the gas inside the cylinder due to the exhaust gas flow, \( v_{gc,e,\text{out}} \), is given by Eq. (15) where \( \dot{m}_E \) is the exhaust mass flow rate, \( \rho \) is the density of the gas inside the cylinder and \( A_c \) is the area of the cross section of the cylinder normal to the cylinder axis.

\[
v_{gc,e,\text{out}} = \frac{\dot{m}_E}{\rho A_c}
\]

(15)

If the axis of the cylinder intercepts and is normal to the axis of revolution of the crankshaft the distance between the axis of revolution of the crankshaft and the axis of the piston pin, \( s \), is given by Eq. (16) where \( a \) is the crank radius, \( \theta \) is the crank angle and \( l \) is the connecting rod length. The connecting rod length is the distance between the axis of the big end bearing and the axis of the small end bearing of the connecting rod.

\[
s = a \cos \theta + (l^2 - (a \sin \theta)^2)^{1/2}
\]

(16)

The instantaneous piston speed, \( S_p \), is given by Eq. (17).

\[
S_p = \frac{ds}{dt}
\]

(17)

The value of \( v_g \) is given by Eq. (18). By convention \( v_g \) is positive when the velocity of the gas is from the cylinder head to the crankshaft and is negative if it is from the crankshaft to
the cylinder head. \( S_p \) is positive when it is from the crankshaft to the cylinder head and is negative when it is from the cylinder head to the crankshaft.

\[
v_g = \frac{1}{2} (v_{gc,l,in} + v_{gc,l,out} + v_{gc,E,in} + v_{gc,E,out} - S_p)
\]

(18)

Applying Eq. (18) \( v_g \) is equal to zero if \( v_{gc,l,in}, v_{gc,l,out}, v_{gc,E,in}, v_{gc,E,out} \) and \( S_p \) are all equal to zero as it happens when the piston is at TDC in the end of the compression stroke when all intake valves and all exhaust valves are closed. By using Eq. (8) we get a zero convective heat transfer coefficient. This is not realistic because the average speed of the gas relative to the cylinder walls never gets zero in the cylinder. To solve this problem we consider a cut off velocity \( v_{g,co} \) below which the \( v_g \) never goes below. The value chosen for the cut off velocity is reasonable but arbitrary and is given by Eq. (19).

\[
v_{g,co} = \frac{1}{2} S_p
\]

(19)

The velocity of the gas to enter in Eq. (8) to calculate the convective heat transfer coefficient is given by Eq. (20).

If \( v_g > v_{g,co} \) then \( v_g = \frac{1}{2} (v_{gc,l,in} + v_{gc,l,out} + v_{gc,E,in} + v_{gc,E,out} - S_p) \) else \( v_g = v_{g,co} \)

(20)

The dynamic viscosity of the gases to be used in Eq. (8) was calculated using Eq. (21) for the air-fuel mixture where \( T \) is in K (Heywood, 1988) and Eq. (22) for the combustion products where \( \phi \) is the air-fuel mixture equivalence ratio (Heywood, 1988).

\[
\mu_{air} (\text{Pa} \cdot \text{s}) = 3.3 \times 10^{-7} \times T^{0.70}
\]

(21)

\[
\mu_p = \frac{\mu_{air}}{1 + 0.027\phi}
\]

(22)

The viscosity of the mixture of gases in the cylinder, \( \mu \), composed of fresh air-fuel mixture and combustion products was calculated using Eq. (23) where \( \tilde{x}_b \) is the molar fraction of combustion products in the mixture.

\[
\mu = \mu_{air}(1 - \tilde{x}_b) + \mu_p \tilde{x}_p
\]

(23)

The thermal conductivity of the unburned gases inside the cylinder, \( k_u \), is given by Eq. (24) where \( y_u \) is the coefficient of isentropic expansion of the mixture of unburned gases inside the cylinder and is given by Eq. (25) where \( c_{p,u} \) is the mass specific heat at constant pressure of the mixture of unburned gases inside the cylinder and \( c_{v,u} \) is the mass specific heat at constant volume of the mixture of unburned gases inside the cylinder. \( \mu_u \) is the dynamic viscosity of the mixture of unburned gases and is considered equal to \( \mu_{air} \) as given by Eq. (21) at the temperature of the mixture of unburned gases.

\[
k_u = \frac{9y_u - 5}{4} \mu_u c_{v,u}
\]

(24)
\( \gamma_u = \frac{c_{p,u}}{c_{v,u}} \)  \( (25) \)

c\(_{p,u}\) is given by Eq. (26) where \( \bar{c}_{p,u}\) is the molar specific heat at constant pressure of the mixture of unburned gases inside the cylinder and \( M_u \) is average molecular weight of the mixture of unburned gases inside the cylinder.

