Proceedings IRF2018: 6th International Conference Integrity-Reliability-Failure
Lisbon/Portugal 22-26 July 2018. Editors J.F. Silva Gomes and S.A. Meguid
Publ. INEGI/FEUP (2018); ISBN: 978-989-20-8313-1

PAPER REF: 7120

NUMERICAL SOLUTION FOR A TRANSIENT PROBLEM OF A SANDSTONE LAYER ON A SOIL FOUNDATION UNDER VERTICAL LOAD USING BEM

Svetlana Litvinchuk, Akeksandr Ipatov[(*)], Aleksandr Boev
Research Institute for Mechanics, Lobachevsky State University of Nizhni Novgorod, Russia
[(*)]*Email*: ipatov@mech.unn.ru

ABSTRACT

This work presents a numerical analysis of wave propagation in a poroviscoelastic media. A problem of a sandstone layer on a soil foundation under vertical load is considered. Sandstone is assumed to be poroelastic, soil is assumed to be poroviscoelastic. The poroviscoelastic formulation is treated as a combination of Biot's theory of poroelasticity and elastic-viscoelastic correspondence principle. Kelvin-Voigt model is employed to introduce viscoelastic effects of porous material skeleton. The three dimensional boundary-value problem in poroviscoelastic formulation is numerically solved using boundary element method (BEM) and boundary integral equation (BIE) method. Results are obtained in Laplace domain. Modified Durbin's algorithm of numerical inversion of Laplace transform is used to perform solutions in time domain.

Keywords: poroviscoelasticity, poroelasticity, viscoelasticity, boundary element method (BEM), boundary integral equation (BIE)

INTRODUCTION

Wave propagation in composite porous materials has applications in many branches of science and technology, such as seismic methods in the presence of sandstones and soils. The implementation of the solid viscoelastic effects in the theory of poroelasticity was first introduced by Biot (Biot, 1956). The viscoelastic deformations in solid skeleton of poroelastic media may be accounted through the use of correspondence principle. Because the complexity of the inertial viscosity and mechanical coupling in porous media most transient response problems can only be solved via numerical methods. Classical formulations for BIE method with their discretized realization and traditional BEM are successful approaches for solving three-dimensional problem (Ugodchikov, 1986). A study of wave propagation in a poroviscoelastic halfspace was previously presented by Igumnov L.A. (Igumnov, 2017).

GOVERNING EQUATION AND NUMERICAL METHOD

The set of differential equations of poroelasticity for displacements $\bar{u}_i$ and pore pressure $\bar{p}$ in Laplace domain (s - transform parameter) take the following form (Schanz, 2001):

$$G\bar{u}_{i,jj} + \left(K + \frac{1}{3}G\right)\bar{u}_{j,ij} - (\alpha - \beta)\bar{p}_{,i} - s^2(\rho - \beta\rho_f)\bar{u}_i = -\bar{F}_i\,, \quad \frac{\beta}{s\rho_f}\bar{p}_{,ii} - \frac{\phi^2 s}{R}\bar{p} - (\alpha - \beta)s\hat{u}_{i,i} = -\bar{a}$$

$$\beta = \frac{k\rho_f\phi^2 s^2}{\phi^2 s + s^2 k(\rho_a + \phi\rho_f)}\,, R = \frac{\phi^2 K_f {K_s}^2}{K_f(K_s - K) + \phi K_s(K_s - K_f)}$$

where G, K - material constant from elasticity, ϕ - porosity, k - permeability, α - Biot's effective stress coefficient, ρ, ρ_a, ρ_f - bulk, apparent mass and fluid densities respectively, $\overline{F}_i, \overline{a}$ - bulk body forces per unit volume.

Poroviscoelastic solution is obtained from poroelastic solution by means of the elastic-viscoelastic correspondence principle. Kelvin-Voigt model is applied to skeleton moduli:

$$\overline{K}(s) = K\left[1+\frac{s}{\gamma}\right],\ \overline{G}(s) = G\left[1+\frac{s}{\gamma}\right].$$

Integral representation and BIE with integral Laplace transform are used. Fundamental and singular solutions are considered in term of singularity isolation. The numerical scheme is based on the Green-Betty-Somilliana formula. Regularized BIE are considered in order to introduce BE discretization (Ugodchikov, 1986). The problem is solved in Laplace domain. Modified Durbin's method (Durbin, 1974) is used to obtain solution in time domain.

$$f(0) \approx \sum_{l=1}^{n}\left[\frac{(F_{l+1}+F_l)\Delta_l}{2\pi}\right],\ \Delta_l = \omega_{l+1}-\omega_l$$

$$f(t) \approx \frac{e^{\alpha t}}{\pi t^2}\sum_{l=1}^{n}\left[\frac{F_{l+1}-F_l}{\Delta_l}\left(\cos(\omega_{l+1}t)-\cos(\omega_l t)\right)+\frac{G_{l+1}-G_l}{\Delta_l}\left(\sin(\omega_{l+1}t)-\sin(\omega_l t)\right)\right]$$

$$F_l = \mathrm{Re}\left[\bar{f}(\lambda+i\omega_l)\right],\ F_{l+1} = \mathrm{Re}\left[\bar{f}(\lambda+i\omega_{l+1})\right],\ G_l = \mathrm{Im}\left[\bar{f}(\lambda+i\omega_l)\right],\ G_{l+1} = \mathrm{Im}\left[\bar{f}(\lambda+i\omega_{l+1})\right]$$

Numerical example of a sandstone layer on a soil foundation under vertical load is considered. Viscoelastic model parameter influence on dynamic responses of boundary functions is studied. The comparison of transient responses for displacements and pore pressure are presented.

ACKNOWLEDGMENTS

The work was financially supported by Russian Scientific Foundation (grant 16-19-10237).

REFERENCES

[1] Biot M. A.: Theory of propagation of elastic waves in a fluid-suturated porous solid. J. Acoust. Soc. Am., 1956, 28(2), pp. 168-191.

[2] Durbin F.: Numerical inversion of Laplace transforms: an efficient improvement to Dubner and Abate's method. The Computer Journal. 1974, 17, pp. 371-376.

[3] Igumnov L.A., Ipatov A.A. The dynamic analysis of a slab on a poroviscoelastic halfspace under vertical load via BEM. Problems of Strength and Plasticity, 2017, 71(3), pp. 357-365.

[4] Schanz M. Wave Propagation in Viscoelastic and Poroelastic Continua, Springer, Berlin, Germany, 2001.

[5] Ugodchikov A.G., Hutoryanskii N.M.: Boundary element method in deformable solid mechanics, Kazan State University, Kazan, Russia, 1986.