\[ c_{p,u} = \frac{\bar{c}_{p,u}}{M_u} \]  \( (26) \)

\( \bar{c}_{p,u}\) is given by Eq. (27) where \( \bar{x}_{i,u}\) is the mole fraction of component \( i\) of the mixture of unburned gases inside the cylinder and \( \bar{c}_{p,i}\) is the molar specific heat at constant pressure of component \( i\) of the mixture of unburned gases inside the cylinder.

\[ \bar{c}_{p,u} = \sum_{i=1}^{n} \bar{x}_{i,u} \bar{c}_{p,i} \]  \( (27) \)

\( M_u \) is given by Eq. (28) where \( \bar{x}_{i,u}\) is the mole fraction of component \( i\) of the mixture of unburned gases inside the cylinder and \( M_{i,u}\) is the molecular weight of component \( i\) of the mixture of unburned gases inside the cylinder.

\[ M_u = \sum_{i=1}^{n} \bar{x}_{i,u} M_{i,u} \]  \( (28) \)

c\(_{v,u}\) is given by Eq. (29) where \( R_u\) is the universal gas constant.

\[ c_{v,u} = \frac{\bar{c}_{p,u} - R_u}{M_u} \]  \( (29) \)

The thermal conductivity of the burned gases inside the cylinder, \( k_b\), is given by Eq. (30) where \( \gamma_b\) is the coefficient of isentropic expansion of the mixture of burned gases inside the cylinder and is given by Eq. (31) where \( c_{p,b}\) is the mass specific heat at constant pressure of the mixture of burned gases inside the cylinder and \( c_{v,b}\) is the mass specific heat at constant volume of the mixture of burned gases inside the cylinder. \( \mu_b\) is the dynamic viscosity of the mixture of burned gases inside the cylinder and is given by Eq. (22) at the temperature of the mixture of burned gases inside the cylinder.

\[ k_b = \frac{9\gamma_b - 5}{4} \mu_b c_{v,b} \]  \( (30) \)

\[ \gamma_b = \frac{c_{p,b}}{c_{v,b}} \]  \( (31) \)

c\(_{p,b}\) is given by Eq. (32) where \( \bar{c}_{p,b}\) is the molar specific heat at constant pressure of the mixture of burned gases inside the cylinder and \( M_b\) is average molecular weight of the mixture of burned gases inside the cylinder.
\[ c_{p,b} = \frac{\tilde{c}_{p,b}}{M_b} \]  

(32)

\( \tilde{c}_{p,b} \) is given by Eq. (33) where \( \tilde{x}_{i,b} \) is the mole fraction of component \( i \) of the mixture of burned gases inside the cylinder and \( \tilde{c}_{p,i} \) is the molar specific heat at constant pressure of component \( i \) of the mixture of burned gases inside the cylinder.

\[ \tilde{c}_{p,b} = \sum_{i=1}^{n} \tilde{x}_{i,b} \tilde{c}_{p,i} \]  

(33)

\( M_b \) is given by Eq. (34) where \( \tilde{x}_{i,b} \) is the mole fraction of component \( i \) of the mixture of burned gases inside the cylinder and \( M_{i,b} \) is the molecular weight of component \( i \) of the mixture of burned gases inside the cylinder.

\[ M_b = \sum_{i=1}^{n} \tilde{x}_{i,b} M_{i,b} \]  

(34)

\( c_{v,b} \) is given by Eq. (35) where \( R_u \) is the universal gas constant.

\[ c_{v,b} = \frac{\tilde{c}_{p,b} - R_u}{M_b} \]  

(35)

The thermal conductivity of the mixture gases in the cylinder, \( k \), composed of fresh air-fuel mixture and combustion products is calculated using Eq. (36) where \( \tilde{x}_b \) is the mole fraction of combustion products in the mixture, \( k_u \) is the thermal conductivity of the unburned gases and \( k_b \) is the thermal conductivity of the burned gases.

\[ k = k_u (1 - \tilde{x}_b) + k_b \tilde{x}_b \]  

(36)

The values of constants \( a \) and \( b \) used in Eq. (8) in this work are presented in Table 1.

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<td></td>
<td></td>
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<tr>
<td>0.64</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 1 - Values of \( a \) and \( b \) used in Eq. (8) in this work.

AREAS OF SURFACES

The area of the intake valves \( A_{IV} \) considered for heat transfer from cylinder walls to the gases in the cylinder is given by Eq. (37) where \( n_{IV} \) is the number of intake valves and \( D_{IV} \) is the diameter of the head of each intake valve.

\[ A_{IV} = n_{IV} \frac{\pi}{4} D_{IV}^2 \]  

(37)
The area of the exhaust valves \( A_{EV} \) considered for heat transfer from cylinder walls to the gases in the cylinder is given by Eq. (38) where \( n_{EV} \) is the number of exhaust valves and \( D_{EV} \) is the diameter of the head of each exhaust valve,

\[
A_{EV} = n_{EV} \frac{\pi}{4} D_{EV}^2
\]  

(38)

The area of the piston crown \( A_{p} \) considered for heat transfer from cylinder walls to the gases in the cylinder is given by Eq. (39) where \( k_{p} \) is the ratio of the area of the piston crown to the cylinder cross section area normal to the cylinder axis and \( B \) is the cylinder bore,

\[
A_{p} = k_{p} \frac{\pi}{4} B^2
\]  

(39)

The area of the cylinder head \( A_{ch} \) considered for heat transfer from cylinder walls to the gases in the cylinder is given by Eq. (40) where \( k_{ch} \) is the ratio of the area of the cylinder head to the cylinder cross section area normal to the cylinder axis and \( B \) is the cylinder bore,

\[
A_{ch} = k_{ch} \frac{\pi}{4} B^2
\]  

(40)

The area of the lateral wall of the cylinder \( A_{cyl} \) considered for heat transfer from the cylinder walls to the gases in the cylinder is given by Eq. (41) where \( l \) is the connecting rod length, \( a \) is the crank radius and \( s \) is the distance between the piston pin axis and the crankshaft axis or revolution as given by Eq. (16) and \( B \) is the cylinder bore.

\[
A_{cyl} = \pi B (l + a - s)
\]  

(41)

RESULTS

Figure 1(a) presents the evolution of the predicted total heat flux from the combustion chamber walls to the gas inside the cylinder with the crank angle during an engine cycle for a Peugeot TU3JP-KFW engine at 5500 rpm and wide-open throttle (WOT). Figure 1(b) presents the evolution of the predicted heat fluxes from the cylinder head, piston crown and exhaust valve to the gas inside the cylinder with the crank angle during an engine cycle for a Peugeot TU3JP-KFW engine at 5500 rpm and WOT.

In this method the convective heat transfer coefficient is assumed equal for all combustion chamber wall surfaces at a given time instant but it changes with the crank angle. The surface temperatures considered for the different combustion chamber wall surfaces are assumed constant during one cycle. Table 2 presents the surface temperatures considered for the different combustion chamber wall surfaces.

<table>
<thead>
<tr>
<th>Wall</th>
<th>Cylinder head</th>
<th>Piston crown</th>
<th>Intake valve</th>
<th>Exhaust valve</th>
<th>Cylinder liner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature /K</td>
<td>413.15</td>
<td>523.15</td>
<td>523.15</td>
<td>923.15</td>
<td>383.15</td>
</tr>
</tbody>
</table>
This study shows that the total heat flux from the combustion chamber walls to the gases in the cylinder changes strongly with the crank angle and can be either positive or negative depending on the crank angle. This study shows that there are differences between the heat fluxes from the distinct zones of the combustion chamber walls considered in this model to the gases in the cylinder at each crank angle. The only cause for the heat flux differences at each crank angle between the different combustion chamber wall surfaces is the different surface temperature of each of the different zones of the combustion chamber walls considered.

Fig. 1 - (a) Evolution of the predicted total heat flux from the cylinder walls to the gas inside the cylinder with the crank angle, during an engine cycle, for a Peugeot TU3JP-KFW engine at 5500 rpm and WOT. (b) Evolution of the predicted heat fluxes from the cylinder head, piston crown and exhaust valve to the gas inside the cylinder with the crank angle, during an engine cycle, for a Peugeot TU3JP-KFW engine at 5500 rpm and WOT.

Fig. 2 - (a) Evolution of the predicted convective heat transfer coefficient from the cylinder walls to the gas inside the cylinder with the crank angle, during an engine cycle, for a Peugeot TU3JP-KFW engine at 5500 rpm and WOT. (b) Evolution of the predicted thermal conductivity of the unburned gases, burned gases and mixture of unburned and burned gases in the cylinder, for a Peugeot TU3JP-KFW engine at 5500 rpm and WOT.
CONCLUSIONS

This study shows that the total heat flux from the combustion chamber walls to the gases in the cylinder changes strongly with the crank angle and can be either positive or negative depending on the crank angle. This study shows that there are differences between the heat fluxes from the distinct zones of the combustion chamber walls considered in this model to the gases in the cylinder at each crank angle. The only cause for the heat flux differences at each crank angle between the different combustion chamber wall surfaces is the different surface temperature of each of the different zones of the combustion chamber walls considered.

ACKNOWLEDGMENTS

The author gratefully acknowledges the assignment of the Peugeot TU3JP-KFW engine by Automóveis do Mondego, Lda, to carry out this study.

REFERENCES

